POWER SPECTRUM METHOD FOR BRIDGE SEISMIC ANALYSIS AND ITS APPLICATIONS IN CHINA

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ABSTRACT :

In China，the seismic design of highway bridges must comply with <Specifications of Earthquake Resistant Design for Highway Engineering JTJ004-89> (1989) since 1990, which is valid only for bridges with the main span shorter than 150m. The spatial effects between seismic excitations acting on different piers, such as the wave passage effect or the incoherence effect, were not mentioned, and only the response spectrum Method and the time history scheme were specified for seismic analysis, with the power spectrum method excluded. The above < Specifications 1989> (its bridges part) will soon be updated by <Specifications of Earthquake Resistant Design for Highway Bridges>, published in September, 2008. In this <Specifications 2008>, some issues about this alternation will be described.
In <Specifications 2008>, design principles are given for highway bridges with net-span longer than 150m. Therefore, the seismic spatial effects should be taken into account for such bridges. The random vibration based power spectrum method (PSM) is added as an optional means for structural seismic analysis. As the Pseudo Excitation Method (PEM) developed in China has been widely accepted as an accurate and efficient algorithm of PSM, some details about PEM are also involved in <Specifications 2008> and will be explained in this paper.

KEYWORDS: Long-span, bridge, power spectrum method, pseudo-excitation

1. INTRODUCTION

China is facing an unprecedented development in the bridge engineering. As an earthquake-frequent country, most of bridges built in China must proceed with earthquake resistant design. So far, however, civil engineers are still using <Specifications of Earthquake Resistant Design for Highway Engineering JTJ004-89>(1989), briefly called <Specifications 1989> later in this paper. As it is valid only for bridges with the main span shorter than 150m, the spatial effects between seismic excitations acting on different piers, such as the wave passage effect or the incoherence effect are completely out of consideration. In 1999, Ministry of Communications of China decided to update this Specifications with Chong-Qing Communications Research and Design Institute assigned responsible for the overall work.
It is known that although the random vibration theory based power spectrum method(PSM) has be recommended by Eurocode EC－8 (1995), it has not yet been widely used due to the complexity of random vibration analysis in terms of PSM for complex bridge structures. In the past 30 years, many progresses have been achieved (e.g., Der Kiureghian and Neuenhofer 1992, Ernesto and Vanmarcke 1994). Unfortunately, the developed methods remain not efficient or reliable enough for being widely accepted by the earthquake engineering community. Since 1985, the authors of this paper and their colleagues have developed an accurate and very efficient method, known as Pseudo Excitation Method, to deal with such problems (Lin and Zhang 2004, 2005). Up to date, this method has been widely used in the earthquake- or wind-resistant analyses of long-span bridges, dams, platforms, convention centers, etc. and formally published by hundreds of engineers, experts, scholars or research students in China. PEM is accurate because all cross-correlation terms between the participating modes and between all excitations have been included. It is also very easy to use because the random vibration analyses are transformed into deterministic dynamic analyses. The most important advantage of PEM is its extremely high efficiency: for complex problems, the seismic analysis using PEM can be even faster than using the conventional Response Spectrum Method (RSM). Therefore this method is now very popular in China, and has been adopted by <Specifications of Earthquake Resistant Design for Highway Bridges> published in September 2008, and so will be called <Specifications 2008> in this paper. Some related issues and details will be given in this paper.

2. BASIC PRINCIPLE OF PEM

In this section, the basic principle of PEM is described only with the random excitations assumed to be stationary, as it is enough for the use of <specifications 2008>, although PEM is also very efficient for problems with non-stationary random excitations.

