EXTENSION OF N2 METHOD TO PLAN IRREGULAR BUILDINGS CONSIDERING ACCIDENTAL ECCENTRICITY

Gennaro Magliulo[1], Giuseppe Maddaloni[2], Edoardo Cosenza[3]

[1] *Assistant Professor, University of Naples "Federico II", Department of Structural Engineering (DIST), Naples, Italy*

[2] *Assistant Professor, University of Naples "Parthenope", Department of Technology (DiT), Naples, Italy*

[3] *Professor, University of Naples "Federico II", Department of Structural Engineering (DIST), Naples, Italy*

Email: gmagliul@unina.it, giuseppe.maddaloni@uniparthenope.it, cosenza@unina.it

ABSTRACT:

The paper deals with the topic of analyses performed according to modern code provisions, in particular Eurocode 8 (EC8) rules. Elastic, non linear static and non linear dynamic analyses of a plan irregular multi-storey r/c frame building designed according to Eurocode 2 (EC2) and EC8 provisions are carried out. The elastic analysis is performed by the computer program SAP2000, while the non linear analyses by CANNY99. A set of 7 earthquakes (each considering both the horizontal components), fully satisfying the EC8 provisions, are used as input of non linear dynamic analyses.
The problem of extension of N2 method to plan irregular buildings, which makes up for the underestimate of seismic demand on stiff side, is focused: three methods, which take into account the accidental eccentricity provided by modern codes, are proposed. The results, in terms of pushover curves, frame top displacements and interstorey drifts, are compared with ones obtained by nonlinear dynamic time-history analyses. Non linear static analyses are carried out both applying the "modal" and "uniform" force pattern. The "orthogonal effects", evaluated by SRSS rule, result to be negligible.

KEYWORDS: Plan irregularity, torsional effects, non linear analyses; natural records, Eurocode 8.

1. INTRODUCTION

N2 method is one of the most spread methods for non linear static analysis: it is provided by EC8 (CEN, 2003) and other modern seismic codes. Since Fajfar published it (Fajfar, 2000), its research group is working on its extension to plan irregular buildings, in order to make up for the underestimate of seismic demand on stiff side. They recently (Fajfar et al., 2006) proposed to combine the results obtained by pushover analysis of a 3D structural model with the results of a linear dynamic spectral analysis: the former results control the target displacements and the distribution of deformations along the height of the building, whereas the latter results define the torsional amplifications. In particular, the Fajfar procedure is characterised by 4 steps: (1) pushover analyses are performed by using a 3D mathematical model, seismic forces are applied at mass centers (CM), independently in two horizontal directions, in each direction with + and – sign; target displacement (displacement demand at CM at roof level) is determined for each of two horizontal directions (the largest value of two values obtained for + and – sign); (2) a modal response spectrum analysis of the 3D mathematical model is performed, considering the excitation in two horizontal directions independently and the results are combined according to the SRSS rule; (3) the correction factor, applied to relevant results of pushover analyses, is determined in order to take into account torsional effects; it is defined, for each horizontal direction separately, as the ratio between two normalized roof displacements, respectively obtained by elastic modal analysis and by pushover analysis; (4) the normalized roof displacement is the ratio between roof displacement at an arbitrary location and roof one at CM; if the normalized roof displacement obtained by elastic modal analysis is smaller than 1.0, the value 1.0 is used.
Such procedure, as proposed, does not consider that different models are to be computed due to different positions of centre of mass corresponding to accidental eccentricities.
In the paper three different methods are proposed in order to apply to a plan irregular building the N2 method as formulated by Fajfar, taking into account the accidental eccentricity; such methods, characterised by different levels of accuracy, are described, applied to a three-storey r/c frame building and compared. The results are shown in terms of pushover curves, frame top displacements and interstorey drifts.
Dynamic analyses are performed using a set of natural records fully satisfying EC8 provisions: average and standard deviation of results are considered.
The results reported herein, for sake of brevity, concern a single structure; however they are confirmed by analyses performed at the same seismic level on different buildings.

