FORCE-DISPLACEMENT RELATIONSHIPS FOR R/C MEMBERS IN SEISMIC DESIGN

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ABSTRACT :

For a long time, the structural model of r/c framed structures has been defined assigning to each member a conventional stiffness, related to the properties of the geometrical cross-section. The more recent seismic codes suggest to evaluate stiffness taking into account the effect of cracking but do not provide operative indication on how do it. The paper proposes a fibre model able to describe the behaviour of a concrete member, starting from the moment-curvature analysis of reinforced concrete sections and taking into account different aspect (as tension shift due to shear force, anchorage deformation and plastic hinge width). By means of it, more reliable force-displacements relationships can be evaluated for beams and columns. The obtained curves can be used to define the equivalent elastic secant flexural stiffness to be adopted in design computer programs to describe the elastic behaviour of cracked members as a function of geometry of the section, reinforcement and axial force.

KEYWORDS: Seismic design, r/c structures, stiffness, cracked section, moment-curvature

1. INTRODUCTION

It is well known that the deformation of r/c members is non linearly related to internal actions and to the amount of reinforcement, because of cracking and non linear behaviour of materials. In spite of this, the analysis of r/c framed structures is usually performed assigning to each member a conventional, constant stiffness, evaluated by the geometrical cross-section properties. This approach was necessary in order to reduce computational effort and to avoid the necessity of iterative analyses (stiffness depends on internal actions and reinforcement – reinforcement depends on internal action values – internal actions depend on stiffness). Only in the last decades some researchers [e.g., Paulay, 1997] started to proclaim the necessity of sweeping this simplification and assigning more reliable values to stiffness. Most recent seismic codes acknowledge it and recommend to evaluate stiffness taking into account the effect of cracking, but their provisions need to be critically examined. The suggestion given by Eurocode 8 ("stiffness may be taken to be equal to one-half of the corresponding stiffness of the un-cracked elements") appears to be as conventional as the traditional approach. Furthermore, its effect appears questionable, because it leads to reduce seismic forces and, consequently, internal actions in all the members. Slightly different is FEMA 356 approach: the proposed reduced value – with respect to the un-cracked element stiffness – is equal to one-half for beams and for columns under tension or with compression due to design gravity loads smaller than 30% of the resistance provided by concrete section; it is equal to 70% for columns with a compression larger than 50%. This leads once again to a reduction of seismic forces but at the same time it increases internal actions in columns with respect to beams, thus helping in respecting capacity design criterion. Nevertheless also these provisions appear to be oversimplified and unable to describe the actual behaviour of r/c members.
It is opinion of the authors that the use of "exact" stiffness might be not extremely relevant in the case of new buildings designed by force-based approach. Indeed, the over or under-estimation of the stiffness of a single member lead respectively to larger or smaller values of internal actions and, correspondingly, of reinforcement, thus contributing to increase or reduce stiffness and therefore partially balancing the error committed. More relevant might be its effect in the case of existing buildings designed for vertical loads only, in which reinforcement is disposed without any account for seismic actions.

In any case, the introduction of new criteria for assigning member stiffness should be based on more reliable and acknowledged studies. Up to know, many researchers discussed this problem, pointing out different aspects to be taken into account for a proper evaluation of member stiffness [e.g., Priestley et al., 2007]. Nevertheless, their aim was to present a general view of the problem and to show the influence of some parameters by means of specific and simplified examples. The research group of the Authors aims at developing a model able to thoroughly describe the behaviour of a concrete member, which will be by itself a tool for designers who need to assign more correct values to member stiffness. An extended, parametric use of the model might also lead to simplified formulations able to define the value of member stiffness as a function of geometrical characteristics (size of section, reinforcement), material properties, loading type and level (value of axial force, value of bending moment expected), kind of analysis (elastic, non linear). The paper presents the general approach followed in developing the model, with a related computer program, and some preliminary results obtained.