2.1. A structure subjected to a single or uniform random excitation

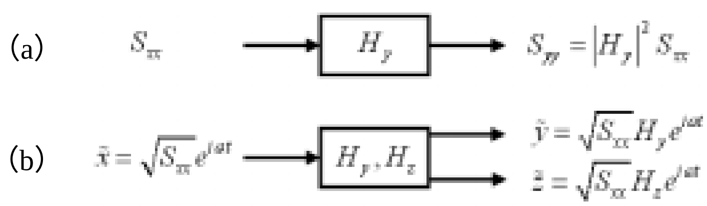

Figure 1 Basic Principle of PEM

Consider a linear system subjected to a zero-mean stationary random excitation $x(t)$ with a given PSD $S_{xx}(\omega)$, see Figure 1 (a). Suppose that for arbitrarily selected responses $y(t)$ and $z(t)$, the auto-PSD $S_{yy}(\omega)$ and cross-PSD $S_{yz}(\omega)$ are desired. $H_y(\omega)$ and $H_z(\omega)$ are their frequency response functions, i.e., if $x(t)$ is replaced by a sinusoidal excitation $e^{i\omega t}$, the harmonic responses of $y(t)$ and $z(t)$ would be $H_y e^{i\omega t}$ and $H_z e^{i\omega t}$, respectively. Clearly, if $x(t)$ is replaced by a pseudo sinusoidal excitation (Lin and Zhang 2004, 2005)

$$\tilde{x} = \sqrt{S_{xx}(\omega)}\ e^{i\omega t} \tag{2.1}$$

the responses would be $\tilde{y} = \sqrt{S_{xx}(\omega)} H_y(\omega) e^{i\omega t}$ and $\tilde{z} = \sqrt{S_{xx}(\omega)} H_z(\omega) \mathrm{e}^{i\omega t}$, see Figure 1 (b). It can be readily verified that

$$\tilde{y}^* \tilde{y} = \sqrt{S_{xx}(\omega)} H_y^*(\omega) \mathrm{e}^{-i\omega t} \cdot \sqrt{S_{xx}(\omega)} H_y(\omega) \mathrm{e}^{i\omega t} = \left|H_y(\omega)\right|^2 S_{xx}(\omega) = S_{yy}(\omega) \tag{2.2}$$

$$\tilde{y}^* \tilde{z} = \sqrt{S_{xx}(\omega)} H_y^*(\omega) \mathrm{e}^{-i\omega t} \cdot \sqrt{S_{xx}(\omega)} H_z(\omega) \mathrm{e}^{i\omega t} = H_y^*(\omega) S_{xx}(\omega) H_z(\omega) = S_{yz}(\omega) \tag{2.3}$$

If $\boldsymbol{y}(t)$ and $\boldsymbol{z}(t)$ are two arbitrarily selected random response vectors of the structure, and $\tilde{\boldsymbol{y}} = \mathbf{a}_y\, e^{i\omega t}$ and $\tilde{\boldsymbol{z}} = \mathbf{a}_z\, e^{i\omega t}$ are the corresponding harmonic response vectors due to the pseudo excitation (2.1), it can also be proved that the PSD matrices of $\boldsymbol{y}(t)$ and $\boldsymbol{z}(t)$ are

$$\mathbf{S}_{yy}(\omega) = \tilde{\mathbf{y}}^* \tilde{\mathbf{y}}^T = \mathbf{a}_y^* \mathbf{a}_y^T \tag{2.4}$$

$$\mathbf{S}_{yz}(\omega) = \tilde{\mathbf{y}}^* \tilde{\mathbf{z}}^T = \mathbf{a}_y^* \mathbf{a}_z^T \tag{2.5}$$

This means that the auto- and cross-PSD functions of two arbitrarily selected random responses can be computed using the corresponding pseudo harmonic responses.
Now, consider a structure subjected to a single seismic random excitation. Its equations of motion are

