

DISPLACEMENT-BASED SEISMIC DESIGN OF REGULAR REINFORCED CONCRETE SHEAR WALL BUILDINGS

JagMohan Humar¹

¹*Distinguished Research Professor, Dept. of Civil and Environmental Engineering
Carleton University, Ottawa, Canada, K1S 5B6
Email: jhumar@ccs.carleton.ca*

ABSTRACT

A displacement based method for the seismic design of reinforced concrete shear wall buildings of regular shape is presented. For preliminary design, approximate estimates of the yield and ultimate displacements are obtained, the former from simple empirical relations, and the latter to keep the ductility demand within ductility capacity and to limit the maximum storey drift to that specified by the codes. For a multi storey building the structure is converted to an equivalent single-degree-of-freedom system using an assumed deformation shape that is representative of the first mode. The required base shear strength of the system is determined from the inelastic demand spectrum corresponding to the ductility demand. In subsequent iterations a pushover analysis for the force distribution based on the first mode is used to obtain better estimates of yield and ultimate displacements taking into account stability under P- Δ effect. A multi-mode pushover analysis is carried out to find more accurate estimates of the shear demand.

KEYWORDS: displacement-based design, seismic design of shear walls, modal pushover analysis, capacity demand diagram, higher mode effect

1. INTRODUCTION

Experience during recent earthquakes has led to the recognition that in addition to the objective of life safety during a rare earthquake seismic design must meet performance objectives related to limits on downtime and economic loss caused by more frequent earthquakes. The process of seismic design that aims to meet one or more performance objectives (POs) is commonly referred to as performance based seismic design (PBSD). A simplified version of the methodology for PBSD is proposed in the SEAOC Vision 2000 report (SEAOC 1995).

The Vision 2000 report specifies a set of discrete performance levels, ranging from fully operational to near collapse, which the structure may be required to meet under specified levels of earthquake hazard. The earthquake hazard is determined from a probabilistic seismic hazard analysis (PSHA) and expressed in terms of the annual frequency of exceedance or the return period. A PO is a combination of the seismic hazard and the expected performance level under such hazard. Quantitative performance levels are defined through limiting values of measurable response parameters, such as storey drifts, floor velocities and accelerations, element deformation and ductility demands, and damage indices. For the present study we will focus on structural and nonstructural damage. Because element deformations and ductility demands can be related to storey displacements and drifts, it is evident that both the structural and nonstructural damage could be controlled by limiting the storey drifts and displacements.

A significant amount of research on DBSD has been carried out over the past 15 years. The essential concepts of DBSD were developed by Freeman and others (Freeman et al 1975) and later refined by Priestley and Calvi (1997), Fajfar (1999) and Chopra and Goel (2001). In the present study we apply a modified form of capacity-spectrum method, in which the demand is expressed by an inelastic spectrum and the capacity by the realistic force displacement relationship, to the design of new structures. The study adapts the previously reported concepts to define a practical displacement-based method for the design of new buildings. The study also highlights the fact, noted in some of the earlier research, that the ductility capacities identified in the seismic codes are rarely mobilized; the design is instead governed by the limit on ductility capacity, displacement limit required to ensure stability under P-Delta effects, or the drift limit usually specified in the codes to limit structural and

non-structural damage. The present study deals with the design of shear wall structures of reinforced concrete that are symmetric and regular in layout. The DBSD method presented here is combined with multi-mode pushover analyses to account for the higher mode effects.

2. DISPLACEMENT-BASED SEISMIC DESIGN

In developing the essential steps of DBSD we assume that the seismic hazard is represented by a uniform hazard spectrum (UHS) for the site under consideration. We will also assume that the primary PO is the achievement of the basic objective defined in Vision 2000 report. It calls for life-safety performance under a seismic hazard corresponding to an earthquake with 10% probability of exceedance in 50 years, or a return period of 475 years, but limited to a maximum value of two-thirds of the maximum considered earthquake (MCE) having a 2% probability of exceedance in 50 years, or a recurrence interval of 2475 years. The secondary PO may be to ensure that the structure remains operational under a more frequent earthquake, one with a recurrence interval of 72 years or a 50% chance of exceedance in 50 years.