2. APPROACH TO THE EVALUATION OF R/C MEMBER BEHAVIOUR

The simplest way for taking into account the effect of cracking is to use an elastic model for materials, with a null concrete tension strength. In the case of pure bending it gives linear moment-curvature and moment-rotation relationships, because the neutral axis depth remains constant as the bending moment increases. When a constant axial force is present and bending moment increases, the above cited relationships are linear in the first stage, when the cross-section is fully in compression (or in tension, depending on the sign of axial force), and then become nonlinear because of the progressive variation of the compressed part of the section. This approach is obviously over-simplified. Indeed, it neglects concrete tension strength, which exists before cracking, tension stiffening effects, present in the early stages after cracking, nonlinear behaviour of materials, at higher values of strain, and many other aspects which will be discussed later on. Nevertheless it should be considered an interesting reference point for more exact models, provided that a proper value of homogenisation factor $n = E_s/E_c$ is used (the conventional value $n = 15$ accounts for viscous effects which are important for dead loads but must not be included when seismic response is analysed).
The starting point for a thorough analysis of the behaviour of a r/c member is the definition of stress-strain relationships for concrete and steel (section 3). When dead loads only are considered, they should include concrete tension strength, tension stiffening and viscous effects. When seismic effects only are considered, as in the model developed by the Authors, the previous aspects must be swept, while it assumes particular relevance the difference between cover (unconfined concrete) and core (confined concrete). Next stages of the research might deal with the opportunity, or not, of considering two different models, one for the application of dead loads and another one for the subsequent application of seismic effect to the structure already deformed.
Stress-strain relationships are used in a fibre model of the cross-section, in order to obtain moment-curvature relationship. This is performed by means of a standard procedure, which is based on traditional assumptions: Navier-Bernoulli plane-section hypothesis and perfect bond between steel and concrete.

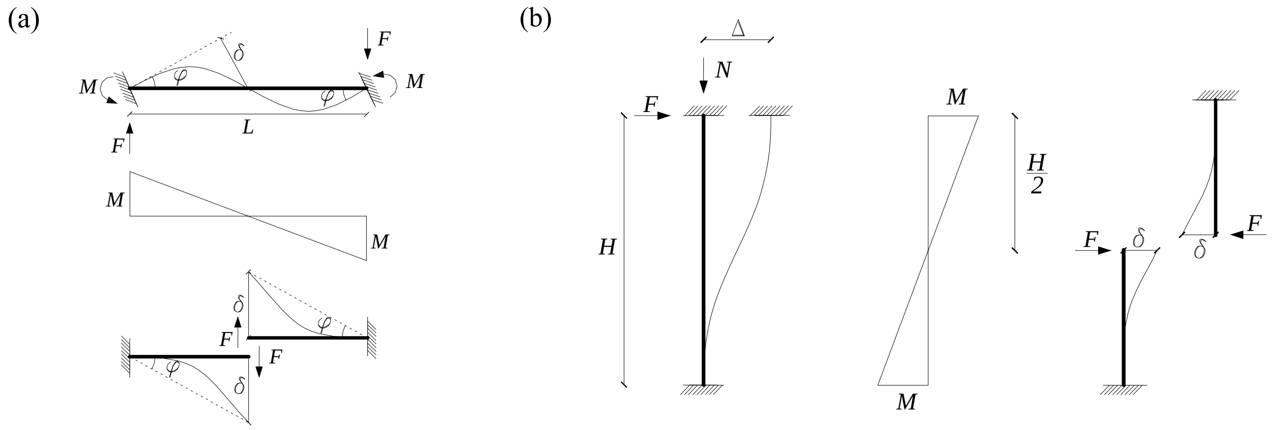

Figure 1 Basic models: (a) beam elements; (b) column elements

The analysis of a member (beam or column) may be easily performed by dividing it into a large number of small elements, evaluating internal actions in each element and its subsequent deformation, integrating the obtained curvatures along the member. The model developed by the Authors includes tension shift effects, anchorage deformations and plastic hinge length. In general, any moment distribution might be considered. The performed analyses are referred to two schemes, typical for framed structures under horizontal forces: beam elements with equal rotation at both ends and column elements with relative displacements and null rotation of the ends. Both cases may be conducted to the analysis of a simpler scheme, a cantilever subjected to a force *F* applied at the free end (Fig. 1).