2. GEOMETRY OF THE BUILDING AND ELASTIC ANALYSIS

The geometry of the analysed three-storey r/c frame building is reported in Figure 1. The interstorey height is equal to 3.2 m at all levels; at the first storey the columns section dimensions are 40×65 cm^2, while all the beams are 40×60 cm^2; at the second storey such dimensions are respectively 40×60 cm and 40×55 cm, at the third 40×55 cm and 40×50 cm. Column dimensions are kept larger than beam ones in order to take into account the capacity design. In Figure 1, 2 m wide balconies are shown as hatched areas.
Elastic analyses are performed by the computer program SAP2000 (CSI, 2004), according to Eurocode rules and considering a design spectrum soil B type 1 with a design ground acceleration on type A ground, a_g=0.35$_g$, taken from the Italian seismic code OPCM 3431. The design is performed according to the High Ductility Class rules, a behaviour factor equal to 2.4 is computed; such value takes into account that according to EC8 the building is “torsionally flexible” and irregular in elevation (the reduction of lateral stiffness from 1st to 2nd storey is larger than 40%).
Concrete characteristic compressive cylinder strength equal to f_{ck}=30 N/mm^2 and steel characteristic yielding strength equal to f_{yk}=430 N/mm^2 are adopted.

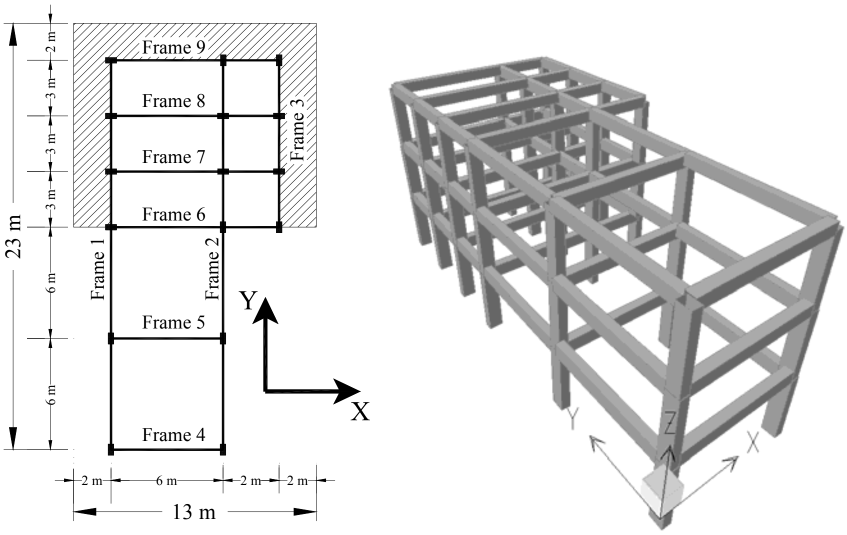

Figure 1 Geometry of the building

3. NON LINEAR MODEL

Non linear analyses are performed by means of the computer program CANNY99 (Li, 1996). Non linearity regards flexural rotations, while all the other deformations are assumed linear. Both beams and columns are characterised by lumped plasticity models; in the latter case for each section two independent non linear springs are assigned, one for each orthogonal direction. No axial force-bending moment interaction is considered at plastic hinge.
Bending moment springs are characterised by a tri-linear skeleton curve, defined by cracking and yielding moment and corresponding rotations; the post-yielding stiffness is assumed equal to zero. An elastic-perfectly plastic steel stress-strain diagram is considered, characterised by an yielding strength equal to 530 N/mm^2, computed as mean of tests on more than 200 bars made by steel called FeB44K performed at the laboratory of Department of “Scienza delle Costruzioni” of the University of Naples Federico II.
The yielding and the ultimate rotations are evaluated as provided by EC8 (CEN, 2004) equations (A.10b) and (A.1) respectively, where the already cited average values are assigned to concrete maximum (f_c=38 N/mm^2) and steel yielding (f_y=530 N/mm^2) strength.
The hysteretic model is Takeda type; the pinching effect is also taken into account.

