TORSIONAL IRREGULARITY OF BUILDINGS

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ABSTRACT :

In the present work; the recently introduced parameter Q which is a ratio of the effective modal masses is modified and used to define the torsional irregularity of buildings. The code (ASCE 7-05) proposed ratio for the definition of the torsional irregularities is compared with the modified Q ratio. Parametric analyses are utilised for this investigation. 300 one storey and 300 five storey buildings having eccentricities in two orthogonal directions at the same time are analyzed by "time history method" under 8 historical earthquakes, 4800 analyses are carried out in total. To eliminate human errors from the calculations, a computer program capable of making above-mentioned analyses in one-run is coded. The code proposed parameter of the torsional irregularity is the ratio of the maximum drift of a floor corner to the average drift of the considered edge of the floor. Since this ratio is calculated under the static loading, it does not consider the eccentricity in the direction parallel to the earthquake excitation direction. It is found that, the use of this code parameter resulted in maximum of 101% difference in the drifts of the corner columns with regard to the ratio of the eccentricity in the direction of the excitation to the radius of gyration. For that case, the use of modified Q ratio resulted in maximum difference of 59%. Moreover, the calculation of the modified Q ratio depends on but does not require the determination of the eccentricities instead; it is calculated by means of the effective modal mass values of the building. The torsional coupling of 3D buildings in the inelastic range is currently being investigated by authors.

KEYWORDS: Torsional coupling; Torsional irregularity; Effective modal mass.

1. INTRODUCTION

Most of the studies regarding the torsional coupling have focused mainly on the uncoupled system parameters (Chandler et al., 1994; De La Llera and Chopra, 1994; De-La-Colina, 1999; Duan and Chandler, 1997; Humar and Kumar, 1998; Kan and Chopra 1981; Marusic and Fajfar, 2005; Paulay, 1998; Stathopoulos and Anagnostopoulos, 2005; Tso and Dempsey, 1980). The main parameter affecting the design of torsionally coupled asymmetric multi-storey buildings has been the plan eccentricity whose evaluation may not be preferable (Goel and Chopra 1993; Harasimowicz and Goel, 1998). Regarding the torsional irregularities, most of the codes have similar definitions (ASCE 7-05, 2005; Ozmen et al. 1998; Tezcan and Alhan, 2001).

In the present study, torsional irregularity of buildings is investigated. The recently introduced parameter Q (Ozhendekci, 2003) was used to investigate the error of the CQC (Complete Quadratic Combination) rule for torsionally coupled multi-storey buildings. Here the ratio Q is modified and used to represent the torsional irregularity of buildings, which is a system parameter of the coupled building and does not require the evaluation of the eccentricities. The definition of the Q ratio is repeated in order to preserve the completeness of the study. Afterwards the mentioned modification is given here.

Parametric analyses are used for the torsional irregularity investigation. 300 nos of one storey and 300 nos of five storey stiffness symmetric buildings in their plans are analyzed by time history method under 8 historical earthquakes, 4800 analyses are done in total. To eliminate human errors from the calculations, a computer program capable of making above-mentioned analyses in one-run is coded.

The code proposed parameter of the torsional irregularity is the ratio of the maximum displacement drift of a floor corner to the average displacement drift of the considered edge of the floor. Since this ratio is calculated under the static loading, it does not consider the eccentricity in the direction parallel to the earthquake excitation direction (e.g., the torsional moment caused by the statically loaded lateral forces is calculated by using the eccentricity only in the direction perpendicular to the excitation direction). The buildings used in the parametric analyses have eccentricities in the two orthogonal directions simultaneously. The effect of the eccentricity in the direction parallel to the excitation direction is also given here.

The modified Q ratio and the code (ASCE 7-05) proposed ratio is compared with regard to torsional irregularity. The ratio of the maximum displacement of the top floor corner of the torsionally coupled building to the maximum displacement of the uncoupled building is chosen as an indicator of the torsional irregularity. Considering the obtained results of the parametric analyses, it has been showed that the dispersion with regard to the ratio between the eccentricity, which is parallel to the excitation direction, and the radius of gyration is reduced if the modified Q ratio is used instead of the code proposed parameter in representing the torsional irregularity.

Using the envelopes of the parametric analyses results, relationships between the modified Q ratio and the response increase because of torsional coupling are given. These relationships can be used to describe torsional irregularity of buildings with smaller variation than that of the code proposed parameter.