3. STRESS-STRAIN RELATIONSHIPS

The behaviour of steel bars is usually schematised by an elastic–perfectly plastic relationship. The actual ultimate strain of steel is really high, usually about 0.10 or more, but excessive strains may lead to slip between longitudinal reinforcement and concrete; furthermore, and even more important, highly strained bars become susceptible to buckling under reversed loading. It is thus necessary to limit steel strain to a conventional value ε_{su}, depending on material properties and also on longitudinal spacing of transverse reinforcement. In the developed computer program this limit is considered a variable which may be assigned by the user. In the examples, the value $\varepsilon_{su} = 0.04$ is used, according to Eurocode suggestions. In next stages of the research, a more refined model which includes strain-hardening will be considered.
As regards concrete, several stress-strain relationships have been proposed to describe the behaviour of unconfined and confined concrete (Fig. 2). The models proposed by Sargin [1971], Park et al. [1982], Mander et al. [1988] have been included in the computer program. The tensile strength of concrete is neglected. The ultimate compressive strain ε_{cu} is assumed equal to 0.0035 for unconfined concrete; it is evaluated by means of equation 3.1 [Priestley et al., 2007] in the case of confined concrete.

$$\varepsilon_{cu} = 0.004 + 1.4 \frac{\rho_v \, f_{yh} \, \varepsilon_{shu}}{f'_{cc}} \tag{3.1}$$

In the above relations, ρ_v is the volumetric confinement ratio, f_{yh} and ε_{shu} the yield stress and the ultimate strain of the hoops (equal to 0.10) and f'_{cc} the peak stress of the confined concrete.

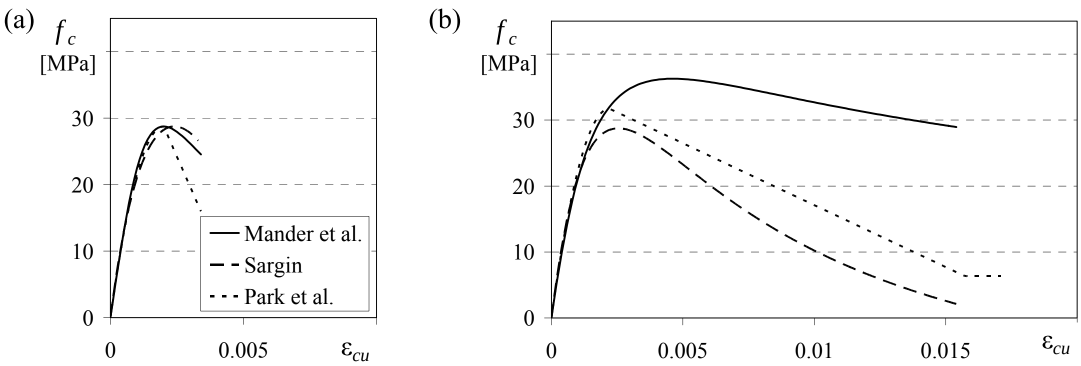

Figure 2 Stress-strain relationships: (a) unconfined concrete; (b) confined concrete

The unloading and reloading response of concrete is also taken into account by means of a linear path extending from the unloading strain to the plastic offset strain; this one is evaluated according to Palermo and Vecchio [2003].
Finally, the average value of material strength is adopted in all the stress-strain models instead of the usual design value, i.e. characteristic value divided by a reduction factor. This choice is considered important for a better understanding of actual nonlinear behaviour of members. It does not significantly affect flexural strength of beams, which mainly depends on steel strength (less variable, because of the quality of industrial process of production). On the contrary, it influences *M-N* column strength in a really relevant way; e.g., an axial force,

which seems excessive when design value of compressive concrete strength is used, becomes scarcely relevant with respect to average compressive strength (which is more than double than design strength). The Authors believe that this aspect needs great consideration when comparing elastic design behaviour to nonlinear response of a structure.