The first step in the design process is to obtain estimates of the yield displacement and the acceptable ultimate displacement of a SDOF model of the structure. For a multi-storey structure this will require the selection of an assumed displacement shape and may require a pushover analysis which provides the relationship between the roof displacement and base shear. The ratio of the ultimate and yield displacements of the SDOF model provides the ductility requirement. The seismic demand curve is now obtained by determining the inelastic spectrum corresponding to the calculated ductility and plotting it in the A-D format. The required capacity or strength is next determined by entering the demand curve at the acceptable ultimate displacement and measuring horizontally to get the spectral acceleration. The product of the spectral acceleration and spectral mass of the equivalent SDOF system provides the strength or the base shear capacity of the structure. It may be noted that while the graphical method just outlined is useful in developing an understanding of the underlying principle, it is not essential to the design process and numerical computations can be as effectively employed to determine the required strength.

2.1 Estimates of yield and ultimate displacement

The yield displacement is defined as the roof displacement at yield. For a cantilever shear wall, assuming that the curvature varies linearly across the height, the yield displacement Δ_y is given by

$$\Delta_y = \phi_y H^2 / 3 \quad (2.1)$$

where ϕ_y is the effective yield curvature and H the height of the wall. For preliminary design an approximate value of ϕ_y can be obtained from empirical relationship, for example, that given by Priestley and Kowalsky (1998).

$$\phi_y \approx 2.0 \varepsilon_y / l_w \quad (2.2)$$

where ε_y is the yield strain of reinforcing steel and l_w is the length of the wall in cross section.

The ultimate displacement corresponding to the life safety condition is the roof displacement after a plastic hinge develops at the base of the wall. This displacement consists of two components, elastic displacement up to yield and plastic displacement following yield, and is given by

$$\Delta_u = \Delta_y + \theta_p (H - 0.5L_p) \quad (2.3)$$

where Δ_u is the ultimate displacement, L_p is the length of the plastic hinge, and θ_p is the plastic drift given by

$$\theta_p = (\phi_u - \phi_y) L_p \quad (2.4)$$

At life safety limit the acceptable ultimate displacement would be governed by one of the following limits.

2.1.1 Drift limit specified in guidelines and codes

Such a limit is prescribed to fulfill the selected PO. As an example, Vision 2000 report states that for structures with concrete shear walls the drift limit at life safety level is meant to guard against structural collapse,

non-structural damage that may jeopardize life safety, and instability due to P- Δ effect and may be taken as about 2.5% under an earthquake with 10% probability of exceedance in 50 years or a return period of 475 years.

For a cantilever shear wall the largest storey drift, θ_u occurs at the roof and is given by

$$\theta_u = \frac{\phi_y H}{2} + \theta_p \quad (2.5)$$

Substituting Equation (2.5) into Equation (2.3) with $\theta_u = 0.025$ we get the limiting value of ultimate displacement.

$$\Delta_u = \Delta_y + (H - 0.5L_p)(0.025 - 0.5\phi_y H) \quad (2.6)$$

in which Δ_y obtained from Equation (2.1).

2.1.2 Local ductility capacity limit

The ultimate displacement corresponding to the ductility capacity of an element depends on the inelastic behavior of the shear wall at its most critical cross section. This section is usually located at the base of the wall. The inelastic behavior is governed by the geometry of the section and the characteristics of the materials, specifically the limiting compressive strain in the concrete ε_u . The ultimate curvature is given by $\phi_u = \varepsilon_u/c_u$, where c_u is the depth of neutral axis when the concrete strain is ε_u . The ultimate displacement is obtained by inserting this value of ϕ_u in Equation (2.4) and then substituting the resulting θ_p in Equation (2.3). The limiting strain in concrete, ε_u is usually assumed as being 0.015 for confined concrete and 0.004 for unconfined concrete.

