HIERARCHY OF DIFFICULTY CONCEPT: COMPARISON BETWEEN LINEAR AND NON LINEAR ANALYSES ACCORDING TO EC8

Gennaro Magliulo[1], Giuseppe Maddaloni[2], Edoardo Cosenza[3]

[1] *Assistant Professor, University of Naples "Federico II", Department of Structural Engineering (DIST), Naples, Italy*
[2] *Assistant Professor, University of Naples "Parthenope", Department of Technology (DiT), Naples, Italy*
[3] *Professor, University of Naples "Federico II", Department of Structural Engineering (DIST), Naples, Italy*
Email: gmagliul@unina.it, giuseppe.maddaloni@uniparthenope.it, cosenza@unina.it

ABSTRACT:

The paper deals with the topic of analyses performed according to modern code provisions, in particular Eurocode 8 (EC8) rules. Elastic, non linear static and non linear dynamic analyses of a multi-storey r/c frame building designed according to Eurocode 2 (EC2) and EC8 provisions are carried out. A suite of 7 earthquakes (each with both the horizontal components), fully satisfying the EC8 provisions, are used as input of non linear dynamic analyses; this allows to rigorously compare for the same design spectrum the demand computed by non linear dynamic analysis to the one associated to elastic modal response spectrum analysis and to non linear static analysis. Non linear static and dynamic analyses are performed adopting a lumped plasticity model, characterised by a non linear moment rotation relationship with a tri-linear skeleton curve and a Takeda like hysteretic behaviour which also takes into account the pinching effect.
The results show that the 3 methods of analyses provided by EC8 for new buildings respect a correct hierarchy in terms of safety, i.e. simpler is the analysis safer is the result returned (hierarchy of difficulty): the building designed according to elastic analysis is verified by both non linear static and dynamic analyses and displacements and total chord rotations demanded by non linear static methodology are larger than those demanded by non linear dynamic one.

KEYWORDS: Hierarchy of difficulty, non linear analysis, natural records, seismic design, Eurocode 8.

1. INTRODUCTION

The modern seismic codes, as EC8 (CEN, 2003) and International Building Code (IBC, 2000), allow the designer to use different analysis methodologies, in particular: 1) lateral force and 2) multi-modal elastic ones and 3) static and 4) dynamic non linear ones. Their level of reliability decreases from n.4) to n.1) and, consequently, the safety margin with respect to the same limit state should increase according to the same order. This has not yet clearly shown, at least in terms of methodologies strictly fulfilling the EC8 provisions.
In the paper the results of elastic, non linear static and non linear dynamic analyses are reported and commented; the analysed structure, a four-storey r/c frame building, completely fulfils the EC8 design provisions, along with the adopted suit of accelerograms (Iervolino et al., 2008).
The results shown herein are, for sake of brevity, limited to a single structure, but they are confirmed by analyses performed on different buildings.

2. GEOMETRY OF THE BUILDING AND ELASTIC ANALYSIS

The geometry of the building is reported in Figure 1; it is a four-storey r/c frame building. The bottom interstorey height is equal to 4 m, while at the other levels it is equal to 3.2 m; at the first storey the dimensions of the sections of all the columns are 40x65 cm^2, of all the beams are 40x60 cm^2, at the second storey such dimensions are respectively 40x60 cm^2 and 40x55 cm^2, at the third 40x55 cm^2 and 40x50 cm^2, while at the top level they are 40x50 cm^2 both for column and beam sections. A low variation of the element dimensions between adjacent floors is assigned in order to favour the structure vertical regularity, while column dimensions are kept larger than beam ones in order to take into account the capacity design. The stairs are sustained by non horizontal beams, unless at the first flight, which is characterised by a slab sustained by a r/c wall on one side and by the beam at middle height of the first storey on the other side, in order to separate them by the structure and to avoid concentrations of shear at the bottom half storey.