4. MOMENT-CURVATURE RELATIONSHIP

In order to describe the moment curvature relationship for r/c rectangular sections, a fibre model has been implemented. The section is considered as constituted by two regions: the core (confined concrete) and the cover (unconfined concrete). Having assigned neutral axis slope, each concrete fibre, either belonging to core or cover, is parallel to the neutral axis and is defined by the area $A_{c,i}$ and by the coordinate of its geometrical centre with respect to the reference axes *x'* (parallel to neutral axis) and *y'*. The origin G of the reference coordinate system is coincident to the geometrical centre of section. Steel bars too are described by the area $A_{s,i}$ of cross-section and by their coordinate (see Fig. 3).
For a given curvature χ, assuming a tentative neutral axis location, the corresponding linear distribution of strain is evaluated. The stress in each concrete fibre and steel bar are calculated by the proper stress-strain relationship, and consequently the corresponding axial force *N* is evaluated:

$$N = \sum_{i=1}^{n_{\text{cov}}} \sigma_{c,i}(\varepsilon_i) A_{c,i} + \sum_{i=1}^{n_{core}} \sigma_{c,i}(\varepsilon_i) A_{c,i} + \sum_{i=1}^{n_s} \sigma_{si}(\varepsilon_i) A_{si} \tag{4.1}$$

where n_{cov}, n_{core} and n_s are the number of fibres in cover, fibres in core and the number of rebars, respectively. Neutral axis location is modified until translational equilibrium is verified (*N* equal to the assigned axial force). The resisting moments corresponding to the assigned value of curvature are then calculated:

$$M_x = \sum_{i=1}^{n_{\text{cov}\,er}} \sigma_{c,i}(\varepsilon_i) \cdot A_{c,i} \cdot y_i + \sum_{i=1}^{n_{core}} \sigma_{c,i}(\varepsilon_i) \cdot A_{c,i} \cdot y_i + \sum_{i=1}^{n_s} \sigma_{s,i}(\varepsilon_i) \cdot A_{s,i} \cdot y_i \tag{4.2a}$$

$$M_y = \sum_{i=1}^{n_{\text{cov}\,er}} \sigma_{c,i}(\varepsilon_i) \cdot A_{c,i} \cdot x_i + \sum_{i=1}^{n_{core}} \sigma_{c,i}(\varepsilon_i) \cdot A_{c,i} \cdot x_i + \sum_{i=1}^{n_s} \sigma_{s,i}(\varepsilon_i) \cdot A_{s,i} \cdot x_i \tag{4.2b}$$

The ultimate curvature is reached when a fibre belonging to core, or a steel bar, attains ultimate strain. Note that the failure of a cover fibre (i.e. the achievement of ultimate strain for unconfined concrete) does not stop curvature increment, but it eliminates the fibre contribution to cross-section strength.
The choice of stress-strain model may significantly influence the moment-curvature relationships, in particular when axial forces are applied (i.e., for columns). As an example, a 30×60 cm rectangular cross-section, with a longitudinal reinforcement ratio equal to 0.0105 and a transverse reinforcement constituted by hoops with a 8 mm diameter and 10 cm spacing, has been considered.

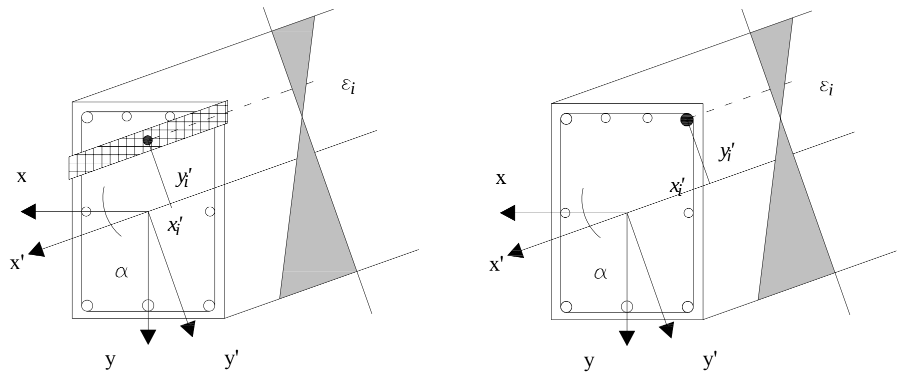

Figure 3 Fibre model

Figure 4a compares the moment-curvature relationships obtained by applying different stress-strain models, under uniaxial bending with an axial compressive force $N = -2000$ kN. All the diagrams have almost the same shape for small curvatures. On the contrary, the softening branch, which starts with the abrupt loss of strength due to the failure of cover fibres, is strongly influenced by the adopted material model. A comparison with experimental values might help in selecting the more proper model for materials. The value of ultimate curvature and the slope of softening branch are strongly influenced by the axial force on the section, as shown in figure 4b which is referred to the above mentioned section, analysed by using Mander's model for concrete.