2.1.3. Limit to preclude instability caused by P- Δ effect.

The P- Δ effect, which is a function of the axial load and the height of the wall, decreases the stiffness of the structure and modifies its elastic and inelastic responses. If the structure is idealized by an elasto-plastic force displacement relationship this effect will cause the stiffness to become negative as soon as yield takes place. This negative stiffness may cause instability in the structure and, theoretically, the structure should not be stressed beyond yield to prevent any chance of P- Δ instability. In other words, the structure should be designed to remain elastic. Nevertheless, it has been observed in previous studies (Krawinkler and Seneviratna 1998) that the structure usually remains stable as long as the excursion into the zone of instability is not excessive. For example, a 10 to 15% reduction in the base shear strength caused by P- Δ effect may be quite acceptable. It was observed in the example presented here that the ratio between the ultimate base shear and yield base shear was about 0.90. This limited excursion into the negative slope region is not likely to lead to instability.

At the beginning of design the structure would normally be designed to meet the drift limits specified by the code as well as that based on ductility capacity. During subsequent iterations in design the excursion into the zone of instability can be evaluated by taking the ratio of the base shear at maximum displacement to that at yield, and the design suitably modified if this excursion is too large.

2.2 Equivalent SDOF system

In order to apply the DBSD method to a MDOF system the latter should first be represented by an equivalent SDOF system. This requires the selection of a displaced shape for the structure. Any logical shape, including an inverted triangular shape, which is similar to the first mode shape, could be selected. Assuming that the selected shape is represented by the vector ϕ , the following parameters are calculated:

$$\Gamma = \frac{(\phi^T \mathbf{M} \mathbf{1})}{(\phi^T \mathbf{M} \phi)}, \quad M^* = \frac{(\phi^T \mathbf{M} \mathbf{1})^2}{(\phi^T \mathbf{M} \phi)} \quad (2.7)$$

where \mathbf{M} is the mass matrix, $\mathbf{1}$ is the unit vector, Γ is the modal participation factor, and M^* is the effective modal mass. The yield and target displacements for the equivalent SDOF system are given by

$$\delta_y = \frac{\Delta_y}{\Gamma \phi^r}, \quad \delta_u = \frac{\Delta_u}{\Gamma \phi^r} \quad (2.8)$$

where ϕ^r is the value of ϕ at the roof.

2.3 Inelastic demand spectrum

The seismic demand is represented by a UHS. For the example presented later in this paper, we will use a life-safety level spectrum that is appropriate for certain regions in California and for a rock site, or Site Class B, as defined in IBC. The design spectral values are taken as being 2/3 those produced by the maximum considered earthquake (MCE). The design spectral acceleration is 1g for period ranging from 0 to 0.4 s. Beyond 0.4 s the spectral values are given by $S_a(T) = 0.4/T$. The UHS is converted to the acceleration-displacement (A-D) format so that it can be plotted on the same graph as the capacity diagram referred to earlier.

The UHS represents elastic response of a SDOF system. When the structure develops inelastic deformations the demand curve is expressed in the form of a constant ductility inelastic spectrum. The inelastic spectrum provides the value of spectral acceleration S_{ay} , such that when the structure having the corresponding yield strength is subjected to the design earthquake it develops the specified ductility μ . The ratio of S_a to S_{ay} is denoted by R_y . The construction of inelastic demand spectrum from the known elastic demand spectrum requires the definition of a relationship between R_y , μ , and T . An equation proposed by Krawinkler and Nassar (Chopra and Goel 2001) will be used in this study. This equation is based on the response of bilinear systems and is given by

$$R_y = [c(\mu - 1) + 1]^{1/c} \quad (2.9)$$

$$c = \frac{T^a}{1 + T^a} + \frac{b}{T} \quad (2.10)$$

and parameters a and b depend on post-yield stiffness. In the present work we assume that the force-displacement relationship is elasto-plastic, in which case $a = 1$ and $b = 0.42$. The displacement of the inelastic system is given

$$S_d = \frac{\mu}{R_y} \left(\frac{T}{2\pi} \right)^2 S_a \quad (2.11)$$

where S_a is the elastic spectral acceleration at the period T . The inelastic spectrum can thus be defined for a constant ductility factor with displacements obtained from Equation (2.11) and accelerations given by S_a/R_y .