4. NON LINEAR STATIC ANALYSES

Non linear static analyses (NLSA) are performed according to EC8, considering a force distribution proportional for each of the 2 orthogonal directions to the first modal shape in the relative direction by mass distribution (“modal” pattern). The first and the third modes of the building are translational and the square root of the ratio of torsional stiffness to lateral stiffness in each of the 2 orthogonal directions is smaller than the radius of gyration of floor mass in plan (see equation (4.1b) in EC8) (CEN, 2003); consequently the structure is torsionally flexible and the application of a specific procedure for the estimation of torsional effects is necessary.
Eight analyses, 2 opposite signs for 4 different positions of the centre of mass, are performed. In Figure 2, 2 of 8 pushover curves in terms of adimensionalized top displacement vs base shear are shown. The bi-linear capacity curve is also presented along with capacity and demand points: “mech” indicates the mechanism of the structure, “ULS” (Ultimate Limit State) and “NC” (Near Collapse Limit State) the attainment in at least one hinge of the rotation value 3/4 Θ_u and Θ_u respectively and “t.ULS” the demand corresponding to the EC8 elastic design acceleration spectrum soil B type 1, a_g=0.35g; Θ_u is the total chord rotation capacity computed according to EC8 empirical formula (A.1) (CEN, 2004).
Figure 2 shows that in the examined case differences between NLSA performed on the same model and along the same direction but with opposite sign are negligible; this conclusion justifies the possibility to consider only one NLSA between the two ones with opposite sign and, in particular, the one with the largest target displacement.

In order to evaluate the torsional effects three variants of the Fajfar method are proposed and presented in the following; they consider different models characterised by different accidental eccentricities, indicating how to associate modal response spectrum analyses (MRSA) with non linear static ones.

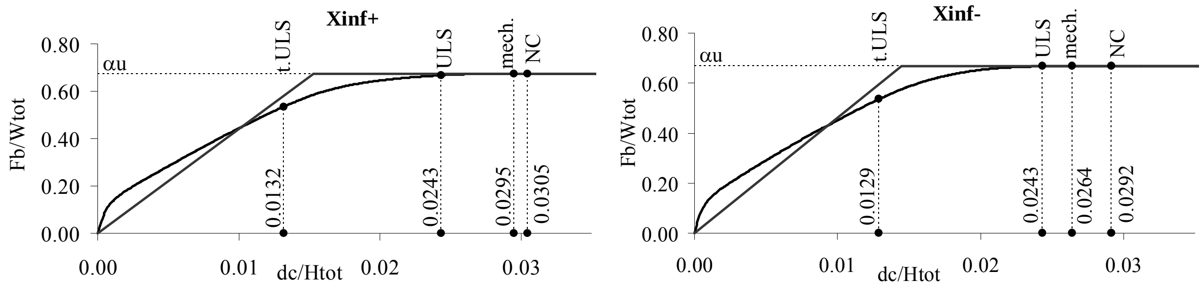

Figure 2 Capacity curves of 2 considered models resulting by a vertical force “modal” pattern

4.1 *Method n.1*

- Four modal response spectrum analyses are performed (called X_{inf}, X_{sup}, Y_{dx}, Y_{sx}, clearly depending on the direction of forces and on the position of centre of mass), one for each position of CM; obviously the load is applied along Y direction when CM is moved along X axis and, conversely, it is applied along X direction when CM is moved along Y axis;
- The results (in particular top displacements of each frame and of each centre of mass) obtained by each MRSA are combined by the SRSS rule:

$$\sqrt{u_{X\,\sup}^2 + u_{Ydx}^2}\;;\quad \sqrt{u_{X\,\sup}^2 + u_{Ysx}^2}\;;\quad \sqrt{u_{X\,\inf}^2 + u_{Ydx}^2}\;;\quad \sqrt{u_{X\,\inf}^2 + u_{Ysx}^2} \tag{4.1}$$