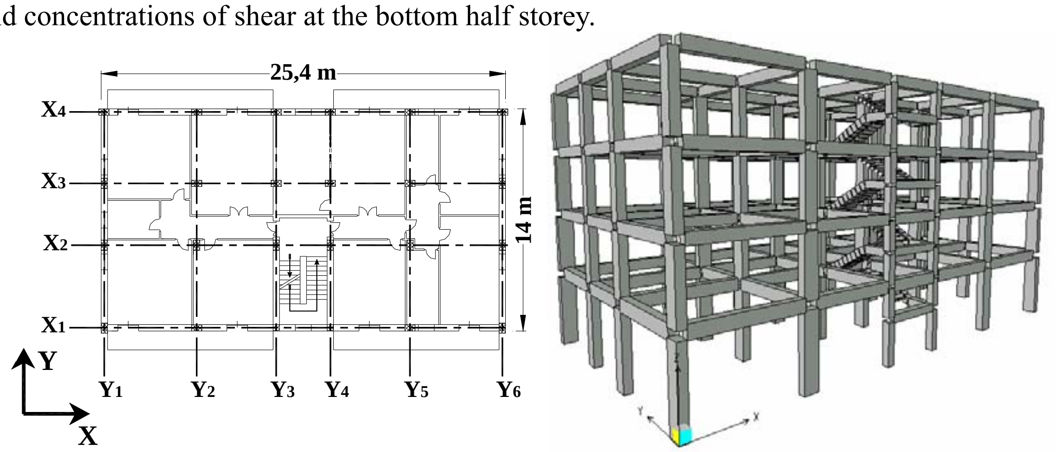

Figure 1 Geometry of the analysed building

Beams and columns are modelled as massless one dimension finite elements; the mass is concentrated at the floor levels, assumed rigid in their own planes, consequently the structure is characterised by 3 DOFs for each floor. From the 1st to the 4th floor the masses are respectively 419 t, 396 t, 385 t and 361 t, while the polar moment of inertia is computed considering a uniform mass distribution on the floor area, equal to 16.4x25.4 m^2. As imposed by the code, a 5% accidental eccentricity is considered; consequently 4 models are analysed with the centre of mass placed in 4 different positions, indicated in the following with respect to the initial position: "dx" @(1.25,0), "sx" @(-1.25,0), "sup" @(0,0.68), "inf" @(0,-0.68).
In order to take the cracking into account, the beam stiffness is assigned to be one-half of the corresponding uncracked beam one; the column stiffness is not reduced. The periods of the first 3 modes of such model are: 0.571 sec (X dir.), 0.526 sec (Y dir.) and 0.495 sec (rot.), while the periods of the first 3 modes of the uncracked structure are: 0.485 sec (X dir.), 0.462 sec (Y dir.) and 0.417 sec (rot.).
The building is designed according to Eurocodes 0 (CEN, 2002a), 1 (CEN, 2002b), 2 (CEN, 2004a) and 8 (CEN,

2003), by modal response spectrum analysis. A design spectrum soil B type 1 with a design ground acceleration on type A ground, a_g =0.35g, taken from the Italian new seismic code (OPCM 3431, 2003), is considered. The design is performed according to the High Ductility Class rules; a behaviour factor equal to 5.85 is computed, also considering that the frame is regular in elevation.
Concrete characteristic cubic strength equal to f_{ck}=30 N/mm^2 and steel characteristic yielding strength equal to f_{yk}=430 N/mm^{2}are adopted.
The orthogonal effects are considered by the 30% rule: 8 combinations for each of the 4 models are computed.
It is to be noted that the dimensions of the frame elements above reported are assigned also in order to satisfy the damage limitation requirement, in particular the limitation for buildings having non structural elements of brittle materials attached to the structure.
As provided by the code, all nodes of the frame structure, unless the top floor ones and those at foundations, satisfy the design condition:

$$\sum M_{Rc} \geq 1.3 \cdot \sum M_{Rb} \tag{2.1}$$

where ΣM_{Rc} and ΣM_{Rb} are the sum of design values of moments of resistance of respectively the columns and the beams framing the joint; this is satisfied along both the orthogonal directions and considering both the signs of seismic action.
In order to satisfy such condition, the new Italian Seismic Code (OPCM 3431, 2003) imposes to multiply the design value of the column bending moment at the node by a coefficient α computed as:

$$\alpha = \gamma_{Rd} \cdot \frac{\sum M_{Rb}}{\sum M_{Sc}} \tag{2.2}$$

where γ_{Rd} =1.3 is the coefficient of equation (2.1) and ΣM_{Sc} is the sum of the column design bending moments at the top and at the bottom of the node. The application of the beam-column capacity design by the coefficient (2.2) can lead to overestimate the column moment of resistance if it is computed for all the design combinations; indeed some combinations, even though characterised by a low column bending moment at the node, can give an high value of coefficient α.

