ON THE SEISMIC DESING OF INELASTIC ASYMMETRIC SYSTEMS

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ABSTRACT :

Design codes incorporate seismic torsional effects via elastic static or dynamic analyses, and seem to favor three-dimensional elastic modal procedures. However, several studies concluded that current code provisions may overestimate or underestimate the design shear forces, leading to inconsistent ductility demands. In particular, it has been reported that the modal method underestimates the design forces for members affected favorably by torsion. This paper presents an analytical study of a model consisting of a rigid deck with uniformly distributed mass, resting on three vertical bilinear inelastic elements in the direction of the input ground motion. The input consists of 87 records from hard to medium hard Californian sites, and also 66 records from the soft lakebed of Mexico City. The parameters studied are: ratio of uncoupled torsional to translational frequencies, Ω; normalized static eccentricity, e_s/r; elastic symmetric natural period, T; and design target ductility, μ_t. The IBC and the Mexico City Codes are evaluated by statistical analyses of ductility demands in asymmetric systems. It is found that on average the IBC provisions overestimate all design forces while the static regulations of Mexico City overestimate design forces for the flexible side elements, but underestimate the forces for the stiff side elements. The three-dimensional modal analysis always underestimates the design shear for elements on the rigid side. New amplification and reduction factors for the static design eccentricity are proposed that lead to ductility demands which closely approximate the target ductility in all elements.

KEYWORDS: Inelastic, Torsion, Fixed-Base, Single-Story, Ductility

1. INTRODUCTION

The examination of structures that performed poorly or collapsed during strong seismic events, some of recent construction and designed according to modern design regulations, indicates that in a number of cases the seismic design might not have properly addressed torsional effects (ATC 1984; Meli 1986). Many studies on torsional seismic effects consider unidirectional models. Efforts conducted using bidirectional models and excitations have reached some contradictory conclusions on the impact of resisting elements in two orthogonal directions. Some researchers have concluded that orthogonal resisting elements are always beneficial by increasing the total torsional stiffness (Stefano et al 1998; Humar and Kumar, 1999). On the other hand, other studies conclude that orthogonal elements effectively decrease the torsional response only if they remain elastic and that one-dimensional models are appropriate when the orthogonal elements are expected to yield (Riddell and Santa-Maria 1999).

Recognizing that currently prescribed static and dynamic elastic methodologies may overestimate or underestimate the design shear forces for the resisting elements of inelastic asymmetric structures, some researchers have proposed new formulas for static design eccentricities with the help of a set of charts (Chandler 1996), while others recommend including the elastic period, T, the static eccentricity e_s, and the inelastic global reduction factor R in coefficients for static design eccentricities (De-la-Colina 1999). In addition, Myslimaj and Tso (2002) and De la Llera and Chopra (1996) concluded that acceptable torsional response requires that the shear center and center of stiffness lie at opposite sides of the center of mass. Similarly, Escobar and Ayala (1998) proposed that the center of shear yield forces should stay between the centers of mass and rigidity.

Nevertheless, most current design provisions still stipulate three-dimensional elastic static and modal analysis to account for seismic torsion, even though it has been documented that for inelastic structures the three-dimensional modal method underestimates the design forces for members affected favorably by torsion (Bazan et al, 1989). The lack of a proven design procedure to attain satisfactory torsional inelastic behavior is attested by contradicting code specifications and discordant conclusions of studies on the inelastic seismic behavior of three-dimensional structures. This paper presents the results of work aimed at resolving some of these issues.