- For each combination and for each frame the following normalized displacement is computed:

$$\eta_{MRSA,i} = \frac{u_{MRSA,i}}{u_{MRSA,CM}} \tag{4.2}$$

  where $u_{MRSA,i}$ is the “i” frame top displacement and $u_{MRSA,CM}$ is the CM top displacement; consequently, 4 $\eta_{MRSA,i}$ are computed for each frame and their maximum is assumed as the reference $\eta_{MRSA,i}$;
- Eight non linear static analyses are performed, two signs (+ and -) for 4 models and for each model the NLSA with the maximum target displacement (t.ULS) is considered; consequently, 4 normalised displacements are obtained for each frame:

$$\eta_{NLSA,i} = \frac{u_{NLSA,i}}{u_{NLSA,CM}} \tag{4.3}$$

  where $u_{NLSA,i}$ is the “i” frame top displacement and $u_{NLSA,CM}$ is the CM top displacement;
- For each frame 4 correction factors are computed:

$$\beta_i = \frac{\eta_{MRSA,i}}{\eta_{NLSA,i}} \tag{4.4}$$

- The seismic demand related to frames on stiff side of the structure is amplified multiplying all relevant quantities obtained by the 4 non linear static analyses by the corresponding correction factors;
- The maximum value of demand parameters among the 4 ones given by non linear static analyses is considered and compared with the capacity one computed according to EC8 (CEN, 2004).

4.2 *Method n.2*

- Four modal response spectrum analyses (called X_{inf}, X_{sup}, Y_{dx}, Y_{sx}) are performed, as at the 1st step of method n.1;

- For each CM position and for each frame, the normalized displacement (Eqn. 4.2) is computed. Consequently, four $\eta_{MRSA,i}$ for each frame are obtained;
- Eight non linear static analyses are performed, two signs (+ and -) for 4 models and for each model the NLSA with the maximum target displacement (t.ULS) is considered; consequently, 4 normalised displacements (Eqn. 4.3) are obtained for each frame, as at step 4 method n.1;
- Four correction factors (Eqn. 4.4), corresponding to the 4 positions of centre of mass, are computed for each frame;
- As at method n.1, 6th step, the seismic demand related to frames on stiff side of the structure is amplified multiplying all relevant quantities obtained by the 4 non linear static analyses by the corresponding correction factors;
- The results of the four amplified NLSA are combined according to the SRSS rule:

$$\sqrt{E^2_{X\,sup}+E^2_{Ydx}}\;;\;\sqrt{E^2_{X\,sup}+E^2_{Ysx}}\;;\;\sqrt{E^2_{X\,inf}+E^2_{Ydx}}\;;\;\sqrt{E^2_{X\,inf}+E^2_{Ysx}} \tag{4.5}$$

  where, as already said, “X” and “Y” indicate analysis direction, while “sup”, “inf”, “dx” and “sx” the centre of mass position;
- The maximum value of demand parameters among the 4 ones given by SRSS combinations is considered and compared with the capacity one computed according to EC8 (CEN, 2004).

4.3 Method n.3

- The first 2 steps are coincident with method n.2;
- Eight non linear static analyses are performed, two signs (+ and -) for 4 models and, consequently, 8 normalised displacements (Eqn. 4.3) are obtained for each frame (as for methods n.1 and n.2 displacements at t.ULS are considered);
- Eight correction factors (Eqn. 4.4), corresponding to 4 positions of centre of mass and both the signs (+ and -), for each frame are computed: to each $\eta_{MRSA,i}$ correspond 2 $\eta_{NLSA,i}$;
- The seismic demand related to frames on stiff side of the structure is amplified multiplying all relevant quantities obtained by the 8 non linear static analyses by the corresponding correction factors;
- The results of the 8 amplified NLSA are combined according to the SRSS rule:

$$\sqrt{E^2_{+X\,sup}+E^2_{+Ydx}}\quad\sqrt{E^2_{+X\,sup}+E^2_{-Ydx}}\quad\sqrt{E^2_{-X\,sup}+E^2_{+Ydx}}\quad\sqrt{E^2_{-X\,sup}+E^2_{-Ydx}}\quad\sqrt{E^2_{+X\,sup}+E^2_{+Ysx}}\quad\sqrt{E^2_{+X\,sup}+E^2_{-Ysx}}$$
$$\sqrt{E^2_{-X\,sup}+E^2_{+Ysx}}\quad\sqrt{E^2_{-X\,sup}+E^2_{-Ysx}}\quad\sqrt{E^2_{+X\,inf}+E^2_{+Ydx}}\quad\sqrt{E^2_{+X\,inf}+E^2_{-Ydx}}\quad\sqrt{E^2_{-X\,inf}+E^2_{+Ydx}}\quad\sqrt{E^2_{-X\,inf}+E^2_{-Ydx}}$$
$$\sqrt{E^2_{+X\,inf}+E^2_{+Ysx}}\quad\sqrt{E^2_{+X\,inf}+E^2_{-Ysx}}\quad\sqrt{E^2_{-X\,inf}+E^2_{+Ysx}}\quad\sqrt{E^2_{-X\,inf}+E^2_{-Ysx}} \tag{4.6}$$

  where, as already said, “X” and “Y” indicate analysis direction, “+” and “-“ its sign and “sup”, “inf”, “dx” and “sx” the centre of mass position;
- The maximum value of demand parameters among the 16 ones given by SRSS combinations is considered and compared with the capacity one computed according to EC8 (CEN, 2004).

Method n.3 provides results, which for sake of brevity are not shown, almost coincident with the ones provided by method n.2. This shows that, even when torsional effects are considered, the sign of pushover does not affect the results and, consequently, it is sufficiently accurate to consider the pushover with the maximum target displacement between the two pushovers with opposite sign.

4.4 Comments on SRSS combination rule

Table 1 shows the increment of frame top displacements due to the evaluation of orthogonal effects by the SRSS rule (Eqn. 4.5) of method n.2. In the 1st column the frames are listed, frames 1, 2 and 3 are parallel to Y axis as reported in Figure 1, while frames from 4 to 9 are parallel to X axis; the 2nd and the 3rd columns show for the frames 4 to 9 and 1 to 3 respectively the top displacements resulting from (Eqn. 4.5), while the 4th and the 5th

columns present the corresponding displacements obtained by method n.2 before applying the combination (Eqn. 4.5); the 6th and the 7th column present the variation in percentage of the results.
It is evident that such variations are negligible; consequently the 6th step of method n.2, i.e. the application of combination (Eqn. 4.5), could be suppressed for computing such parameters.

Table 1 Estimation of orthogonal effects evaluated by SRSS combination rule: frame top displacements computed according to method n.2

	SRSS applied		**SRSS NOT applied**		**Variation [%]**	
Frame	u_x [m]	u_y [m]	u_x [m]	u_y [m]	u_x [-]	u_y [-]
1	-	0.126	-	0.126	-	0.24
2	-	0.124	-	0.124	-	0.01
3	-	0.133	-	0.133	-	0.14
4	0.148	-	0.147	-	0.19	-
5	0.138	-	0.137	-	0.07	-
6	0.127	-	0.127	-	0.01	-
7	0.127	-	0.127	-	0	-
8	0.129	-	0.129	-	0.02	-
9	0.132	-	0.132	-	0.05	-

5. STEP BY STEP NON LINEAR DYNAMIC ANALYSES

Both the horizontal components of a set of 7 earthquakes, i.e. 14 natural records, are used for non linear dynamic analyses whose results are shown herein; according to the selection procedure presented in Iervolino et al., 2008, they satisfy the EC8 provisions. According to the code, if the response is obtained from at least 7 non linear time histories analyses, as for the performed analyses, the average of response quantities should be used as the design value of the action effect E_d in relevant verifications.
Figure 3 shows the elastic spectrum of the 14 records used, along with the average spectrum (smooth thick line), the EC8 elastic design spectrum (black thick line) and the curve whose ordinates are equal to 90% of such spectrum ones (black thin line); SF indicates the value of factors used to scale the records for the purpose to matching code spectrum: only one record is scaled by a low SF equal to 1.08.