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = -\mathbf{M}\mathbf{e}\ddot{x}_g(t) \tag{2.6}$$

in which: the ground acceleration $\ddot{x}_g(t)$ is a stationary random process; its PSD $S_{\ddot{x}_g}(\omega)$ is known and; $\boldsymbol{e}$ is a given constant vector, indicating the distribution of inertia forces. Let the pseudo ground acceleration be

$$\ddot{\tilde{x}}_g(t) = \sqrt{S_{xx}(\omega)}\ e^{i\omega t} \tag{2.7}$$

Substituting Eqn.2.7 into Eqn.2.6 gives the pseudo equations of motion

$$\mathbf{M}\,\ddot{\tilde{\mathbf{y}}} + \mathbf{C}\,\dot{\tilde{\mathbf{y}}} + \mathbf{K}\,\tilde{\mathbf{y}} = -\mathbf{M}\,\mathbf{e}\sqrt{S_{\ddot{x}_g}(\omega)}\ e^{i\omega t} \tag{2.8}$$

Using the first q normalized modes for mode-superposition leads to (Clough and Penzien 1993)

$$\tilde{\mathbf{y}}(t) = \mathbf{a}_y(\omega) e^{i\omega t} = \sum_{j=1}^{q} \gamma_j H_j\ \boldsymbol{\varphi}_j\ \sqrt{S_{xx}(\omega)} e^{i\omega t} \tag{2.9}$$

in which ω_j, $\boldsymbol{\varphi}_j$, ς_j, H_j and γ_j are the j-th natural angular frequency, mass normalized mode, damping ratio, frequency response function and mode participation factor, respectively. According to PEM

$$\mathbf{S}_{yy}(\omega) = \tilde{\mathbf{y}}^* \tilde{\mathbf{y}}^T = \mathbf{a}_y^*(\omega) \mathbf{a}_y^T(\omega) \tag{2.10}$$

Substituting Eqn. 2.9 into Eqn.2.10 and expanding it gives the conventional algorithm

$$\mathbf{S}_{yy}(\omega)=\sum_{j=1}^{q}\sum_{k=1}^{q}\gamma_j\gamma_k\ \boldsymbol{\varphi}_j\ \boldsymbol{\varphi}_j^T\ H_j^*(\omega)H_k(\omega)S_a(\omega) \qquad (2.11)$$

This means these two equations are mathematically identical to each other. However the computational effort required by Eqn.2.10 is approximately only $1/q^2$ of that required by Eqn. 2.11. Therefore, Eqn. 2.10 is also known as the fast CQC algorithm (Lin, J.H. 1992).

2.2. Structures subjected to multiple random excitations

Consider a linear structure be subjected to a number of stationary random excitations, which are denoted as an m dimensional stationary random process vector $\boldsymbol{x}(t)$ with known PSD matrix $\boldsymbol{S}_{xx}(\omega)$. It is a Hermitian matrix and so it can be decomposed, e.g. by using its eigenpairs $\boldsymbol{\psi}_j$ and d_j $(j=1,2,...,r)$, into

$$\mathbf{S}_{xx}(\omega)=\sum_{j=1}^{r}d_j\boldsymbol{\psi}_j^*\boldsymbol{\psi}_j^T \qquad (r\le m) \qquad (2.12)$$

in which r is the rank of $\boldsymbol{S}_{xx}(\omega)$. Next, constitute r pseudo harmonic excitations

$$\tilde{\mathbf{x}}_j(t)=\sqrt{d_j}\boldsymbol{\psi}_j\ e^{i\omega t} \qquad (j=1,2,\cdots,r) \qquad (2.13)$$

By applying each of these pseudo harmonic excitations, selected response vectors $\mathbf{y}_j(t)$ and $\mathbf{z}_j(t)$, which can be displacements, internal forces or other linear responses, may be easily obtained and expressed as

$$\tilde{\mathbf{y}}_j(t)=\mathbf{a}_{yj}(\omega)\ e^{i\omega t} \qquad (2.14)$$
$$\tilde{\mathbf{z}}_j(t)=\mathbf{a}_{zj}(\omega)\ e^{i\omega t} \qquad (2.15)$$