2. ASYMMETRIC SINGLE STORY MODEL

We consider a rigid deck with a uniformly distributed mass supported by three vertical resisting elements oriented in the direction of the input ground motion, as shown in Figure 1. The mass and the initial total elastic stiffness of the system are calculated in terms of the elastic period and the critical damping ratio is taken as 5%. One vertical element is located at the deck center of mass; the other elements are at the same distance from that center. Each element, $j = 1$ to 3, exhibits a bilinear hysteretic behavior with second slope equal to 2 percent of the initial one, yield strength $F_{y,j}$, and initial stiffness k_j. The deck is considered as a stiff diaphragm; thus, its motion is described by two degrees of freedom: translation $u(t)$ and rotation $\theta(t)$ of the center of mass. The seismic input consists of a set of 87 Californian earthquake records from stiff to medium stiff sites, and a set of 66 accelerograms recorded in the soft lakebed of Mexico City. The records of each set were normalized to have the same Arias Intensity. The average spectra of the two sets are depicted in Figure 2.
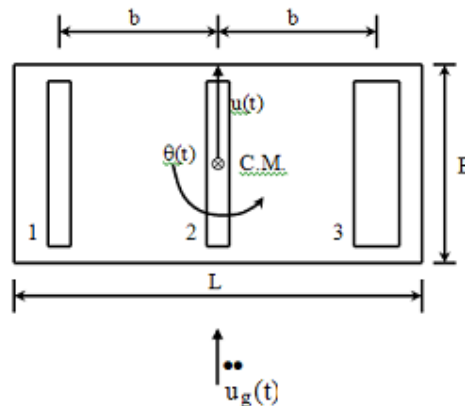

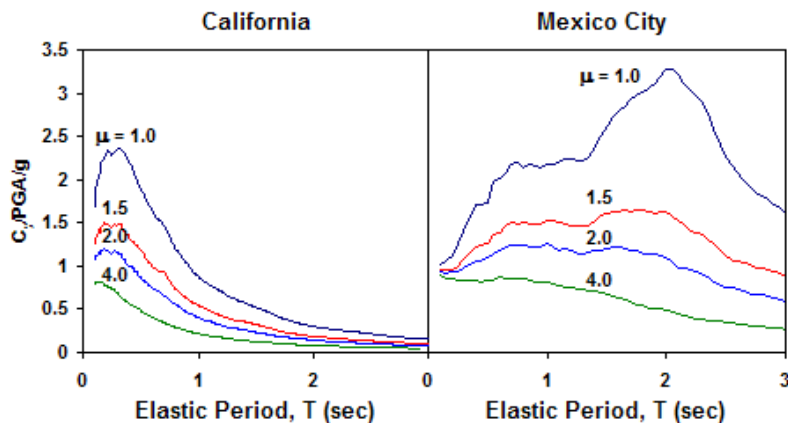Figure 1 Plan view of an asymmetric single-story model

Figure 2 Average elastic and inelastic input spectra: Californian (left) ands Mexican (right) records

3. STRUCTURAL AND RESPONSE PARAMETERS

The structural parameters of the asymmetric models considered are summarized in Table 1 and comprise:

- Ratio, Ω, of uncoupled torsional frequency to translational frequency (Ω = 1.0, 1.3, and 1.6)
- Normalized static eccentricity, e_s/r (r is the deck radius of gyration e_s/r = 0.1 to 0.72)
- Elastic symmetric natural period, T, from 0.1 to 3.0 seconds
- Design target ductility, μ_t (2 and 4)

Table 1 Structural parameters of one-story asymmetric models

Case	Ω	e_s/r	k_1/k	k_2/k	k_3/k
1	1.6	0.72	0.240	0.100	0.660
2	1.6	0.60	0.275	0.100	0.625
3	1.6	0.30	0.360	0.100	0.540
4	1.3	0.72	0.190	0.100	0.710
5	1.3	0.60	0.235	0.100	0.665
6	1.3	0.30	0.340	0.100	0.560
7	1.0	0.72	0.100	0.100	0.800
8	1.0	0.60	0.160	0.100	0.740
9	1.0	0.30	0.305	0.100	0.595
10	1.0	0.10	0.400	0.100	0.500

4. PROCEDURE FOR EVALUATING SEISMIC TORSIONAL EFFECTS

We use the following procedure for evaluating methodologies to account for seismic torsional effects:

1. Select a target ductility, μ_t.
2. For a period, T, calculate the total seismic resistance, V_y,, from the average spectra for the selected μ_t.
3. For each earthquake record, calculate the ductility demand for a symmetric structure with period T.
4. Select an unbalanced stiffness distribution to define an asymmetric system.
5. Distribute V_y (step 2) between the vertical resisting elements using the method under evaluation.
6. For each earthquake record, calculate the ductility demand for each resisting element.
7. Calculate the mean ductility demand of each resisting element, μ_i, and the ratio $R_\mu, = \mu_i/\mu_t$.