2.4 Preliminary design

For preliminary design of the structure, the yield displacement is obtained from Equations (2.1) and (2.2). The ultimate displacement is obtained from Equation (2.6), if the code specified limit governs, or from Equations (2.3) and (2.4) with an appropriate value of φ_u if the ductility capacity governs. The system is now converted into an equivalent SDOF system assuming a reasonable displacement shape, for example, an inverted triangle, and using Equations (2.7) and (2.8). The ductility demand is given by the ratio Δ_u/Δ_y or, equivalently, δ_u/δ_y . The inelastic demand spectrum for the calculated ductility is now obtained. On entering this spectrum with δ_u we obtain the spectral acceleration S_{ay} . The design base shear for the structure is given by $V = S_{ay} M^*$.

The base shear is distributed across the height of the structure in proportion to the element of the vector $\mathbf{M}\boldsymbol{\phi}$ where $\boldsymbol{\phi}$ is the assumed displacement shape vector. An elastic analysis of the structure is now carried out for the storey level forces, determined as above, to obtain the design moments in the shear wall and the reinforcement is selected to provide the required moment capacity.

2.5 Subsequent iterations in design

In order to obtain a more refined design, one or more iterations need to be performed at this stage. The first step in this process is to carry out a moment-curvature analysis of the preliminary wall cross-section using strain compatibility and force equilibrium. Such an analysis provides the moment resisting capacity of the wall cross section, the effective moment of inertia, and the ultimate curvature. These parameters are required for the modal and pushover analyses described in the following paragraphs as well as for obtaining better estimates of the yield and ultimate displacements.

The pushover analysis is carried out for a predefined pattern of lateral forces applied on the wall. It is usual to assume that the lateral forces are given by product of mass matrix, \mathbf{M} , and the first mode shape $\boldsymbol{\phi}_1$. This

assumption implies that a modal analysis must first be carried out in order to obtain the mode shape. The lateral forces needed in the pushover analysis are then obtained from $S_1 = M\phi_1$

The pushover curve provides the relationship between the roof displacement and the base shear. It is idealized by a bi-linear curve and provides the yield displacement Δ_y . At the same time, the moment-curvature analysis of the wall section gives the yield curvature ϕ_y and the ultimate curvature ϕ_u . These parameters permit a more precise evaluation of the acceptable ultimate displacement. The limit corresponding to ductility capacity is obtained from Equations (2.3) and (2.4) with Δ_y equal to that obtained from the pushover analysis. The limit corresponding to code-specified value of storey drift is given by Equation (2.6), again with Δ_y obtained from the pushover analysis. The smaller of the two limits governs the design.

With the new values of Δ_y and Δ_u a revised estimate for the base shear is obtained using the procedure outlined in Section 2.4. If the difference between these two base shears is substantial, a new base moment must be calculated and the wall section redesigned. The process is repeated until convergence is achieved.

2.7 Multi-mode pushover analysis

On convergence the DBSD procedure based on the first mode provides a good estimate of the design moments and to some extent of the drifts. However, the shear forces are not accurate, and the contribution of higher modes must be considered for obtaining better estimates of such forces. The multi-mode pushover analysis (MPA) proposed by Chopra and Goel (2002) provides a simple and reasonably accurate method of considering the higher mode contribution. The MPA procedure uses pushover analyses based on the first few mode shapes, and combines the modal responses so obtained, assuming that they are uncoupled.

3. APPLICATION OF DISPLACEMENT-BASED SEISMIC DESIGN

As an illustration of the DBSD procedure outlined in the previous sections the design of a 12-story reinforced concrete shear wall building located in California is presented. The building is assumed to have a symmetric plan. Torsional vibrations are ignored and only two-dimensional analyses are carried out.

For preliminary design the yield curvature and the roof displacement at yield are calculated from Equations (2.2) and (2.1). The following values are obtained: for 6-meter wall $\phi_y = 5.67 \times 10^{-7}$ and $\Delta_y = 382.7$ mm; for 4-meter wall $\phi_y = 8.5 \times 10^{-7}$ and $\Delta_y = 573.8$ mm.