The corresponding PSD matrices can be computed by means of (Lin, et al. 1994, 2005; Zhong ,W 2004)

$$\mathbf{S}_{yy}(\omega)=\sum_{j=1}^{r}\tilde{\mathbf{y}}_j^*(t)\tilde{\mathbf{y}}_j^T(t)=\sum_{j=1}^{r}\mathbf{a}_{yj}^*(\omega)\mathbf{a}_{yj}^T(\omega) \qquad (2.16)$$
$$\mathbf{S}_{yz}(\omega)=\sum_{j=1}^{r}\tilde{\mathbf{y}}_j^*(t)\tilde{\mathbf{z}}_j^T(t)=\sum_{j=1}^{r}\mathbf{a}_{yj}^*(\omega)\mathbf{a}_{zj}^T(\omega) \qquad (2.17)$$

The way used to decompose $\mathbf{S}_{xx}(\omega)$ into the form of Eqn.2.13 is not unique. In fact, the Cholesky scheme is perhaps the most efficient and convenient way to do it, i.e. $\mathbf{S}_{xx}(\omega)$ is decomposed into

$$\mathbf{S}_{xx}(\omega)=\mathbf{L}^*\mathbf{D}\mathbf{L}^T=\sum_{j=1}^{r}d_j\mathbf{l}_j^*\mathbf{l}_j^T \qquad (r\le m) \qquad (2.18)$$

in which $\mathbf{L}$ is a lower triangular matrix with all its diagonal elements equal to unity and $\boldsymbol{D}$ is a real diagonal matrix with r non-zero diagonal elements d_j. The implementation of Cholesky decomposition for a Hermitian matrix is very similar to that for a real symmetric matrix.

3. SOME REGULATIONS ABOUT PSM IN <SPECIFICATIONS 2008>

The random-vibration-theory-based PSM (power spectrum methods), including the PEM, for stationary random vibration have been adopted as an optional method together with the response spectrum Method (RSM) and time-history method (THM) in <Specifications 2008>, which mainly aims at bridges shorter than 150m. However, for longer highway bridges, some design principles have be added, among which seismic spatial effects have been paid special attention.

3.1. Production of Ground Acceleration PSD

According to <Regulations 2008>, the structural seismic analysis to which the RSM is applicable can also be solved by means of PSM, however the differences of the demands obtained from these two methods must be within 20% of the smaller one. As for the ground acceleration PSD, for very important bridges, i.e. the type A or type B bridges according to <Regulation 2008>, it should be given by specialists by means of geological explorations. For shorter or less important bridges, however, Kaul's suggestion has been adopted for producing such input PSD $S_a(\omega)$ from the response spectrum S of the local site acceleration

$$S_a(\omega) = \frac{T\xi S^2}{\pi^2}\left(\ln\left[\left(-\frac{T}{2T_d}\ln p\right)^{-1}\right]\right)^{-1} \quad (r \le m) \tag{3.1}$$

in which T_d is the seismic duration time, usually taking 20 sec ; ξ is the damping ratio ; $T = 2\pi/\omega$. Kaul(1978) suggests taking the non-surpass probability p =0.85. However, our research shows p =0.5 is more reasonable (Zhang, Y.H. 2007). The maximum differences of the demands due to p =0.5 or 0.85 and by using the conventional RSM are 10.2% or 19.1% , respectively, for any structure with basic period less than 8 sec, under uniform seismic excitations and located in any area of China. If Kaul's formula is replaced by an iteration-based method (Sun, J.J., et al 1990 ; Zhang, Y.H. et al. 2007), this maximum difference will decrease to only 1.2%. A program which performs these conversions has been developed for engineering use.