3. NON LINEAR MODEL

Non linear analyses are performed by means of 2 computer programs, SAP2000 (CSI, 2004) and CANNY99 (Li, 1996). Non linearity regards flexural rotations, while all the other deformations are assumed linear. Both beams and columns are characterised by lumped plasticity models; in the latter case for each section two independent non linear springs are assigned, one for each orthogonal direction. No axial force-bending moment interaction is considered at the plastic hinge.
Bending moment springs are characterised by tri-linear skeleton curve, defined by cracking and yielding moment and corresponding rotations; the post-yielding rotation is assumed equal to zero. Such moments and the corresponding curvatures are computed considering a parabola-rectangle diagram for concrete under compression, characterised by maximum and ultimate strength equal to the medium value for concrete 30/37 according to EC2, i.e. 38 N/mm^2; a strain value at the end of the parabola equal to 0.2% and an ultimate strain equal to 0.35% are assigned. The concrete Young modulus and its maximum tensile strength are also computed according to EC2. An elastic-perfectly plastic steel stress-strain diagram is considered, characterised by a maximum strength equal to 530 N/mm^2, computed as mean of tests on more than 200 bars made by steel called FeB44K performed at the laboratory of Department of “Scienza delle Costruzioni” of University of Naples Federico II. A steel maximum strain equal to 1% and a Young modulus equal to 200000 N/mm^2 are assigned.
The cracking rotation is computed multiplying the corresponding curvature by L_v/3, where L_v is the shear span, i.e. the bending moment / shear ratio, evaluated at the end section where the plastic hinge is located; such internal forces result from considering the building loaded by horizontal forces, whose shape is computed according to EC8 (CEN, 2003) equivalent lateral force procedure taking into account the vertical regularity of the structure. The yielding and the ultimate rotations are evaluated as provided by EC8 (CEN, 2004b) equations (A.10b) and (A.1) respectively, where the already cited medium values are assigned to concrete maximum (f_c) and steel yielding (f_y) strength.
The hysteretic model is Takeda type, even though in CANNY99 the pinching effect is also taken into account.

4. NON LINEAR STATIC ANALYSIS

Non linear static analyses are performed according to EC8, considering a force distribution proportional for each of the 2 orthogonal directions to first modal shape in the relative direction by mass distribution. The first 2 modes of the building are translational and the square root of the ratio of torsional stiffness to lateral stiffness in each of the 2 orthogonal directions is larger than the radius of gyration of floor mass in plan (see equation (4.1b) in EC8 (CEN, 2003)); consequently the structure is torsionally stiff and the application of a particular procedure for the estimation of the torsional effects is not necessary.
Eight push over are performed, 2 opposite signs for 4 different positions of the centre of mass.
In Figure 2 the results of 4 push over curves, obtained considering the positive sign of the applied distribution force, in terms of adimensionalised top displacement vs base shear are shown.

