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A PROCEDURE FOR THE CALIBRATION OF PARTIAL SAFETY FACTORS FOR CAPACITY MODELS

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ABSTRACT:
Worldwide research has now reached a level of integration where an effort towards the harmonization of procedures is absolutely needed. Such harmonization may regard, for example, the various steps that lead to the definition of capacity models to be included in design codes, specifically: definition of the test setup, quantities to be measured, identification of the basic variables influencing the phenomenon, distinction between average values and other fractiles, disaggregation of the model in different parts accounting for mechanics, fine-tuning and randomnesses, and, finally, assessment of the model against the experimental results. Test results and ensuing model developed according to this procedure would naturally lend themselves to be easily shared among the scientific community and would facilitate the task of calibrating the partial coefficients, with the ambitious aim of attaining a uniform reliability level among all capacity equations. This paper proposes a first attempt towards the harmonization of procedures for the development of capacity design equations to be included in codes and the execution of tests. The procedure developed, is applied to the definition of the debonding strength model for FRP, starting from the model proposed in the Italian Instructions CNR DT 200-2004. The procedure has the aim of evaluate the uncertainties of the assumed model (both of the mechanical model and of the basic variables) to obtain a constant level of structural reliability.

KEYWORDS: Design assisted by testing; capacity models; calibration; FRP debonding strength; experimental tests.

1. ASSUMPTIONS

In the following sections the steps to be undertaken for a proper and consistent development of a capacity model, are presented in detail. Inspiration for the procedure came from EN-1990, "EC0 (Eurocode 0). The following three assumptions are made:

- a "sufficient" number of test results is available (this point will be better clarified in the next section);
- all main geometrical and mechanical quantities are measured in the experimental tests carried out to validate the analytical model;
- all random variables are normally distributed.

2. DESCRIPTION OF THE PROCEDURE

2.1. Development of a theoretical strength model

Given a capacity mechanism, an analytical model can be developed based on the *a priori* understanding of the underlying physics of the problem (as opposed to *a posteriori* regression-based models, which are instead based on the outcomes of purposely carried out experimental tests). The model can be given as:

$$r_t = b \cdot g_t\left(\underline{X}\right) \tag{2.1}$$

where: $g_t\left(\underline{X}\right)$ is the capacity model, as function of all basic variables $\underline{X}$ thought to be affecting the phenomenon to be modeled, and b is a least-squares fine-tuning parameter accounting for all other variables (or secondary phenomena) not included in the theoretical mechanical model (e.g., either because they are deemed irrelevant, or because their effect is not perfectly understood, or because they do not fit into the formulation, or because the model is intentionally kept simple).

2.2. Measurement of the basic variables in tests

After having defined a theoretical model, and only at this stage, should it be validated against some experimental results. In the tests, all (geometrical and mechanical) basic variables should be measured for each specimen and should be available for the model's validation. Geometrical quantities are usually easily measured. For as regards mechanical quantities (e.g., material properties), measures should be taken according to one of the following methods:

- extracting a sample from each specimen before testing;
- cutting one or more portions of each specimen;
- non-destructive testing, after calibration on other similar specimens.

If it is not possible to measure all basic variables of each specimen, destructive tests shall be carried out on purposely prepared sets of material specimens (e.g., concrete cubes). Here, mean values of the variables are obtained, as opposed to the previous three items, where point values of the variables are obtained.
In order to be considered as "sufficient", the number of tests to carry out should be as follows:

1) If *point values* of the basic variables are available for each tested specimen, a single test should be carried out for each basic variables set X_i; when validating the model results r_t against the experimental results r_e (see Step 3), comparisons should be made in terms of point values (i.e., test by test, as explained at the next step);
2) If *mean values* of the basic variables are available for each group of tested specimens, a minimum number of 5 tests (a test set) should be carried out for each basic variables set X_i, with i=1...5, in order to get a reasonable estimate X_{km} of the mean values of the basic variables set in the k-th test set; when validating the model results r_t against the experimental results r_e (see Step 3), comparisons should be made in terms of mean values (i.e., test set by test set, as explained at the next step).

2.3. Model-experimental results comparison and fine-tuning

Here, the parameter *b* is used to fine-tune the prediction capability of the theoretical model. One should proceed as follows:

1) The (mean or point) values of the measured properties are placed in the capacity function $g_t(\underline{X})$ to obtain the (mean or point) theoretical capacity value r_t to be compared with the (mean or point) experimental value r_e;
2) The correction coefficient *b* is computed by the least-square method to minimize the difference between theoretical r_t and experimental r_e values:

$$\min\left[\sum_i \left(r_{ti}-r_{ei}\right)^2\right] \quad \text{or} \quad \min\left[\sum_k \left(r_{tkm}-r_{ekm}\right)^2\right] \tag{1.2}$$

where: r_{ti} is the theoretical capacity computed by plugging the point values $\underline{x}_i$ of the basic variables used in test *i* into the function $g_t(\underline{X})$ and r_{ei} is the experimental capacity obtained from the i-th test; r_{tkm} is the theoretical capacity computed by plugging the mean values x_{km} of the basic variables used in test set *k* into the function $g_t(\underline{X})$ and r_{ekm} is the experimental capacity obtained as the mean from the *k-th* test set. At this stage, the correspondence of the test results with the initial model hypotheses should be checked. Particularly, in order to verify the model significance, if the difference between theoretical r_t and experimental r_e values is unacceptably large (say, more than 40% in terms of normalised values), one should try and reduce it by either:

1) reformulating the theoretical model with a better interpretation of the underlying physical phenomena, or
2) enhancing the theoretical model to include previously neglected variables, or
3) increasing the number of (sets of) experiments in order to fine-tune the correction coefficient *b*.