2. DEFINITION OF *Q* RATIO

The dynamic degrees of freedom of the stiffness-symmetric one-storey building are selected as "x" and "θ-axes" (Figure 1). Here it denotes that: CM=center of mass and CR=center of rigidity. The base shear response of this building is

$$v_b(t) = \mathrm{M}_1^x\left(\ddot{z}_1(t) + p(t)\right) + \mathrm{M}_2^x\left(\ddot{z}_2(t) + p(t)\right) \tag{2.1}$$

where $p(t)$ = earthquake acceleration in the x- direction, $\ddot{z}_i(t)$ = i^{th} modal acceleration response and M_i^x =effective modal mass of the i^{th} mode for x-direction excitations and can be expressed as

$$M_i^x = \frac{\left(\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{m}\,\mathbf{B}^x\right)^2}{\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{m}\,\boldsymbol{\phi}_i} \tag{2.2}$$

where $\mathbf{m}$ is the mass matrix and $\boldsymbol{\phi}_i$ is the mode shape of the i^{th} mode. The vector considering the influence direction of the earthquake is $\mathbf{B}^x = \{1 \;\; 0\}^{\mathrm{T}}$, where the superscript "x" denotes the direction of the earthquake.

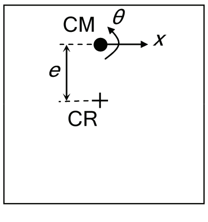

Figure 1. Plan of the one-storey model used for the analytical study and the dynamic degrees of freedom as x and θ-axes.

In Eqn. 2.1, the effect of the eccentricity on the behavior of the building is taken into consideration by the effective modal masses. The effects of these two effective modal masses on the torsional behavior of the building are investigated. Considering x-direction excitations for the calculation of the effective modal masses, the maximum of them is

$$M_l^x = \max(M_1^x, M_2^x) \tag{2.3}$$

where the superscript x denotes the direction of the earthquake and the subscript l is the mode number from which the maximum effective modal mass M_l^x is obtained.

Considering θ -direction excitations, the effective modal mass moments of inertia M_1^θ and M_2^θ are calculated by using $\mathbf{B}^\theta = \{0\ \ 1\}^\mathrm{T}$ in Eqn. 2.2, instead of $\mathbf{B}^x$. The maximum of them is

$$M_j^\theta = \max(M_1^\theta, M_2^\theta) \tag{2.4}$$

where the superscript θ describes the direction of the earthquake and the subscript j is the mode number from which the maximum effective modal mass moment of inertia M_j^θ is obtained.

However, also the j^{th} mode has the effective modal mass M_j^x for the x-direction excitations. This means that base-shear forces or the displacements in the x-direction develop also in the j^{th} mode for x-direction excitations. On the other hand, these displacements develop mainly by torsional motion rather than by translational motion in the j^{th} mode while developing mainly by translational motion in the l^{th} mode. In other words; for small values of M_l^x / M_j^x ratio, the maximum displacement response of corner columns, which is also dependent on the characteristics of the earthquake and the location of the corner column, develops mainly by torsional motion rather than by translational motion and vice versa. In this manner, for high values of this ratio, the behavior of the building becomes torsionally uncoupled (e.g., plane frame behavior).

In the dynamic analysis of three-dimensional structures, it is not appropriate to assign a direction to a mode shape that may includes components of all the directions. However, considering Eqns. 2.3, 2.4 and the above defined behavior of the building, lth and jth modes are labeled as (x) and (θ) modes, respectively. Here, parentheses emphasize the labeling of the modes instead of assigning directions to them. Using this notation, above given ratio can be written as

$$Q = \frac{M_{(x)}^x}{M_{(\theta)}^x} \tag{2.5}$$

and is a measure of the torsional behavior of the building as stated above and given by Ozhendekci (2003). It is underlined that the effective modal masses $M_{(x)}^x$ and $M_{(\theta)}^x$ for x-direction excitations are the ones included in Eqn. 2.1.

Eqns. 2.3 and 2.4 can be defined for multi-storey buildings having eccentricities in the two orthogonal directions. Using the same notation, these equations become

$$M_l^x = \max(M_1^x, M_2^x, \ldots M_n^x) \tag{2.6}$$

and

$$M_j^{\theta} = \max(M_1^{\theta}, M_2^{\theta}, \ldots M_n^{\theta}) \qquad (2.7)$$

where n is the number of the degree of freedom of a multi-storey building.