5. CURRENT CODE PROVISIONS FOR SEISMIC TORSION

In current code static procedures, the eccentricity, e_s, is the distance from the center of mass to the center of stiffness or center of twisting. Direct design forces on the vertical resisting elements result from distributing V_y proportionally to their elastic stiffness. Design eccentricities are stipulated as e_s times an amplification factor, α_1, and then times a reduction factor α_2. The IBC 2006 and the Mexico City 2004 codes prescribe $\alpha_1 = 1.0$, $\alpha_2 = 0.0$ and $\alpha_1 = 1.5$, $\alpha_2 = 1.0$, respectively. The design moments are resolved into horizontal shear forces at the resisting elements, according to their relative elastic stiffness and distance to the center of mass.

The three-dimensional modal analysis requires the calculation of elastic periods and modes of vibration. For each mode, the base shear force is calculated for the modal period and distributed between the resisting elements in accordance with the modal shape. Design shear forces are calculated with a modal combination rule.

5.1 Performance of current static torsional provisions

We first assess the performance of the static method by calculating the ratio R_μ obtained with the eccentricities prescribed in the IBC 2006 and the Mexico City 2004 codes. Initial calculations showed that R_μ values are very similar for ductility demands of 2 and 4. Thus, we conducted our assessment only for a target ductility of 2. Values of R_μ obtained using the IBC provisions for all cases listed in Table 1 are plotted at the left of Figure 3. Except for short-period structures, the IBC provisions overestimate the design force for all elements as R_μ is smaller than unity. The provisions are more conservative for high e_s/r on the flexible side. Systems with small Ω are also more conservative for flexible side elements; the opposite is true for elements on the stiff side. As a rule, long period systems are designed more conservatively, while for short period structures R_μ is greater than one, indicating that the static method of IBC 2006 is not conservative.

R_μ ratios obtained using the static provisions of the 2004 Mexico City Code are presented at the right hand side of Figure 3, and indicate that the Mexican Code is overly conservative for elements on the flexible side for elements on the flexible side, particularly for systems with small Ω or large e_s/r. For stiff side elements, however, design forces can be appreciably underestimated, particularly for short period systems and for long period structures having $\Omega = 1.0$. R_μ values significantly bigger than unity are obtained, sometimes even greater than 2. The reason is that by stipulating $\alpha_2 = 1.0$, the static eccentricity is never reduced, leading to design forces that are smaller than required for elements favorably affected by torsion.
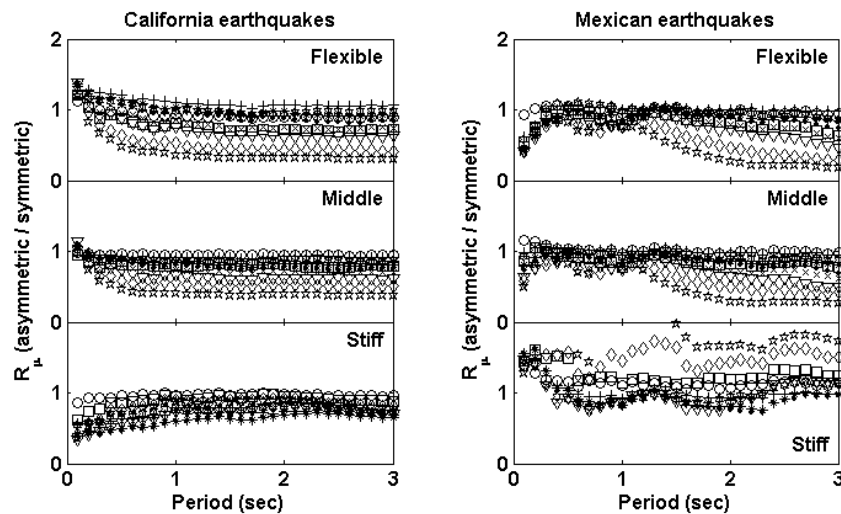

Figure 3 R_μ of systems designed with the IBC 2006 (left) and 2004 Mexico City (right) provision, $\mu_t = 2$.