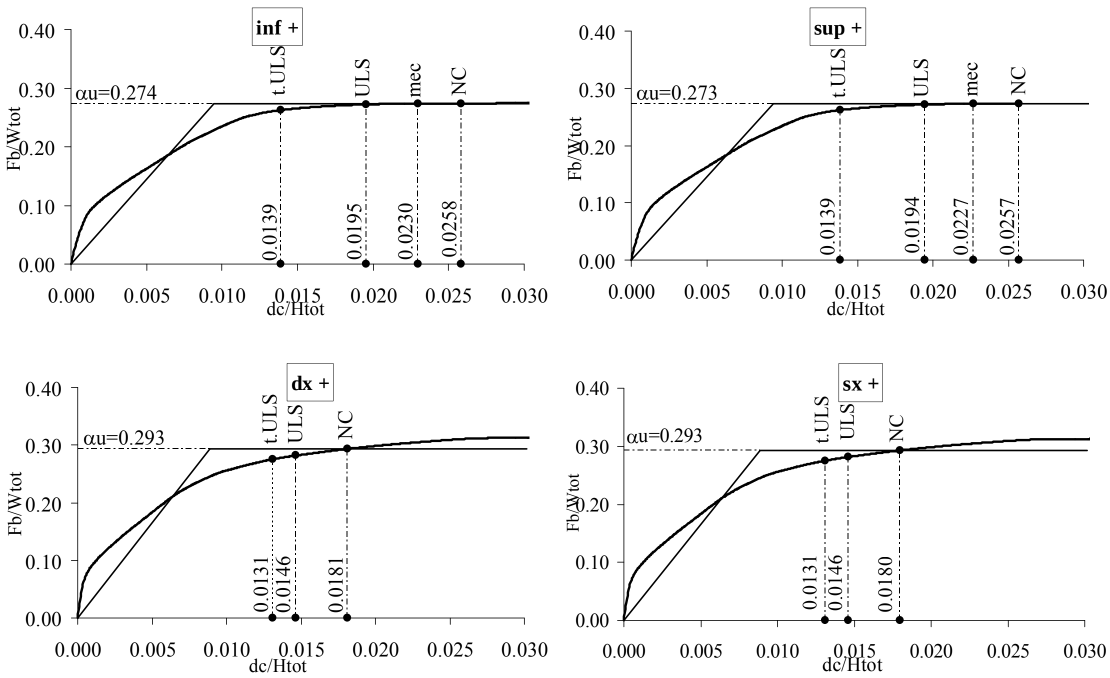

Figure 2 Capacity curves of the 4 considered models

The bi-linear capacity curve is also presented along with capacity and demand points: “mec” indicates the mechanism of the structure, “ULS” (Ultimate Limit State) and “NC” (Near Collapse limit state) the attainment in at least one hinge of the rotation value 3/4 Θ_u and Θ_u respectively and “t.ULS” the demand corresponding to the elastic design acceleration spectrum soil B type 1, a_g =0.35g; Θ_u is the total chord rotation capacity computed according to EC8 empirical formula (A.1) (CEN, 2004b). The capacity curves computed forcing the structure along Y direction show a certain slope and the Near Collapse limit state precedes the mechanism point, evidencing a less ductile behaviour with respect to the orthogonal direction; this is due to the presence along such direction of the inclined beams supporting the stairs. The Ultimate Limit State is always verified, i.e. the demand is lower than the capacity.
According to EC8 provisions, the orthogonal effects of demand parameters evaluated at target points are computed by the Square Root of Sum of Squares rule (SRSS) resulting in 16 combinations; the maximum values obtained by such combinations, considering the results at “t.ULS”, are compared to results of elastic and non linear dynamic analyses. The results are provided in terms of top centre of mass displacements and demand / capacity ratio of total chord rotation computed at beam and column ends.

5. STEP BY STEP NON LINEAR DYNAMIC ANALYSIS

Both the horizontal components of a set of 7 earthquakes (Tab. 1), i.e. 14 natural records, are used for non linear dynamic analyses whose results are shown herein; according to the selection procedure presented in (Iervolino et al., 2008), they satisfy the EC8 provisions: the mean of zero period spectral response acceleration values (calculated from individual time histories) should not be smaller than the value of $a_g \cdot S$ for the site in question; in the range of periods between $0.2T_1$ and $2T_1$, where T_1 is the fundamental period of the structure, in the direction where the accelerogram is applied, no value of the mean 5% damping elastic spectrum, calculated from all time histories, should be less than 90% of the corresponding value of the 5% damping elastic response spectrum; if the response is obtained from at least 7 non linear time histories analyses, the average of response quantities should be used as the design value of the action effect E_d in relevant verifications.

Table 1 Earthquakes used for non linear dynamic analyses

Earthquake code	Earthquake name	Earthquake Country	Date
000187	Northern and central Iran	Iran	16/09/1978
000196	Montenegro	Yugoslavia	15/04/1979
000199	Montenegro	Yugoslavia	15/04/1979
000230	Montenegro (aftershock)	Yugoslavia	24/05/1979
000291	Campano lucano	Italy	23/11/1980
006263	South Iceland	Iceland	17/06/2000
006334	South Iceland (aftershock)	Iceland	21/06/2000

Figure 3 shows the elastic spectrum of the 14 records used, along with the average spectrum (black smooth line), the EC8 elastic design spectrum (black thick line) and the curve whose ordinates are equal to 90% of such spectrum ones (black thin line); SF indicates the value of factors used to scale the records for the purpose to matching code spectrum: only one record is scaled by a low SF equal to 1.08. It is to be noted that the matched code spectrum is the one used for the design of the analysed building, assigning q=1.
According to the direction of registration of the accelerograms, a direction, X or Y, is given to the two components of each earthquake; the analyses are performed applying, for each earthquake, the X component along the longitudinal direction of the building and the Y component along the orthogonal direction.
As for non linear static analyses top centre of mass displacements and demand / capacity ratio in terms of total chord rotation at beam and column ends are shown. For each of the 4 models obtained moving the centre of mass, the average of the 7 maximum results obtained applying the 7 earthquakes is considered along with the average plus the standard deviation and the maximum of the 7 maximum results; the maximum effect among the ones of the 4 models is always considered.