Eqn. 2.5 can be also used for multi-storey buildings provided that Eqns. 2.6 and 2.7 are used instead of Eqns. 2.3 and 2.4, respectively. Hence, for a multi-storey building, as Q goes to infinity, the building behaves as torsionally uncoupled and hence, represents a planar-frame behavior. Since Q is a ratio and denotes the portion of the torsional behavior in the whole behavior of the building; damping is not considered in this definition.

3. MODIFICATION OF THE Q RATIO

The curves given in this section are calculated by algebraic equations derived analytically using the one storey building with the plan given in Figure 2(a). These equations are rather long expressions and do not need to be given here.

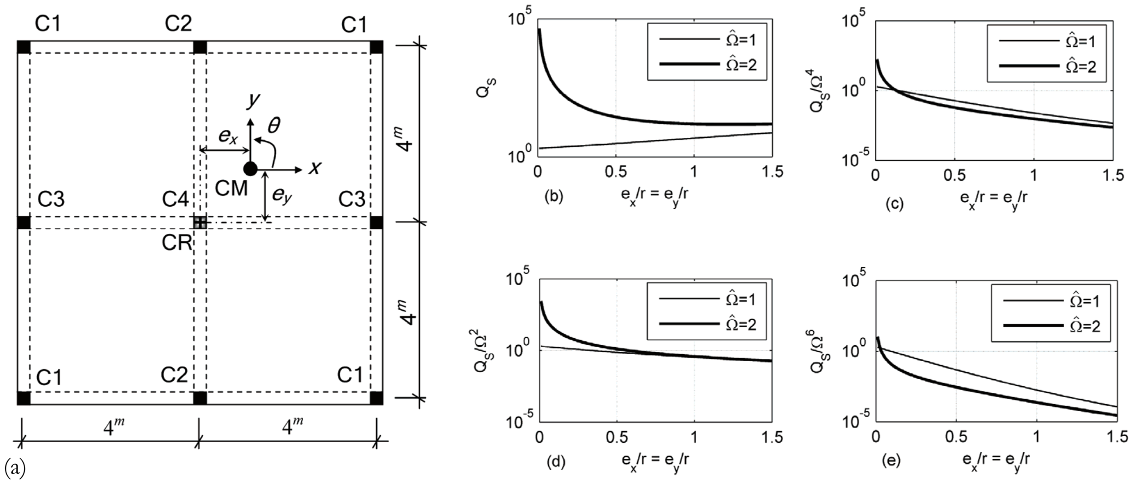

Figure 2. Modification of the Q ratio; (a) used plan view, (b) unmodified, (c) modified with Ω^4, (d) modified with Ω^2, (e) modified with Ω^6

It is known that if the ratio uncoupled torsional natural frequency to uncoupled x-direction natural frequency, $\hat{\Omega}$ is higher than 2, the displacement increase because of torsional coupling is proportional to e/r ratio (Kan and Chopra, 1981). In other words, for sufficiently small eccentricities, the buildings with the ratio $\hat{\Omega} \geq 2$ show planar frame behavior. Considering Figure 2(b), Q ratio increases as the $e_x/r = e_y/r$ ratio decreases for $\hat{\Omega} = 2$. Here r is radius of gyration. Hence, the high values of the Q ratio correspond to the small eccentricity buildings with planar frame behavior for ratio $\hat{\Omega} = 2$ (Figure 2(b)).

It is known that a building with small eccentricity leads to a torsional response comparable to but smaller than that of a building with large eccentricity for $\hat{\Omega} = 1$ (Tso and Dempsey 1980; Kan and Chopra, 1981). In other words, for the ratio $\hat{\Omega} = 1$, the buildings with small eccentricity are torsionally sensitive and this may not mean much dynamic response amplification than that of the buildings with large eccentricity. Considering Figure 2(b), although Q ratio decreases as the e_x/r, e_y/r ratios decrease for $\hat{\Omega} = 1$, which is the inverse of that for $\hat{\Omega} = 2$, it does not change dramatically with regard to e/r for $\hat{\Omega} = 1$. Hence, for both of the cases $\hat{\Omega} = 1$ and $\hat{\Omega} = 2$, it can be said that, small values of the ratio Q correspond to torsionally sensitive buildings and large values of it correspond to torsionally uncoupled (unsensitive) buildings. From now on, the term Q_S is used instead of the term Q.