5.2 Performance of current code dynamic modal provisions

Figure 4 displays R_μ for systems designed with the three-dimensional modal method, using a CQC rule. Adequate design forces result for flexible side elements, whose ductility demands are very close to the target. For stiff side elements, this method yields acceptable design forces only for systems with tuned frequencies; otherwise, the method underestimates the design forces, for both Californian and Mexican earthquakes. Figure 4 also highlights where the modal method is inadequate since the stiff side and middle elements exhibit R_μ significantly larger than 1.0. The reason is that the method incorporates automatically elastic dynamic amplifications of torsional moments, leading to significant force reductions for elements affected favorably by torsion.
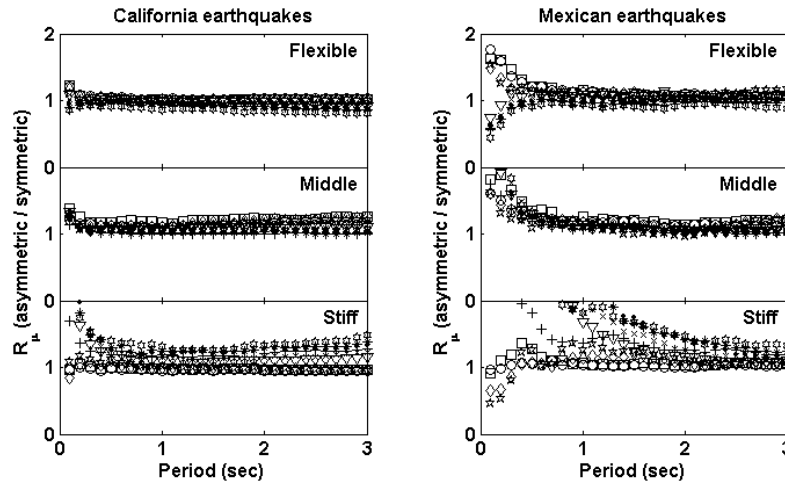
Figure 4 R_μ of systems designed with the three-dimensional modal method, $\mu_t = 2$.

6. PROPOSED STATIC METHOD

This and previous studies have shown that in addition to the fundamental period the ratios e_s/r and Ω play a significant role in the inelastic seismic response of asymmetric structures. Modification factors for static eccentricities are proposed in this section incorporating those parameters. We aim to achieve more uniform ductility demands in the resisting elements, bringing them to be consistently close to the target ductility for all periods. To this end, parametric studies were conducted varying α_1 and α_2 systematically until average ductility demands in all elements were sufficiently close to the target ductility. The proposed formulas for α_1 and α_2 are:

$$\alpha_i = \begin{cases} a - (b - c\Omega)\left(\dfrac{e_s}{r}\right)^d \left(e \cdot \dfrac{T}{T_0}\right) & ;0 < T \leq T_0 \\ a - (b - c\Omega)\left(\dfrac{e_s}{r}\right)^d & ;T \geq T_0 \end{cases} \qquad (6.1)$$

Table 2 Coefficients of the proposed modification factors

Case	i	T_0 (s)	a	b	c	d	e
California	1	0.5	1.2	1.70	0.80	0.30	1.00
	2	0.5	1.0	1.65	0.83	0.65	-
Mexico City	1	2.0	1.5	2.1	0.80	0.50	0.50
	2	2.0	1.0	2.1	0.65	0.65	-