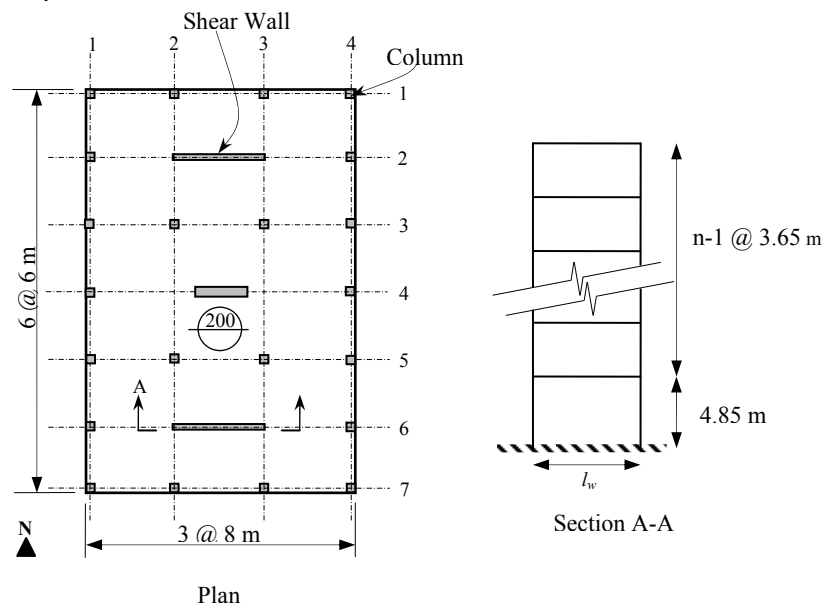


Figure 1: Plan and elevation of 12-storey building

The plan view of the building is shown in Figure 1. The figure also shows a typical elevation. The building has 3 bays, each 8 m wide, in the East-West (E-W) direction and 6 bays, each 6 m wide, in the North-South (N-S)

direction. The first-storey height in each building is 4.85 m; the height of each of the remaining storeys is 3.65 m. The lateral resistance in the East-West direction is provided by three shear walls located in the 2nd, 4th and 6th framing lines. The walls are 6m, 4m, and 6m long, respectively, and are assumed to be 400 mm thick for the first pass of design. The floors consist of reinforced concrete flat slabs, 200 mm in thickness, supported by 22 regularly spaced columns, each of size 500 x 500 mm, as shown in Figure 1. The dead loads consist of: partitions 0.5 kN/m²; electrical, mechanical, ceiling 0.5 kN/m²; and roof insulation and water proofing 0.5 kN/m². The live load is comprised of snow on roof at 2.2 kN/m², and floor load at 2.4 kN/m². The following material properties are assumed: steel yield strength $f_y = 400$ MPa; concrete strength $f_c' = 30$ MPa.

The building is designed for earthquake forces in E-W direction neglecting the accidental torsion effect. The contribution of RC columns to lateral resistance is ignored. The total dead load of the building works out to 72,772 kN, so that the inertia mass is 7,418.2 tonne. The design gravity loads at the base of the walls consisting of the tributary dead load and half of the live load reduced by the tributary area related reduction factor works out to 9,827.8 kN for the 6-meter wall and 8,965.6 kN for the 4-meter wall. The various steps in the design are presented in the following sections.

3.2 Displacement estimates

3.2.1 Yield displacement

The designer has considerable flexibility in choosing the relative strengths of the walls. Once the relative strengths are chosen the relative stiffness values are automatically determined. Here we assume that the strengths of walls are proportional to the square of the wall length. This assumption will yield approximately equal reinforcement ratios for the different walls. The strength of each 6-meter wall works out to $0.409V_b$, and the strength of 4-meter wall as $0.182V_b$, where V_b is the total design base shear. Considering that the three walls can be represented by springs in parallel, the yield displacement of the system may be calculated as follows:

$$\Delta_y = \frac{V_b}{\sum k_i} = \frac{V_b}{\sum V_{yi}/\Delta_{yi}} = \frac{V_b}{0.409V_b/382.7 + 0.182V_b/573.8 + 0.409V_b/382.7} = 407.4 \text{ mm}$$

3.3 Ultimate displacement

3.3.1 Limit prescribed by the codes

In the following we will assume that the storey drift must be limited to 0.025. The roof displacement is now given by Equation (2.6). Using the data for 6-meter wall, assuming that $L_p = l_w/2$, and substituting the values of other parameters into Equation (2.6) we get $\Delta_u = 915.3$ mm. The corresponding value for the 4-meter is $\Delta_u = 832.3$ mm.