In this study, rather than torsional sensitivity of buildings, the displacement increase because of torsional irregularity of buildings is investigated. Hence, the ratio Q_S is modified to this aim. The frequency ratio is defined using the frequencies of the coupled system as

$$\Omega = \frac{\omega_{(\theta)}}{\omega_{(x)}} \tag{3.1}$$

The relationship between the coupled frequency ratio as $1/\Omega^4$ and the uncoupled frequency ratio $\hat{\Omega}$ with regard to e_x / r is given in Figure 3. In this figure, the ratio $1/\Omega^4$ is relatively independent from the ratio e_x / r for buildings with $\hat{\Omega} = 2$ while it is inversely proportional with e_x / r for $\hat{\Omega} = 1$. The modification can be made empirically by multiplying the ratio Q_S by $1/\Omega^4$.

$$Q_R = \frac{Q_S}{\Omega^4} \tag{3.2}$$

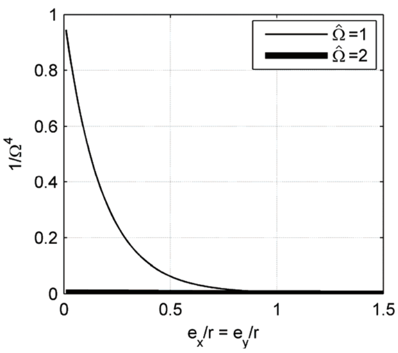

Figure 3. Relationships between the coupled frequency ratio as $1/\Omega^4$ and the uncoupled frequency ratio $\hat{\Omega}$ with regard to e_x / r.

With this modification, the signs of the curves of the ratio Q_R for $\hat{\Omega} = 1$ and for $\hat{\Omega} = 2$ become same (Figure 2(c)). In Eqn. 3.2., using the ratio Ω^4 gives the best matching (Figure 2(c)). For example, the results were not good for the ratios Ω^2 and Ω^6 which can be seen in Figures 2(d), 2(e), respectively. Thus, the Q_R ratio depends on but does not require the evaluation of the eccentricities.

It can be said that, for both of the cases $\hat{\Omega} = 1$ and $\hat{\Omega} = 2$, small values of the ratio Q_R correspond to torsionally irregular buildings and large values of it correspond to torsionally regular buildings. These results are obtained by algebraic equations and are agreed with the results of the parametric analyses. It is noted that parametric analyses include different values of the eccentricities e_x and e_y at the same time and the Q_R ratio as being a dynamical system parameter considers both of them.

Parametric analyses also include the buildings whose lateral stiffnesses are not equal in the two orthogonal directions. Regarding this parameter, for a building of which lateral stiffness in the *y*-direction is two times its lateral stiffness in the x-direction ($\alpha = 2$), the use of $\omega_{(y)}$ instead of $\omega_{(x)}$ in Eqn. 3.1 gives better results. When a building have equal stiffnesses in the two orthogonal directions, using the average of the two frequencies and using $\omega_{(x)}$ yield similar results, however $\omega_{(x)}$ is used for simplicity.

4. RESULTS OF THE PARAMETRIC STUDY

Considering the appropriate parameter values with regard to practical buildings, thus, for 0.5, 1 and 2 values of the ratio α, $\hat{\Omega}$ was chosen from the groups of {0.75, 1.00, 1.25, 1.5}, {1.00, 1.25, 1.5, 1.75}, and {1.5, 1.75, 2.00, 3.00}, respectively. For all of the buildings, the assigned values of the e_x/r and e_y/r ratios were chosen from the values of 0.05, 0.1, 0.2, 0.3 and 0.5.

The displacement increase ratio δ_x/δ_{x_0} is chosen as an indicator of the torsional irregularity. Here δ_x is the maximum displacement of the top floor corner of the coupled building and δ_{x_0} is the maximum displacement of the uncoupled building in the *x*-direction. In this study, the emphasis is given to the maximum values of this ratio. Hence, the figures given here are for the earthquake for which the maximum values of this ratio are obtained.

In the code (ASCE 7-05) the torsional irregularity of buildings is defined by the ratio $\Delta_{\text{max}}/\Delta_{\text{average}}$. Here Δ_{max} is the maximum drift of a floor corner and Δ_{average} is the average drift of the considered edge of the floor in the excitation direction. The building examples used in the study are represented by their $\Delta_{\text{max}}/\Delta_{\text{average}}$ ratios and modified Q_R ratios in the horizontal axes of the figures. It is not possible to give all the results of the parametric analyses here. However, some of them having high variations with regard to e_x/r, which also correspond to practical buildings at the same time, are shown below.