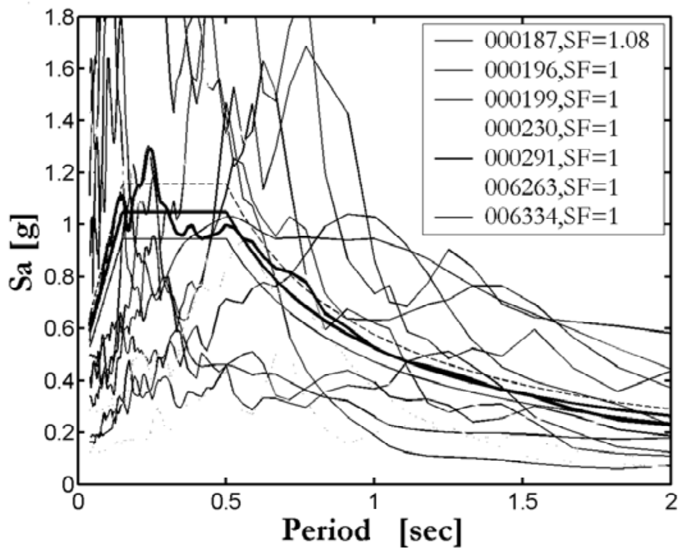

Figure 3 Spectra of the records used for non linear analyses, their average and EC8 spectrum

6. COMPARISONS BETWEEN THE ANALYSIS RESULTS

Non linear dynamic analyses are compared to non linear static ones in terms of maximum displacement and rotational ductility demand and such demand is compared to capacity at the Ultimate Limit State, which correspond to the attainment of the total chord rotation equal to 3/4 Θ_u, as already written.
In Table 2, X and Y top centre of mass displacements are listed: NLSA indicates the maximum values obtained by the 16 SRSS combinations of the results of non linear static analyses at "t.ULS", i.e. the demand corresponding to the elastic design acceleration spectrum soil B type 1, a_g =0.35g; NLDA(average) means that the average among the maximum results of the 7 non linear dynamic analyses is considered and the maximum among the 4 models is reported; as NLDA(av+SD) the average plus the standard deviation instead of the only average of the 7 results for each model is considered, while as NLDA(maximum) the maximum among the maximum results of the 7 non linear dynamic analyses is taken and, as in the other 2 cases, the maximum of the 4 models is presented. Table 2 shows that the displacements obtained by non linear dynamic analyses, averaged, according to EC8, on 7 earthquakes (NLDA(average)), are lower than the ones obtained by non linear static analyses (NLSA). This does not happen when the average plus the standard deviation is considered (NLDA(av+SD)) and, with a large scatter, when the maximum of the 7 maximum displacements is computed (NLDA(maximum)).

Table 2 Top centre of mass X and Y direction displacements

Analysis	U_X [m]	U_Y [m]
NLSA	0.189	0.180
NLDA(average)	0.141	0.137
NLDA(av+SD)	0.247	0.216
NLDA(maximum)	0.345	0.272

In Figures 4 demand/capacity ratios in terms of total chord rotations at beam and column (italic numbers) end sections of frames X1 are presented; for sake of brevity the other frames are not shown, but, obviously, the reported conclusions concern the whole building. NLSA and NLDA(average) ratios are always lower than one; consequently it is confirmed that the building designed according to EC8 provisions by elastic modal response spectrum analysis at the Ultimate Limit State is verified at the same limit state according to both non linear static and dynamic analysis performed according to EC8. Furthermore, also in terms of ductility demand, as already noted in terms of displacements, NLDA(average) provides lower values with respect to NLSA; consequently results show that safety levels associated to different analyses performed according to EC8 are correlated to their accuracy, i.e. the design of elastic analysis, the simplest, is more conservative with respect to safety verification made by non linear static analysis, which is more conservative with respect to non linear dynamic analysis, which is the most complicated.
On the contrary, in the case of NLDA(av+SD), few sections, which do not belong to the two presented frames, show a ductility demand in terms of total chord rotation which overpass the capacity; this happens at many sections if results given by NLDA(maximum) are observed. The variation is obviously due to the variability of the intensity of the seismic input: the variation coefficient, i.e. the standard deviation/mean ratio, of ductility demand averaged on all element ends is equal to 0.665, while its maximum value is 2.35. Lower values can be obtained considering the maximum variation coefficients of the top centre of mass displacement among the 4 models: 0.76 along X direction, 0.60 along Y direction. An evident consequence of the large standard deviation is, as already noted, that the evaluation of the safety level on the base of maximum of the maximum effect of each of the 7 earthquakes can lead to opposite conclusions with respect to considering the average of maximum effects.