3.3.2 Limit on ductility capacity

We will assume that the concrete is not confined so that the strain limit is 0.004. Assuming a neutral axis depth equal to $0.3l_w$, we get for the 6-meter wall $c = 1800$ mm, $\phi_u = 2.22 \times 10^{-6}$ and $\Delta_u = 598.7$.

3.4 Equivalent SDOF system

Because we assumed the yield displacement to be inversely proportional to the wall length and the yield strength to be proportional to the square of wall length the stiffness of the walls will be proportional to the cube of the wall length. It is therefore possible to determine the mode shapes, although the absolute values of the periods can not be determined. A modal analysis, including P- Δ effect, gives the following dynamic characteristics: $\Gamma = 1.485$, $M^* = 4846.5$ tonne. For the equivalent SDOF system, we get $\delta_y = 274.3$ mm, $\delta_u = 403.2$ mm, and $\mu = 1.47$

The demand diagram corresponding to $\delta = 1.47$ is plotted in Figure 2 in A-D format. The intersection of a vertical through $\delta_u = 403.2$ mm and the demand diagram gives the performance point. The capacity diagram is obtained by drawing a horizontal from the performance point up to the yield displacement, $\delta_y = 274.3$ mm and another line from the origin to the yield as shown in Figure 2. From this diagram the design base shear, without accounting for any overstrength, is given by $V = 0.0663 \times 9.81 \times 4846.5 = 3152.2$ kN

When this base shear is distributed according to shape $M\phi_1$ we get a design base moment of 120,110 kNm with P- Δ effect taken into account. The 6-meter wall should thus be designed for moment of $0.409 \times 120,110 = 49,125$ kNm, and the 4-meter wall should be designed for a moment of $0.182 \times 120,110 = 21,860$ kNm.

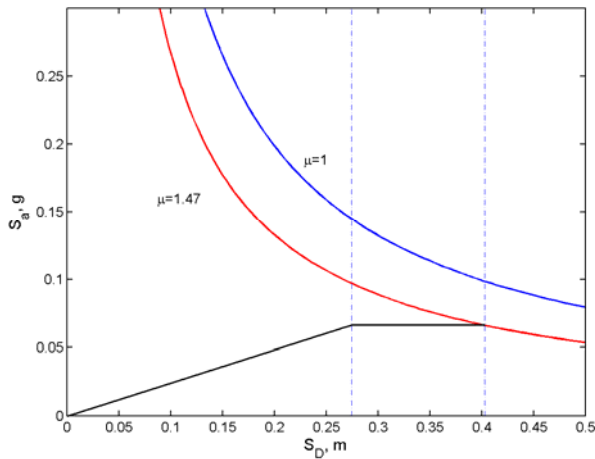


Figure 2: Demand and capacity diagram

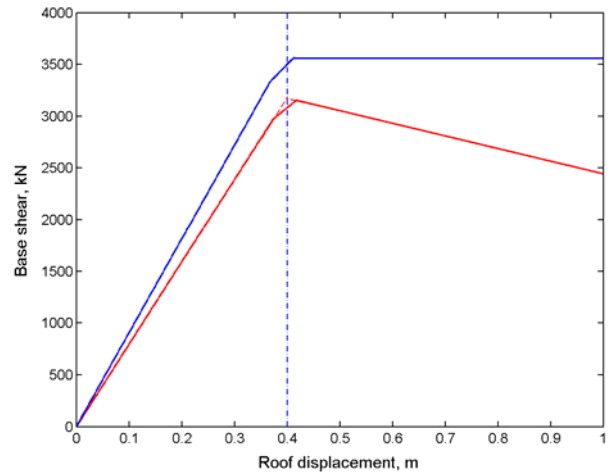


Figure 3: Push over diagrams

3.4 Further design iterations

New estimates of ultimate displacement could be calculated from the curvatures at the effective yield and those corresponding to concrete strain of 0.004, both obtained from a moment curvature analysis. Using Equation (2.3) the ultimate displacement for the 6-meter wall works out to 770.8 mm; the corresponding value for the 4-m wall is 894.6 mm. A modal analysis using the effective moments of inertia just determined and including P- Δ effect gives the values of modal participation factor and the effective modal mass..