4.1. One Storey Buildings

Considering Figure 4(a), the points representing different buildings are on the same vertical line. This is because the ratio $\Delta_{\text{max}}/\Delta_{\text{average}}$ is a parameter (or a solution) of the statical properties of a building. In other words, this ratio does not depend on the eccentricity e_x (i.e., the moment caused by a lateral force statically loaded at the mass center is equal e_y times the value of this force). The buildings with the same ratio of $\Delta_{\text{max}}/\Delta_{\text{average}} = 1.26$ can have different e_x/r ratios of 0.05 to 0.5 (Figure 4(a)). For example, while the building with the ratio $e_x/r = 0.5$ has no displacement increase ($\delta_x/\delta_{x_0} = 0.98$) due to torsional coupling, the building with the ratio $e_x/r = 0.05$ has displacement increase of 101% ($\delta_x/\delta_{x_0} = 2.01$) (Figure 4(a)). It is noted that both buildings are classified as having “1a Torsional Irregularity” according to ASCE 7-05 because of their same ratio of $\Delta_{\text{max}}/\Delta_{\text{average}} = 1.26$. If modified *Q* ratio is used to represent these buildings, maximum difference with regard to e_x/r ratio is 59% instead of 101% (Figure 4(b)). Hence there is 101-59=42 reduce in the dispersion if ratio Q_R is used. This is because the ratio Q_R is a dynamical parameter and considers the dynamical properties of the buildings.

For the ratio $\hat{\Omega} = 1.25$, corresponding differences of the use of the ratios Q_R and $\Delta_{\text{max}}/\Delta_{\text{average}}$ are 45% and 74%, respectively (Figure 4(c),(d)). Considering Figure 5(a),(b) for the ratio $\hat{\Omega} = 1.50$, both ratios have identical results.

For the values of this ratio bigger than 1.75 use of $\Delta_{\text{max}}/\Delta_{\text{average}}$ ratio may give better results. For the two extreme cases of the analyses, with the ratios $\alpha = 0.5$, $\hat{\Omega} = 0.75$ and $\alpha = 2$, $\hat{\Omega} = 3$, the use of the ratio $\Delta_{\text{max}}/\Delta_{\text{average}}$ may give better results. Further modification including those cases is not made, instead the use of the $\Delta_{\max}/\Delta_{\text{average}}$ ratio is proposed.

4.2. Five Storey Buildings

Not all the results of five storey buildings are given here since the results of one storey and of five storey buildings differ slightly (Figure 4(b) and 5(d)). The ratios δ_x/δ_{x_0} of five storey buildings are smaller than that of one-storey buildings.