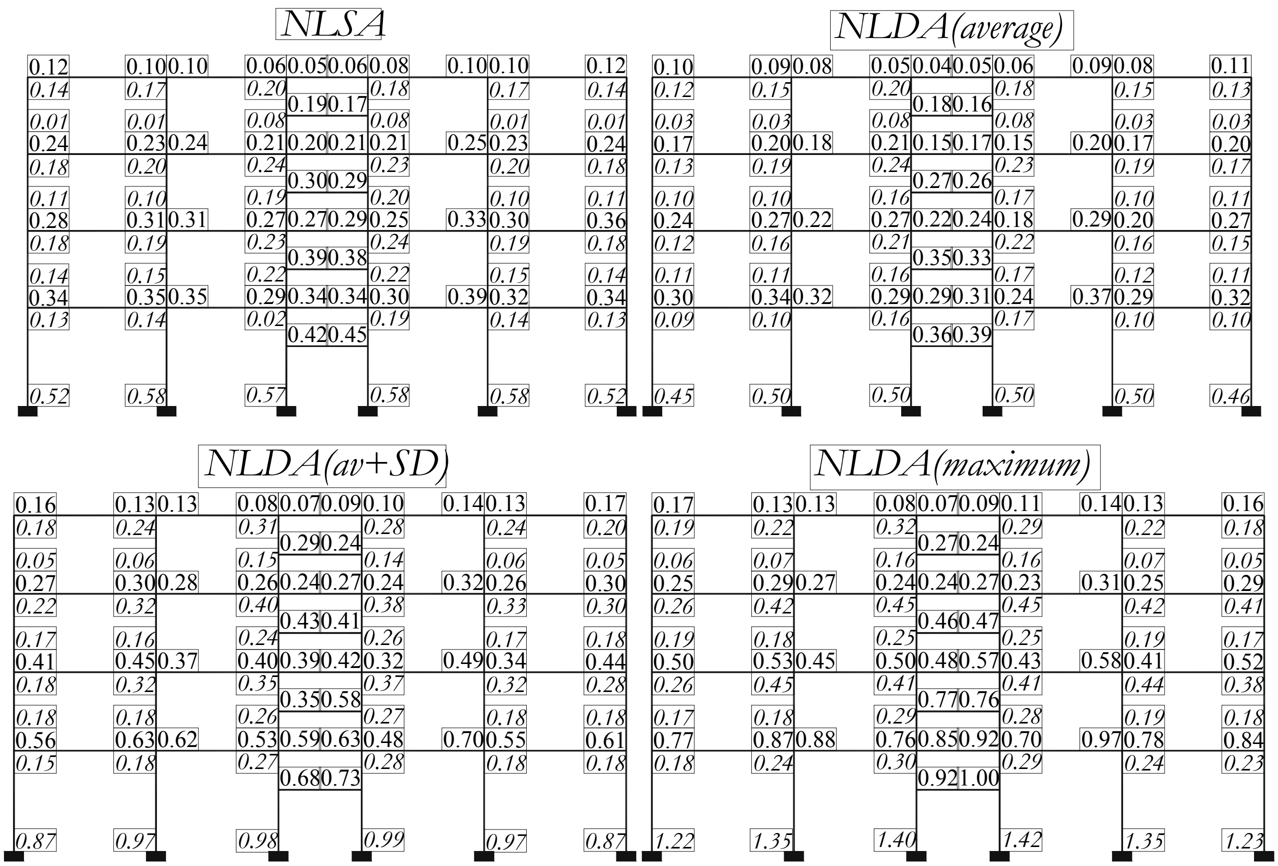

Figure 4 Frame X1: demand/capacity ratio in terms of maximum rotations at element ends

7. CONCLUSIONS

A 4-storey, r/c frame building is designed according to EC8 provisions by modal response spectrum analysis considering soil B type 1 with a design ground acceleration on type A ground, a_g =0.35g. Non linear static and dynamic analyses are performed adopting a lumped plasticity model, characterised by a non linear moment rotation relationship with a tri-linear skeleton curve and a Takeda like hysteretic behaviour which also takes into account the pinching effect. The input adopted for non linear dynamic analysis is selected in order to rigorously satisfy the EC8 provisions ; this allows to rigorously compare for the same design spectrum the demand computed by non linear dynamic analysis to the one associated to elastic modal response spectrum analysis and to non linear static analysis.