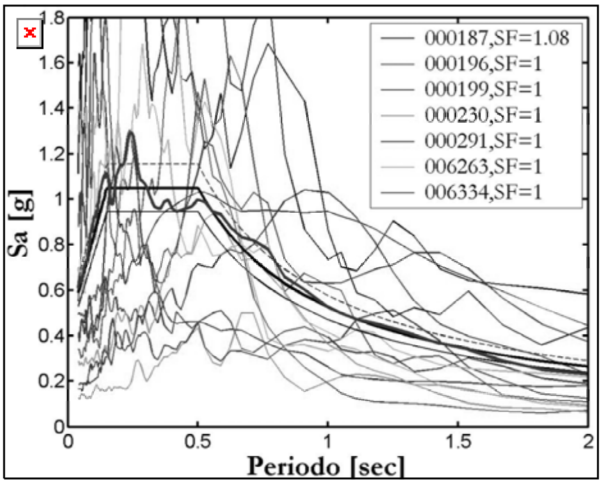

Figure 3 Spectra of records used for non linear analyses, their average and EC8 spectrum

The matched code spectrum is the one used for the design of the analysed building, assigning *q*=1.
As for non linear static analyses, top centre of mass and frame displacements are shown. For each of the 4 models obtained moving the centre of mass, the average of the 7 maximum results obtained applying the 7 earthquakes is

considered along with the average plus the standard deviation and the maximum of the 7 maximum results; the maximum effect on the 4 models is always considered.

6. COMPARISONS BETWEEN THE ANALYSES RESULTS

In this chapter results of methods n.1 and n.2 are compared to the ones of non linear dynamic analyses (NLDA) in terms of displacements; as already written, the capacity at Ultimate Limit State corresponds to the attainment of a chord rotation equal to 3/4 Θ_u.

In Figures 4 and 5 the following symbols are used: NLSA indicates results of non linear static analyses at "t.ULS" without considering torsional effects, i.e. the maximum of the 4 SRSS combinations (Eqn. 4.5), where the effects are not amplified by correction factors; NLSA(meth. n.1) and NLSA(meth. n.2) indicate the results of method n.1 and 2 respectively; NLDA(average) means that the average among the maximum results of the 7 non linear dynamic analyses is considered and the maximum among the 4 models is reported; as NLDA(av+SD) the average plus the standard deviation instead of the only average of the 7 results for each model is presented, while NLDA(maximum) means that the maximum among the maximum results of the 7 non linear dynamic analyses is taken and, as in the other 2 cases, the maximum of the 4 models is shown.

Figure 4 shows frame top displacements, absolute and normalized with respect to CM displacement, along X and Y direction. In Figure 5 the intersorey drifts are reported, divided by the corresponding interstorey height.

It can be observed that NLSA(meth. n.1) gives displacements larger than NLSA and NLSA(meth. n.2), in particular for frames 4 and 5; this is due to the assumption of the maximum $\eta_{MRSA,i}$ in order to compute the four correction factors for each frame. This allows the NLSA(meth. n.1) displacements to better approximate the displacements of non linear dynamic analysis when the standard deviation is also considered and, in particular, to estimate on safe side the displacements due to torsion.