3.2. Response PSD analyses for bridges

Appendix C.1 of <Specifications 2008> summaries the use of PEM as follows:

3.2.1 For bridges subjected to uniform ground motion

Divide the effective frequency domain $[\omega_L, \omega_U]$, with equal interval $\Delta\omega$, by m frequency points. If the PSD of the stationary ground acceleration $\ddot{x}_g(t)$, denoted as $S_a(\omega)$, is known, then constitute the pseudo acceleration $\ddot{\tilde{x}}_g(t) = \sqrt{S_a(\omega)}e^{i\omega t}$, and compute a pseudo response (displacement, internal force, and so on) of interest, denoted as $\tilde{y}(\omega,t) = Y(\omega)e^{i\omega t}$, in which $Y(\omega) = Y_r(\omega) + iY_i(\omega)$ is a complex number. Then the PSD of this response would be

$$S_y(\omega) = |Y(\omega)|^2 = Y_r^2(\omega) + Y_i^2(\omega) \quad (r \le m) \tag{3.2}$$

The lower and up bounds ω_L and ω_U of the effective frequency domain can be determined according to

$$\omega_L \leq 0.7\omega_1\,, \qquad \omega_U \geq 1.2\omega_q \tag{3.3}$$

3.2.2 For bridges subjected to non-uniform ground motion with wave passage effect considered

If the apparent wave speed along the bridge is v (it can represent v_p for P-waves, or v_s for S-waves), then the pseudo ground acceleration vector for all N piers would be

$$\{\ddot{x}_g\} = \{1, e^{-i\omega T_2}, \cdots, e^{-i\omega T_N}\}^T \sqrt{S_a(\omega)} e^{i\omega t} \tag{3.4}$$

in which T_j is the time for the seismic waves traveling from the first pier to the *j*th one. Denote X_j as the x-direction (along the bridge) coordinate of the *j*th pier, then

$$T_j = \left(X_j - X_1\right)/v \tag{3.5}$$

Under the harmonic excitation vector Eqn.3.4, any selected response can be easily solved, denoted as $\tilde{y}(\omega,t) = Y(\omega)e^{i\omega t}$. Its auto-PSD can still be calculated by Eqn. 3.2.

3.3. Computation of Demands from response PSD

For any response y of the bridge, if its PSD $S_y(\omega)$ has been calculated, then the demand (i.e. the expected extreme value) of y, denoted as $\hat{y}$, can be computed by the following steps:

3.3.1 Calculate the ith *spectral moment of* y *for i=0, 2:*

$$\lambda_i = \int_0^\infty \omega^i S_y(\omega) d\omega \approx \int_{\omega_L}^{\omega_U} \omega^i S_y(\omega) d\omega \approx \sum_{l=1}^{m} \omega_l^i S_y\left(\omega_l\right) \Delta\omega \tag{3.6}$$

in which λ_2 is the second spectral moment of y, $\lambda_0 = \sigma_y^2$ is the *0*th spectral moment, i.e. the variance，while σ_y is the standard deviation of y.

3.3.2 Computation of Demands from the spectral moments

Assume seismic excitations be zero-mean stationary random process, its linear response y will have the same probabilistic characteristics. Denote y_e as the extreme value of y, and define dimensionless parameters $\eta = y_e / \sigma_y$, $\nu = \sqrt{\lambda_2/\lambda_0}/\pi$, the expected value of η would approximately be (Davenport, 1961)

$$E(\eta) \approx \left(2\ln \nu T_d\right)^{1/2} + \gamma / \left(2\ln \nu T_d\right)^{1/2} \tag{3.7}$$

in which，the Euler constant $\gamma = 0.5772$, and the expected value of y_e is approximately

$$\hat{y} = E(y_e) = E(\eta)\sigma_y \tag{3.8}$$

The expected extreme value $\hat{y}$ is the quantity corresponding to the demand given by the RSM. According to our research, demands according to Davenport(1961) are mostly quite close to those according to Vanmarcke (1972). This conclusion agrees with Gupta and Trifunac(1998).