2.4. Definition of a probabilistic capacity model

A probabilistic capacity model r should be now defined to include the inevitable model error (that is, the error remaining even after the least-square fine-tuning):

$$r = r_t \cdot \delta = b \cdot g_t(\underline{X}) \cdot \delta \quad (2.2)$$

where δ is the model error, represented by a random variable with Normal distribution with mean value $\delta_m = 1$ and standard deviation σ_δ.

For the estimation of the latter, two different cases might occur:

1) The values X_i of the basic variables X are available as point values for each test i:

1.a) The error is evaluated for each test as:

$$\delta_i = \frac{r_{ei}}{r_{ti}} = \frac{r_{ei}}{b \cdot g_t(\underline{X}_i)} \quad (2.3)$$

where r_{ei} and r_{ti} are the point values of the experimental and theoretical capacity, respectively, for the *i-th* test.

1.b) The standard deviation of the error is estimated through the sample standard deviation (here, it is considered that $\delta_m = 1$):

$$\hat{\sigma}_\delta = \sqrt{\frac{\sum_{i=1}^{n}(\delta_i - 1)}{n-1}}, \qquad n = \text{number of tests} \quad (2.4)$$

2) The values X_{km} of the basic variables X are available as mean values for each set of k tests:

2.a) The error is evaluated for each tests set as:

$$\delta_k = \frac{r_{e_{km}}}{r_{t_{km}}} = \frac{r_{e_{km}}}{b \cdot g_t(\underline{X}_{km})} \quad (2.5)$$

where $r_{e_{km}}$ and $r_{t_{km}}$ are the mean values of the experimental and theoretical capacity, respectively, for the *k-th* test set.

2.b)The standard deviation of the error is estimated through the sample standard deviation (here, it is considered that $\delta_m = 1$):

$$\hat{\sigma}_\delta = \sqrt{\frac{\sum_{k=1}^{m}(\delta_k - 1)}{m-1}}, \qquad m = \text{number of test sets} \quad (2.6)$$

For what concerns the basic variables variation, if the test population is fully representative of the population, the coefficients of variation V_{X_i} of the basic variables can be directly determined from the test data. In most cases, the coefficients of variation are determined based on a priori knowledge.

2.5. Estimation of mean and variance of the capacity model

The characteristic value of the capacity model should be sought starting from its statistics, under the normality assumption. For the above-defined probabilistic function $r = r_t \cdot \delta = b \cdot g_t(\underline{X}) \cdot \delta$, the first-order approximation of the mean is (since $\delta_m = 1$):

$$r_m = \mathrm{E}(r) \cong b \cdot g_t(\underline{X}_m) \quad (2.7)$$

The first-order approximation of the variance is:

$$\mathrm{Var}(r) = \sum_i \left[c_i^2 \cdot \mathrm{Var}(X_i)\right] + c_\delta^2 \cdot \mathrm{Var}(\delta) + \sum_i \sum_{j \neq i}^{n} \left[c_i \cdot c_j \cdot \mathrm{Cov}(X_i X_j)\right] \quad (2.8)$$

where $c_i = \left.\frac{\partial r}{\partial X_i}\right|_{\underline{X}_m, \delta_m}$ and $c_\delta = \left.\frac{\partial r}{\partial \delta}\right|_{\underline{X}_m, \delta_m}$ are the values of the partial derivatives of the function r with respect to the basic variables $\underline{X}_i$ and to the error δ, respectively, computed at the mean values of $\underline{X}_i$ and δ, and where the covariance $\mathrm{Cov}(X_i X_j)$ is given by:

$$\mathrm{Cov}\left(X_i X_j\right)=\mathrm{E}\left[\left(X_i-X_{im}\right)\left(X_j-X_{jm}\right)\right]=\mathrm{E}\left(X_i X_j\right)-\mathrm{E}\left(X_i\right)\mathrm{E}\left(X_j\right)=\frac{1}{n}\sum_{l=1}^{n}\left(X_{il}-X_{iml}\right)\left(X_{jl}-X_{jml}\right) \tag{2.9}$$

If the basic variables are considered as statistically independent, the last term in the equation. (2.9) is zero.