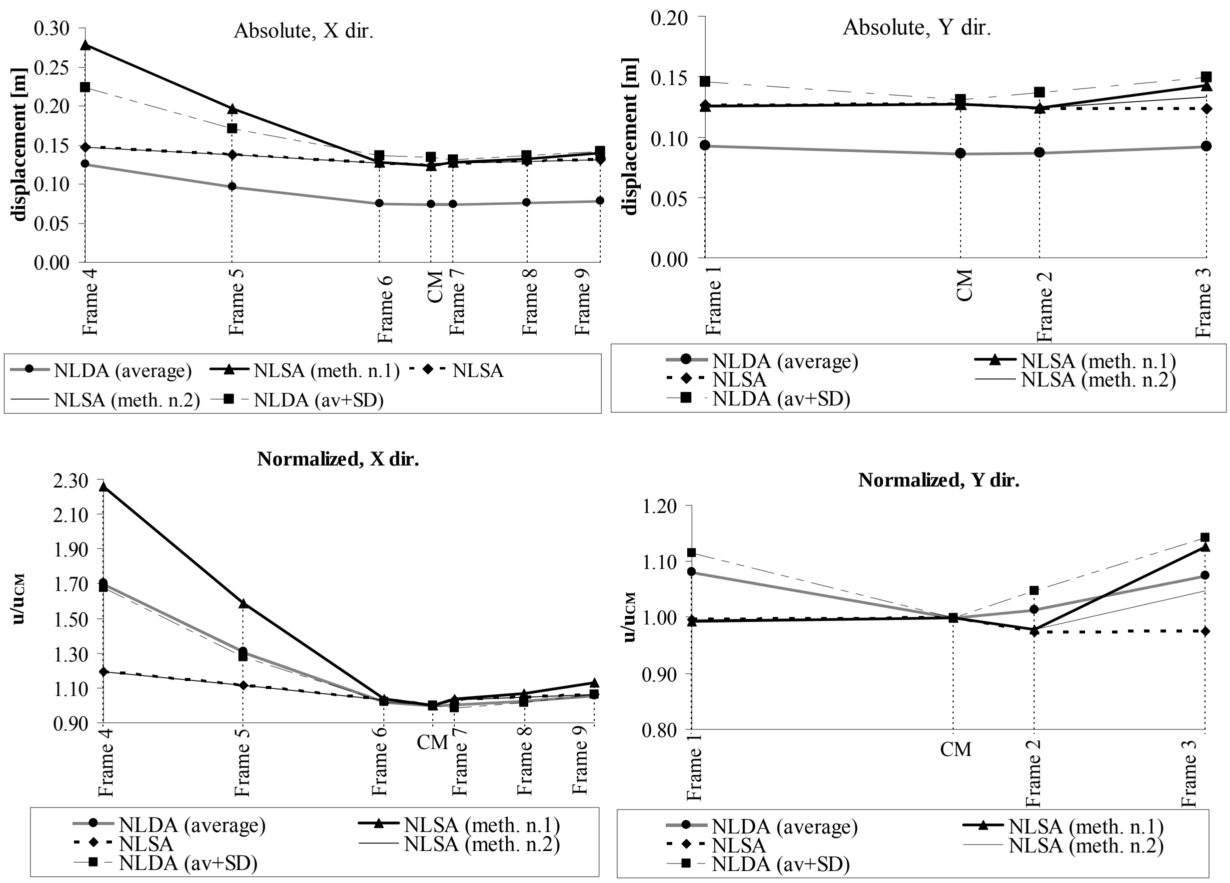

Figure 4 Absolute and normalized frame top displacements along X and Y direction

On the contrary, the positive aspects of NLSA(meth. n.2) are that the correction factor computed for a model comes from $\eta_{MRSA,i}$ and $\eta_{NLSA,i}$ computed for the same model and that the increment of displacements due to torsion effects is low, which is positive if the approximation of NLDA(average) absolute displacements is considered. Figures 4 and 5 also show that NLSA methods provide displacements quite larger than NLDA(average) displacements and that the standard deviation is not negligible with respect average results, as it can be observed by the difference between NLDA(average) and NLDA(av.+SD) displacements.