A large amount of results is obtained; herein, for sake of brevity, they are reported only in terms of top centre of mass displacement and rotational ductility at the element ends. The 5% accidental eccentricity is considered, consequently 4 models are analysed. 28 non linear dynamic analyses (7x4) are performed and the maximum effect among the 4 models is considered; for each model the average of 7 maxima obtained from 7 earthquakes (NLDA(average)), the average plus standard deviation (NLDA(av+SD)) and the maximum of the maxima (NDLA(maximum)) are computed. The results of elastic analysis are the maximum among the 32 combinations due to 8 combinations (the 30% rule is adopted) by 4 models, while the ones of non linear static analysis (NLSA) are the maximum at the Ultimate Limit State of 16 results due to the SRSS rule (4 models by 4 different combinations).

All the results show that at Ultimate Limit State NLDA(average) provides lower effects with respect to NLSA and for both of them the demand is lower than the capacity provided by the code, corresponding to the attainment at element ends of 3/4 Θ_u, where Θ_u is the ultimate chord rotation; this demonstrates that safety level associated to different analyses performed according to EC8 is correlated to their accuracy (hierarchy of difficulty), i.e. the design of elastic analysis, the simplest, is more conservative with respect to safety verification made by non linear static analysis, which is more conservative with respect to non linear dynamic

analysis, which is the most complicated.
On the contrary, at few sections in the case of NLDA(av+SD) and at many sections in the case of NLDA(maximum) the Ultimate Limit State is not verified, i.e. the demand in terms of maximum chord rotation overpasses the capacity. This is due to quite large variation of results due to different accelerograms, also shown by standard deviation/mean ratio of results. Consequently it can be stated that the evaluation of the safety level on the base of maximum of the maximum effect of each of the 7 earthquakes can lead to opposite conclusions with respect to considering the average of the maximum effects.
The results shown in this paper are confirmed by analyses performed on other buildings, which, for sake of brevity, are not reported herein.

ACKNOWLEDGMENT

This research has been partially funded by Italian Department of Civil Protection in the frame of the national project ReLUIS – theme 2.

REFERENCES

CEN. (2002). Eurocode 0: Basis of structural design. EN 1990, Brussels
CEN. (2002). Eurocode 1: Actions on structures – Part 1-1: General actions- Densities, self weight, imposed loads for buildings. EN 1991-1-1, Brussels.
CEN. (2004). Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings. EN 1992-1-1, Brussels.
CEN. (2003). Eurocode 8: Design of structures for earthquake resistance – Part 1: General rules, seismic actions and rules for buildings. Final Draft, prEN 1998-1, Brussels.
CEN. (2004). Eurocode 8: Design of structures for earthquake resistance – Part 3: Assessment and retrofitting for buildings. Draft No 7, prEN 1998-3, Brussels
CSI Computer & Structures Inc. (2004). SAP2000. Linear and Nonlinear Static and Dynamic Analysis of Three-Dimensional Structures. Computer & Structures, Inc., Berkeley, California.
Iervolino I., Maddaloni G., Cosenza E. (2008). Eurocode 8 compliant real record sets for seismic analysis of structures. *Journal of Earthquake Engineering* **12(1)**, 54-90.
International Building Code (2000). International Code Council, Falls Church, Virginia.
Li KN. CANNY99 (1996). Three-dimensional nonlinear dynamic structural analysis computer program package. Technical and Users' manual.
OPCM 3431(2005). Ulteriori modifiche ed integrazioni all'ordinanza del Presidente del Consiglio dei Ministri n. 3274 del 20 marzo 2003, recante "Primi elementi in materia di criteri generali per la classificazione sismica del territorio nazionale e di normative tecniche per le costruzioni in zona sismica". Rome (in Italian).