4. EXAMPLES OF PEM APPLIED TO SEISMIC ANALYSES OF LONG-SPAN BRIDGES

Many papers or doctoral dissertations have been published in China which the dynamic characteristics of long-span bridges under seismic actions are studied with PEM as an important tool. These papers have powerfully promoted the PEM being adopted by the <Specifications 2008>. A few application examples from those papers, most of them were published in 2007 and 2008, are chosen and briefly introduced below.

- Huang, H.X. and Zhang, Z., etc.(2007) of Bridge Engineering Institute, Dalian University of Technology and College of Civil Engineering and Hebei University of Technology study the random seismic responses of a self-anchored and mixed cable-stayed and suspension bridge with seismic spatial effects considered. The peak responses of internal forces and displacements of the bridge under the action of P-, SH- and SV-waves are calculating using PEM, and in particular the responses due to uniform excitations and multi-support excitations are analyzed and compared, and concluded that such spatial effects are of considerable significance, and that PEM is very efficient to deal such problems.
- Zhao, C.H. and Zhou, Z.X. (2007) of Southwest Jiaotong university and Chongqing institute of communication investigate the stationary stochastic response of a CSFT (concrete filled steel tube) arch bridge with the net-span of 336. 28m and the net arrow height of 77.27m under jointly applied longitudinal, vertical and transverse seismic excitations by using PEM. The results show that the influences of the multi-support input on response is very important except for the transverse excitations.
- Wang, L.H., Peng, H.X., JIN, Y.X. and Zhang, H. (2008) at Hunan University study the stochastic responses of cable-stayed arch bridge by using PEM. The dynamic characteristics and seismic response behavior of the bridge subjected to longitudinal excitations, longitudinal plus vertical excitations, transverse plus vertical excitations and three-direction excitations are studied. Compared with the responses for the same bridge without cables, it is concluded that the existence of the cables improves the forced state of the bridge near the skewback of main arch ribs, so that the seismic behavior is increased.
- Dong J. (2008), consulting engineer of Architectural Design and Research Institute of Zhongtian., Guizhou Province, studied the random seismic responses of long-span spatial structures under ground excitations with wave passage effect being involved in order to overcome the incompleteness of the <Specifications 1989> in this respect. The seismic responses of a five-span continuous rigid-flamed bridge were numerically analyzed using PEM, and the responses with or without the wave passage effect be considered were compared, and the importance of the wave passage effect was demonstrated.
- SUN, JM, YE, JH and CHENG, WR(2007) at Shanghai University of Electric Power and Southeast University, published a paper entitled “Application of the pseudo-excitation method with spatial coherence in random vibration analysis of long span space structures” in Journal of Railway Science And Engineering. Four kinds of long-span structures were numerically investigated by using PEM. The author pointed out in the abstract that “it shows the pseudo excitation method is efficient and precise and is applicable to the random seismic response analysis of the complicate structure under multiple support excitations.
- Fan Lichu, etc.（2001）at Tongji University investigated the response characteristics of No.2 Nanjing Yangtze River Bridge under non-uniform seismic action, including wave passage, incoherence and local site effects. Its total length is 1238m. The height of the towers is 195.41m. The first 300 modes of the bridge were used for mode-superposition and fast CQC analysis using PEM. It was concluded that the responses of the bridge under seismic action with spatial variation are bigger than those under uniform seismic action up to 40%, and that the large differences of the site conditions at the bottoms of the towers and piers significantly change the responses of the bridges. This factor should be included in the seismic design of such bridges.
- Zhong,W.X. et al. (2003) study the Hong Kong Tsing-Ma long-span suspension bridge using PEM. It has a main span of 1377 m. The height of the towers is 206 m. The 3D finite element model of the bridge has:

769 nodes, including 29 support ones; 1010 elements and 2254 degrees of freedom. The first 180 modes were used in the mode superposition analysis. It is concluded that if the ground motion is assumed to be uniform, the response due to PEM is very closed to the results due to RSM. However, the results differ significantly from those when the spatial effects are taken into account. Comparatively, the influence of the wave passage effect is more important than that of the incoherence effect.

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