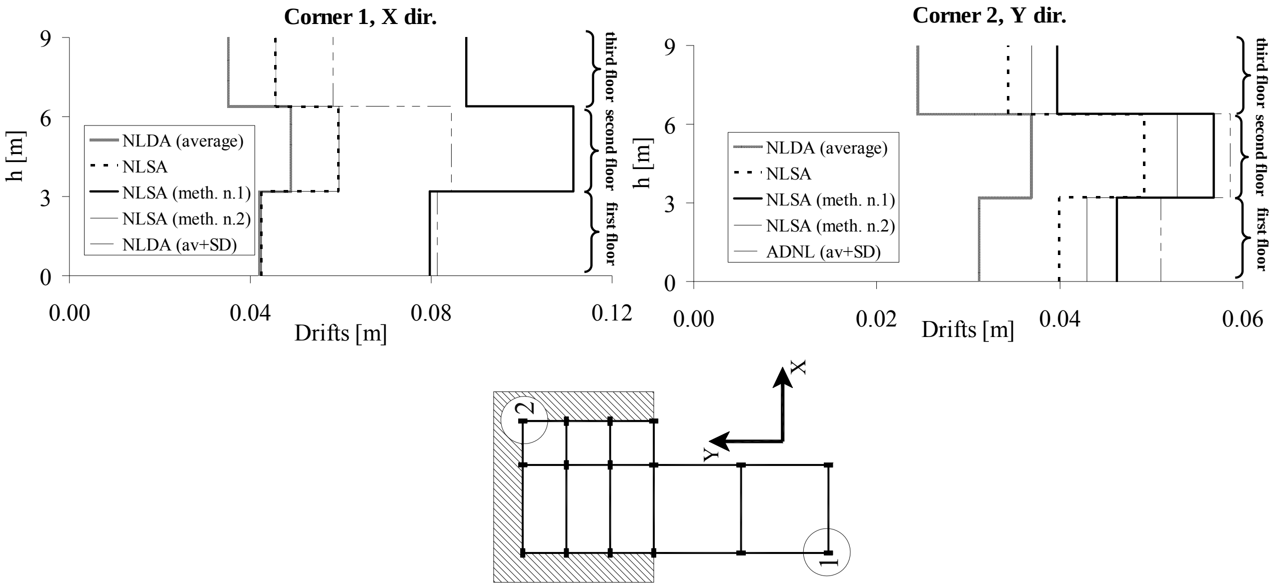

Figure 5 Drifts at corners 1 and 2 along X and Y direction

ACKNOWLEDGMENT

This research has been partially funded by Italian Department of Civil Protection in the frame of the national project ReLUIS – theme 2.

REFERENCES

CEN. (2003). Eurocode 8: Design of structures for earthquake resistance – Part 1: General rules, seismic actions and rules for buildings. Final Draft, prEN 1998-1, Brussels.
CEN. (2004). Eurocode 8: Design of structures for earthquake resistance – Part 3: Assessment and retrofitting for buildings. Draft No 7, prEN 1998-3, Brussels.
CSI Computer & Structures Inc. (2004). SAP2000. Linear and Nonlinear Static and Dynamic Analysis of Three-Dimensional Structures. Computer & Structures, Inc., Berkeley, California.
Fajfar P. (2000). A non linear analysis method for performance-based seismic design. *Earthquake Spectra* **16(3)**: 573-592.
Fajfar P., Marusic D., Perus I. (2006). The N2 method for asymmetric buildings. *First European Conference on Earthquake Engineering and Seismology*. Geneva, Switzerland, 3-8 September.
Iervolino I., Maddaloni G., Cosenza E. (2008). Eurocode 8 compliant real record sets for seismic analysis of structures. *Journal of Earthquake Engineering* **12(1)**, 54-90.
Li KN. CANNY99 (1996). Three-dimensional nonlinear dynamic structural analysis computer program package. Technical and Users' manual.
OPCM 3431(2005). Ulteriori modifiche ed integrazioni all'ordinanza del Presidente del Consiglio dei Ministri n. 3274 del 20 marzo 2003, recante "Primi elementi in materia di criteri generali per la classificazione sismica del territorio nazionale e di normative tecniche per le costruzioni in zona sismica". Rome (in Italian).