Figure 5 presents averages of mean ductility demand of asymmetric structures designed via different methods, for target ductilities of 2 and 4. The proposed amplification and reduction factors provide adequate design forces for elements on the flexible side, and also result in average ductility demands lower than those produced by current static factors, and similar to those from the dynamic method. Moreover, the new factors yield lower coefficients of variation of the ductility demand. For the middle and stiff side elements, the proposed factors provide design forces better than the currently stipulated static and dynamic procedures, producing ductility demands closer to the target.
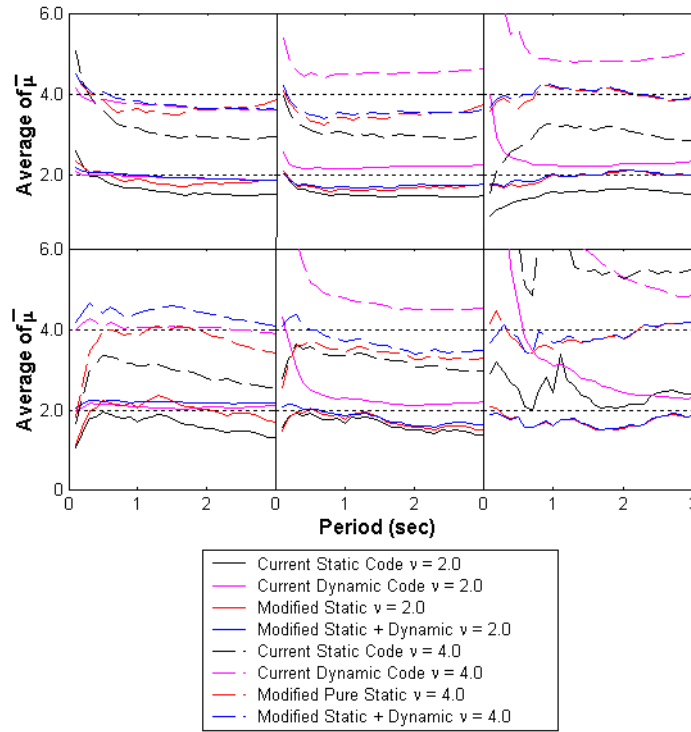
Figure 5 Average of mean ductility demand of asymmetric systems excited by Californian (top) and Mexican earthquake records (bottom)
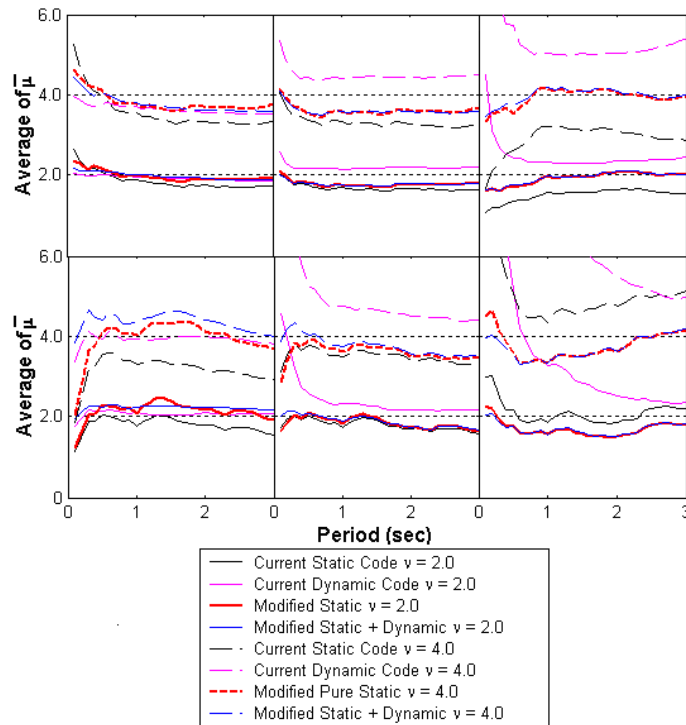