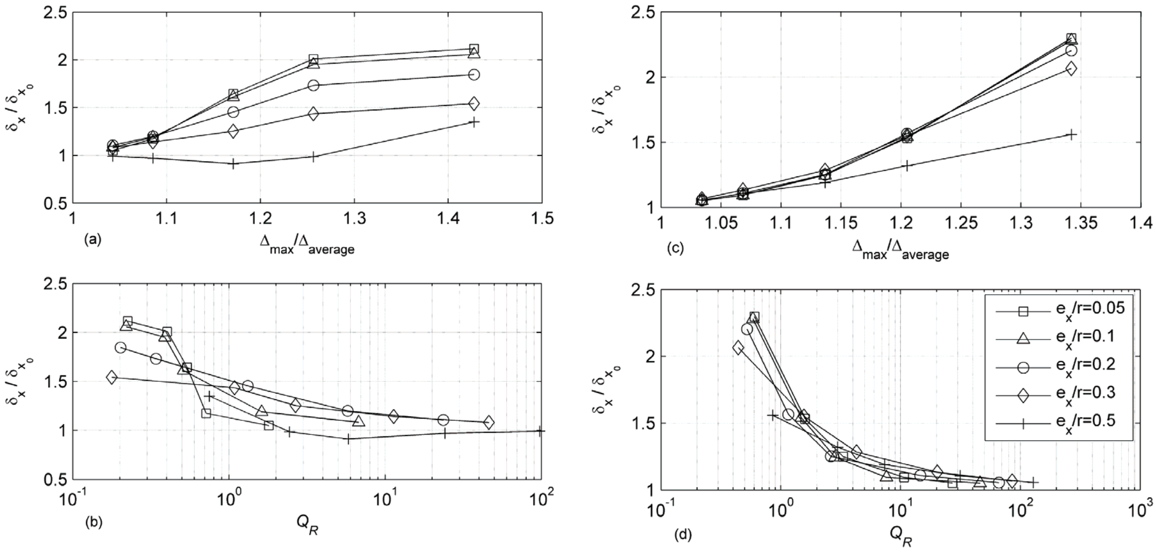

Figure 4. δ_x/δ_{x_0} ratio for the one-storey buildings with ($\alpha=1$, $\hat{\Omega}=1$): (a) $\Delta_{\max}/\Delta_{\text{average}}$ (b) Q_R ($\alpha=1, \hat{\Omega}=1.25$): (c) $\Delta_{\max}/\Delta_{\text{average}}$ (b) Q_R

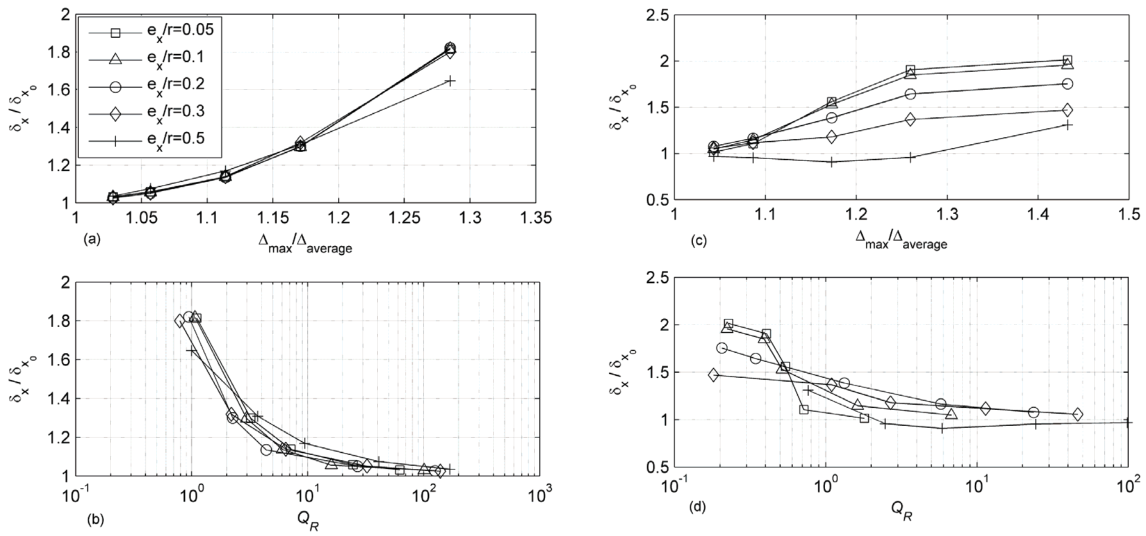

Figure 5. δ_x/δ_{x_0} ratio for the one-storey buildings with ($\alpha=1, \hat{\Omega}=1.5$): (a) $\Delta_{\max}/\Delta_{\text{average}}$ (b) Q_R five-storey buildings with ($\alpha=1, \hat{\Omega}=1$): (c) $\Delta_{\max}/\Delta_{\text{average}}$ (b) Q_R

5. CONCLUSIONS

In most of the codes, the torsional irregularity of buildings is defined by the ratio of the maximum drift of a floor corner to the average drift of the considered edge of the floor in the excitation direction. Since this ratio is calculated under the static loading, it does not consider the eccentricity in the direction parallel to the excitation direction. In other words, it does not consider the dynamical properties of a building. It is found that, especially for buildings with the ratios $\alpha = 1$ and $\hat{\Omega} = 1$, the obtained values of the dynamic amplification because of the torsional irregularity differ greatly with regard to the eccentricity in the excitation direction as 101%. If proposed ratio Q_R, which does not require the evaluation of the eccentricities, is used to represent the torsional irregularity of the buildings, this dispersion reduces to 59%.

Omitting the high values of this dispersion may not be economic. Apart from that, if a building does not have equal eccentricities e_x and e_y in the *x* and *y*-axes, respectively, the accuracies of the design process of the *x* and *y*-directions may not be equal. Accordingly, the design of this building would not be uniform regarding the two orthogonal directions.

The buildings used in the analyses have eccentricities in the two orthogonal directions at the same time. Provided figures also give the dependence of the code proposed parameter on the eccentricity in the excitation direction.

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