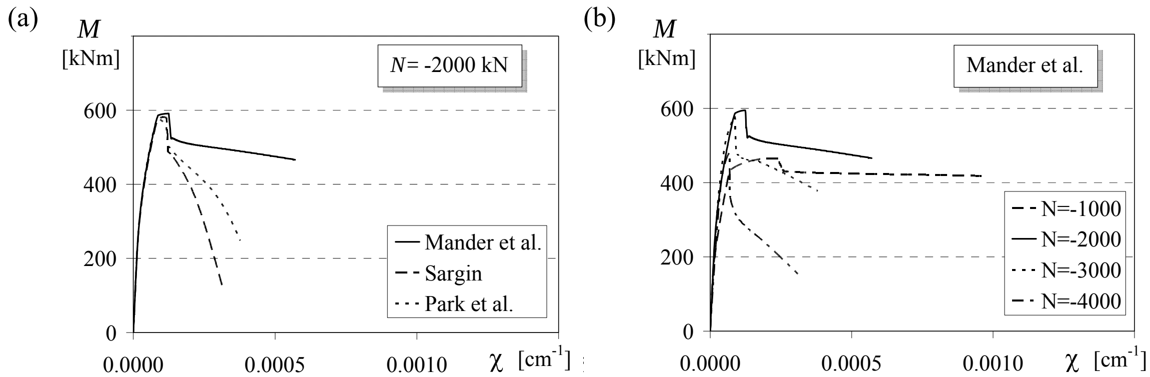

Figure 4 Moment curvature relationships as a function of
(a) constitutive model; (b) axial force on the section

5. FORCE-DISPLACEMENT RELATIONSHIP FOR A CANTILEVER SCHEME

The F-δ analysis is performed on a cantilever subjected to a constant axial force N and an increasing shear force F at the free end (Fig. 5a). The top displacement corresponding to any value of F is calculated by subdividing the member into a finite number of small elements (e.g., one hundred), evaluating the flexural deformation of each element from the M-χ relationship (which may vary along the beam in order to account for rebars distribution) and integrating the deformation all-over the beam length. Although this is a quite standard process, some peculiarities must be pointed out.

As it is well known, the behaviour of a member subjected to shear force and bending moment cannot be analysed with reference to single cross-sections, because the inclination of shear cracks causes stress and strain in longitudinal reinforcement to be larger than what expected by the value of bending moment in the cross-section (the so-called "tension-shift" phenomenon). In the proposed model, this phenomenon is taken into account by evaluating flexural deformation of each element as correspondent to a bending moment larger than the one theoretically acting in the cross-section and obtained by shifting bending moment diagram of the quantity $\Delta z = 0.5\ z \cot\theta$, where z is the internal lever arm and θ is the slope of concrete strut, which may vary in function of the value of shear force (Fig. 5b). A second aspect included in the model is the strain penetration, i.e. the partial pullout of the bars and the gradual dissipation of compressive concrete strains at the base of the cantilever. This is taken into account by considering in the model the flexural deformation of an element having length $L_{SP} = 0.022\ f_y\ d_b$, where d_b is bar diameter [Priestley et. al, 2007], subjected to the maximum bending moment. The loading process is thus performed by gradually increasing F from 0 to the value to $F_{\max} = M_{\max} / l$, where $M_{\max}$ is the maximum moment capacity of the end cross-section and l is the length of the cantilever.