Figure 6 Statistics of mean ductility demand of asymmetric systems for Californian (top) and Mexican (bottom) earthquake records ignoring case 7 and 8
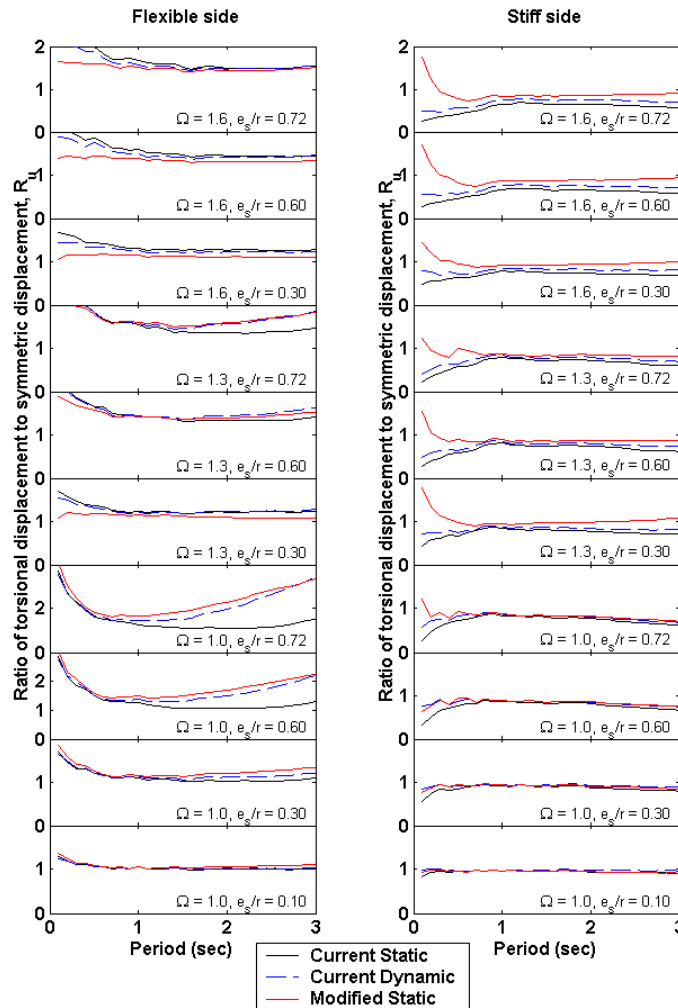
7. MAXIMUM DISPLACEMENT

Figure 7 Ratio of mean maximum relative displacement for Californian records

The maximum displacement of the j-th element, u_{maxj}, is $\mu\, u_{yj}$, in which μ is the target ductility and u_{yj} is the yield displacement. Note that applying amplification and reduction factors to torsional moments results in shear forces and displacements that are greater than in the associated symmetric system. Figure 7 shows the ratio, R_u, of the maximum displacement of the edge elements to the symmetric displacements. The procedures used to design the asymmetric systems were: 1) Current static regulations, 2) Three-dimensional modal analysis, and 3) The proposed method. The maximum displacement of the flexible side element is always greater than the displacement at the stiff side. The maximum displacement of structures with the same ratio Ω, is proportional to e_s/r. Structures with large static eccentricity and/or with tuned uncoupled frequencies ($\Omega = 1$) deform appreciably more than those with separated uncoupled frequencies.

Even for the extreme cases 7 and 8 in Table 1 (with $\Omega = 1.0$ and large e_s/r), it is possible to obtain ductility demands close to the target, but the maximum displacements at the flexible side are very large, up to four times those of symmetric systems. Thus, structures with tuned frequencies or large eccentricities should be avoided. Figure 6 depicts averages of mean ductility demand of asymmetric structures ignoring cases 7 and 8. The

comparison of Figure 5 with Figure 6 reveals that by eliminating extreme cases the averages of mean ductility demand of the flexible side element become appreciably closer to the target. In addition, the coefficients of variation of the mean ductility demand decrease significantly.

8. CONCLUSIONS

This study shows that current static provisions in the IBC 2006 Code, on average, overestimate design forces for all resisting elements of asymmetric systems, while the current static regulations of the Mexico City Code overestimate design forces for elements on the flexible side but underestimate the forces for elements on the stiff side. Furthermore, the currently stipulated three-dimensional modal analysis is acceptable only for the edge elements on the flexible side, underestimating the design forces for other elements.

Since the ratios Ω and e_s/r, and the fundamental period, T, play a decisive role on the inelastic behavior of asymmetric systems, new amplification and reduction factors for use with the static method are proposed as functions of these three parameters. An additional advantage of the proposed static design eccentricities is that they lead to ductility demands that are closer to the target ductility.

Even though it is feasible to bring the ductility demands of systems with tuned translational and torsional frequencies and/or with large static eccentricities to be close to the target ductility, their maximum displacements can be up to four times the values of the associated symmetric system. Therefore, it is recommended that structures with tuned frequencies and/or large static eccentricities be avoided.

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