The effective yield load and ultimate displacement based on P- Δ instability can be worked out from a pushover analysis. Push over analysis is carried out for the distribution of load given by $M\phi_1$ and the resulting pushover curve is shown in Figure 3. The yield displacement is seen to be 400 mm. The idealized curve including the effect of P- Δ , shown by the dashed line, shows that the maximum base shear strength is approximately 3,180 kN. If a 10% reduction in the strength is assumed as the limit beyond which instability may result, the limit strength works out to 2,862 kN and the corresponding displacement is 656 mm as compared to the limit of 770.8 based on ductility capacity. The limit based on the allowable story drift of 0.025 is 832.2 mm. Thus, instability on account of P- Δ controls, the governing ultimate displacement is 660 mm and the ductility requirement is $660/400 = 1.64$. The yield and ultimate displacements for the equivalent SDOF system are 269.3 and 441.8 mm, respectively. From revised demand and capacity curves the spectral acceleration at yield is found to be 0.0538g. The required base shear strength is 2559.2 kN. Further iterations are carried out. They converge at the ductility capacity of 1.57, and design shear of 2800 kN

3.5 Multi-mode analysis

To include the higher mode contributions to the base shear a method based on demand capacity diagrams similar to that used for the first mode response can again be used. For each higher mode ϕ_i the capacity diagram is obtained from a push over analysis for a force distribution proportional to $M\phi_i$. Since the shear wall sections have been designed, the complete push over diagram for any given mode can be obtained and hence the performance point or the ultimate displacement for the given mode can be determined. The response parameters corresponding to this displacement are obtained from the push over data base. The procedure is similar to the one used in design but in the reverse order and is appropriate to the evaluation of a given design.

In general the response in second and higher modes is elastic. This is so in the present case as well. Hence the

higher mode responses are obtained from an elastic modal analysis and combined with the response determined for the first mode. The values in Table 1 show that the higher modes make major contributions to the base shear.

Table 1: Multi-mode analysis

Mode	Period sec	Modal participation Factor Γ	Effective modal Mass M^* ton	Spectral Acceleration g	Base shear kN	Roof displacement m
1	4.205	1.485	4849	0.0615	2925	0.6160
2	0.629	0.714	1490	0.6356	9295	0.0449
3	0.223	0.374	503	1.0000	4932	0.0045
4	0.113	0.260	247	1.0000	2431	0.0007
5	0.068	0.194	139	1.0000	1370	0.0002
SRSS					11272	0.6177

4. CONCLUSIONS

The following conclusions can be drawn from the study presented here:

1. The proposed DBSD presented here is both conceptually straightforward and simple to implement. The method represents an important step in the performance-based design. One advantage of the method is that it can be used to satisfy multiple objectives, and the designer has freedom to choose the value of the quantitative measure of performance level.
2. The preliminary design based on estimates of yield and ultimate displacements based only on the geometry and material properties of the structure, and assuming an appropriate shape for the first mode, provides a reasonable first design. If considered necessary, further iterations may be used to refine the design, but the method converges quite rapidly.
3. The ductility capacity of shear walls corresponding to the limiting strain of 0.015 for confined concrete is rarely mobilized since either the drift limit specified in the codes, or the limit corresponding to P- Δ instability governs. The designer has the freedom to choose any other strain limit and detail the structure accordingly. When a lower strain limit is selected, ductility capacity may govern the design.
4. The ductilities specified in the codes can rarely be mobilized without exceeding the drift limits and ductility capacity. The NBCC 2005, for example, defines a ductility of 3.5 for shear walls, while the IBC specifies a limit of about 2.5 (response modification coefficient R divided by the system overstrength factor), both of which are considerably greater than 1.57, mobilized in the 12-storey building studied here.
5. Modal pushover analyses shows that the shear response is greatly influenced by higher modes.

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