The use of softening constitutive relationships requires to analyse also an unloading phase with increasing top displacement. The problems related to the integration in the case of softening models have been discussed by many researchers. Referring to the theoretical bending moment diagram, the development of inelastic strain along the beam appears to be impossible, because the damage shall occur in a segment of zero length [e.g. see Royer-Carfagni, 2001]. Refined models should take into account the crack spacing and width, as well as the slip of the reinforcing bars [e.g. see Manfredi and Pecce, 1998]. A more common approach is based on the "plastic hinge" concept [Barnard and Johnson, 1965], which assumes damage to be concentrated in a finite region. When maximum moment capacity has been reached, the inelastic and softening deformations occur in this region while adjacent elements follow an elastic unloading branch. Note that the tension-shift phenomenon and the strain penetration already provide a constant-moment zone. In the developed model, when maximum moment capacity is reached the width of this zone is progressively increased up to a value L_P chosen according to references (Fig. 5c). Plastic hinge width may be related to the effective depth *d* of the members cross-section or to the distance from the critical section to the point of contraflexure. In the analyses performed, following this second approach it has been assumed

$$L_P = 0.05\, l + L_{SP} \tag{5. 1}$$

In the unloading phase (Fig. 5d) the elements located within the plastic hinge (part 1) follow the softening branch of the *M*-χ curve, while other elements (part 2) follow an elastic unloading path. This process is conducted by progressively increasing curvature of plastic hinge elements, up to the achievement of the ultimate curvature at the end cross-section; at the same time force *F* decreases from F_{max} to the value $F_u = M_u / l$, where M_u is the bending moment corresponding to the ultimate curvature.

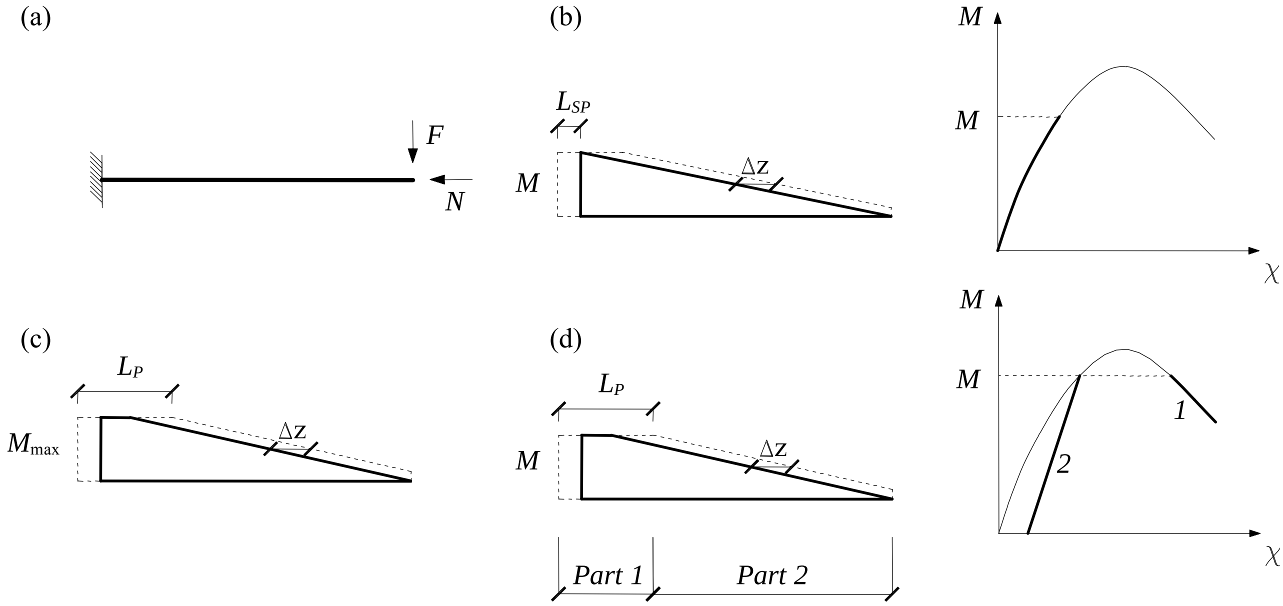

Figure 5 Cantilever model and loading process

6. MOMENT-ROTATION RELATIONSHIP FOR BEAM ELEMENT

Assuming equal rotation φ at both the ends of the beam, the moment-rotation curve for a beam element of length *L* can be easily obtained starting from the *F*- δ curve of the cantilever beam of length *L*/2:

$$M = \frac{L}{2} F\, ; \quad \varphi = \frac{2\delta}{L} \tag{6.1}$$

As an example, figure 6a shows the *M*-φ relationship for a beam with a span length equal to 5.50 m and a rectangular cross-section 30×50 cm, longitudinal reinforcement $A_s = A_s' =$ 6 cm^2 constant along the beam length, hoop diameter and spacing equal to 8 mm and 25 cm respectively, concrete C20/25 and steel B450C (according to European classification of materials). In this case the moment-rotation curve is almost bilinear. Such a behaviour allows linear analysis, performed using secant stiffness *K*, up to the attainment of the yielding moment, or nonlinear analysis with an elastic-perfectly plastic model.

Figure 6b shows the secant stiffness, normalized to the conventional elastic stiffness $K_g = 6E_cI_g/L$ (being I_g the moment of inertia of the gross concrete section about centroidal axis and E_c the modulus of Young of the concrete), as a function of the ratio between bending moment *M* and ultimate moment resistance M_{Rd}. In the analysed scheme the actual stiffness is therefore about 0.20 of the conventional elastic stiffness. Note also that the maximum moment M_{max} is larger than M_{Rd}. Indeed, the *M*-φ relation is obtained with reference to the average value of steel and concrete resistance while M_{Rd} is evaluated with reference to design values; furthermore, the design resistance is obtained without taking into account the confining effect of hoops.

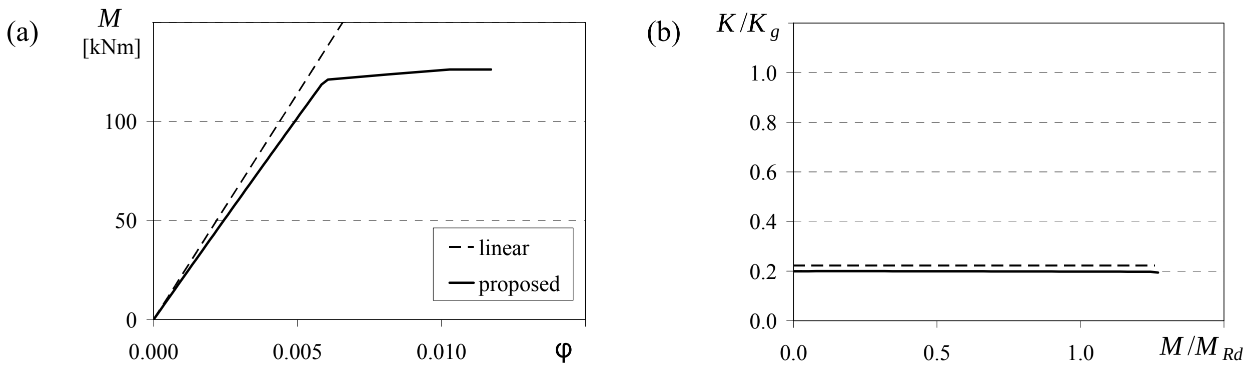

Figure 6 (a) *M*-φ relation for a beam; (b) normalized equivalent secant stiffness

Figures 6a and 6b show also the moment curvature relationship and the stiffness obtained using an elastic model for materials, with a null concrete tension strength. Under these hypotheses, the depth of the neutral axis depends only on the geometry of concrete section and rebars and, consequently, the moment-curvature and moment-rotation relationships are linear. The effective inertia, evaluated with reference to compressed concrete and steel bars homogenised by the ratio $n = E_s/E_c$ between the elastic tangent modulus of steel and concrete, leads to an effective stiffness which is not much grater than the one obtained by the proposed method; the difference mainly depends on the deformation increment due to tension shift.

7. FORCE-DISPLACEMENT RELATIONSHIP FOR COLUMN ELEMENT

Assuming null rotation of the ends, the force displacement curve for a column element of length *h* can be easily obtained starting from the *F*-δ curve of the cantilever beam of length *h*/2 by applying the following relation:

$$\Delta = 2\,\delta \tag{7.1}$$

As an example, figure 7a shows the *F*-Δ relation for a mid-span column of a three storey non seismically designed building, having inter-storey height *h* equal to 3.00 m. The column has rectangular cross-section 30×40 cm, longitudinal reinforcement $A_s = A_s' = 8$ cm^2, hoop diameter and spacing equal to 8 mm and 25 cm respectively, concrete and steel as in the previous example. The compressive axial force on the column is equal to –560 kN. In this case the force-displacement curve is clearly non linear. The use of a secant stiffness *K* is still possible, although less accurate, but its value is strongly dependent on the value of the shear force *F* acting on the structural element, as shown in figure 7b. In this figure, *K* is normalised to the conventional elastic stiffness $K_g = 12E_cI_g/h^3$, while *F* is normalised to the value F_{Rd} corresponding to the reaching of design bending resistance M_{Rd} at the ends of the column. It may be noted that secant stiffness is close to the conventional one for small values of *F* and rapidly decreases for F/F_{Rd} ratios higher than 0.4. Once again, the F/F_{Rd} ratio reaches values larger than 1.0 for the reasons previously explained with reference to beam elements.

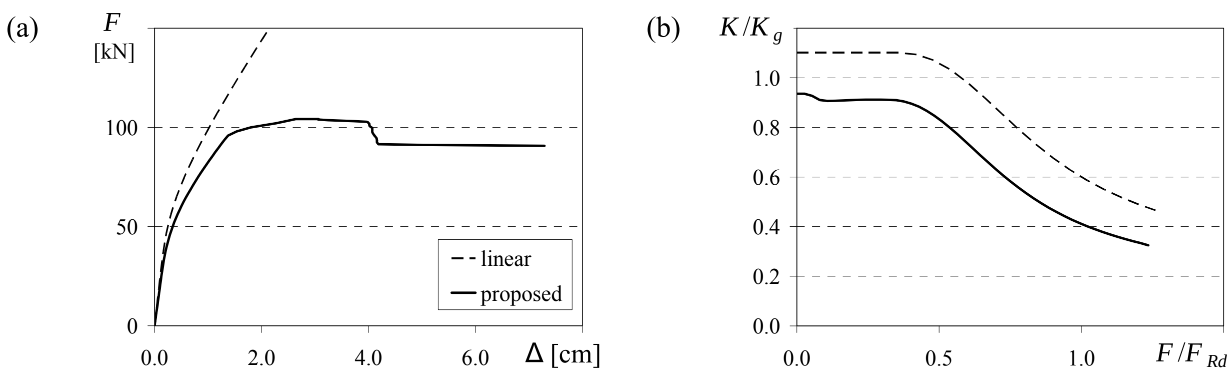

Figure 7 (a) *F*-Δ relation for a column; (b) normalized equivalent secant stiffness

Differently from beam element, the use of an elastic model for materials, with a null concrete tension strength, does not lead to a linear F-δ relationship. Indeed if a constant axial force is present and bending moment increases, the moment-curvature relationship is linear in the first stage, when the cross-section is fully in compression, and then becomes nonlinear because of the progressive variation of the compressed part of the section. Consequently the effective moment of inertia I_{eff} is not constant along the column length. It is however possible to integrate deformation along the column, similarly to the proposed procedure. The obtained F-Δ relationship and the subsequent values of secant stiffness differ from those obtained by the proposed model mainly because of the effect of tension shift.

8. CONCLUSIONS

A comprehensive approach to the structural analysis of r/c structures is very complex and requires tools able to take into account different phenomena as tension shift, bond slip, crack spacing and width etc. The model proposed in the paper might be an useful tool for designers who need to assign more correct values to member stiffness. Furthermore an extended parametric use of the model might also lead to simplified formulations able to define the value of member stiffness as a function of geometrical characteristics, material properties, loading type and level, kind of analysis.

ACKNOWLEDGEMENTS

The Authors wish to thank eng. Tania Di Franco for her important contribution.

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