2.6. *Estimation of the characteristic value of the capacity model*

The expression correctly representing its characteristic value r_k is finally found as:

$$r_k = r_m - 1.64\cdot\left[\mathrm{Var}\left(r\right)\right]^{1/2} \tag{2.10}$$

2.7. *Check of the error properties*

To check if the residuals (model errors) satisfy the initial hypothesis, some tests must be performed:

1) Check of the normality hypothesis of error (normality of residuals): an assumed probability distribution can be verified by:

1.a) Construction of the probability graph

1.b) Execution of a “goodness-of-fit” test as the Kolmogorov-Smirnov test (K-S) or the chi-square (χ^2) test.

2) Check of the hypothesis of omoschedasticity of error: the test for the omoschedasticity of the error allows to verify if the variance of the residuals does not vary with respect to the independent variate; the error variability must be plotted vs. the capacity variability; if the residuals are regularly arranged, the model is well specified.

3. APPLICATION

The procedure explained above is here applied for developing a consistent formula for the characteristic debonding strength of an FRP strengthening. Of course, the procedure may be applied to any capacity formula.

3.1. *Development of a theoretical strength model.*

Several formulations have been developed for the debonding strength of FRP fabrics externally bonded to concrete. A possible one, which is here studied for demonstration purposes, is the one given in the Italian CNR Guidelines, CNR DT-200/2004. There, the debonding strength is expressed as a certain function of some basic variables; here only the functional form is retained, with no consideration of characteristic/design values for the basic variables:

$$f_{fd}=\sqrt{\frac{2k_G k_b\cdot E_f\sqrt{f_c\cdot f_{ct}}}{t_f}} \tag{3.1}$$

where k_G is a least-square fine-tuning parameter (what above was called b); k_b is a geometric model parameter depending on the width of both the strengthened beam and the FRP system; E_f, f_c and f_{ct} are basic random variables: Young’s modulus of FRP strengthening, concrete compression strength, and concrete tensile strength, respectively; t_f is the FRP strengthening thickness.

3.2. *Measurement of the basic variables in tests.*

The procedure is here applied using experimental test results from the literature (Chajes et al. (1996), Miller et al. (1999), Pellegrino et al. (2005), Brosens et al. (2001), Nakaba et al. (2001)) showed in Alessandri et al.. It should be noted that in all collected cases the basic variables E_f, f_c and f_{ct} are given in terms of mean values obtained from tests on samples extracted from the specimens before testing.

3.3. *Model-experimental results comparison and fine-tuning*

The model is fine-tuned by calibration of the coefficient k_G, which should be carried out by comparing the theoretical values of the debonding strength with the experimental ones. Through the least-squares method, the value of k_G is determined so to minimize the difference between the experimental value of the maximum debonding force, $F_{max,e}$, and the theoretical one, given by:

$$F_{max,t} = b_f \cdot t_f \cdot f_{fd} = b_f \sqrt{2k_G k_b \cdot t_f \cdot E_f \sqrt{f_c \cdot f_{ct}}} \tag{3.2}$$

where b_f is the width of the FRP. The obtained value is $k_G = 0.107$. A comparison between theoretical and experimental results is given in Fig. 1. The available experimental tests are grouped according both geometrical and mechanical properties.

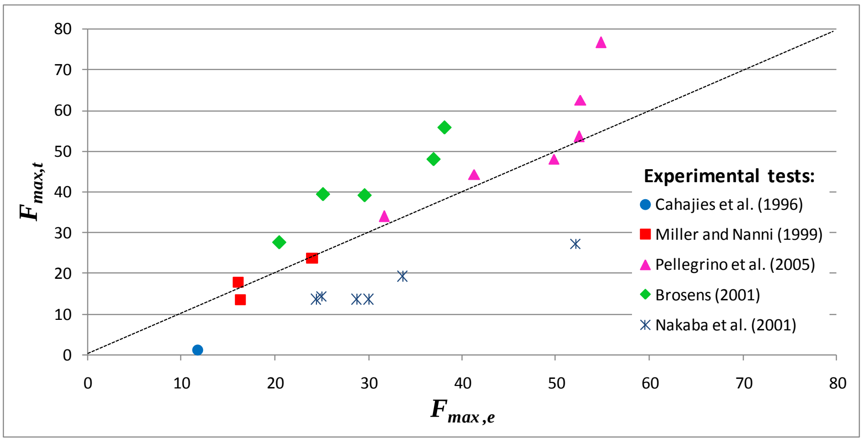

Fig. 1. Comparison between theoretical and experimental results.

3.4. Definition of the probabilistic capacity model

The assumed probabilistic model of the capacity is:

$$f_{fd} = \sqrt{\frac{2k_G k_b \cdot E_f \sqrt{f_c \cdot f_{ct}}}{t_f}} \cdot \delta \tag{3.3}$$

where δ is a random variable with unit mean and standard deviation σ_δ.The error δ in the equation of f_{fd} is evaluated as the ratio between theoretical and experimental values of the maximum force F_{max}; using the tests results, it is possible to calculate the mean value, assumed as unit value, and the variance, given by:

$$\mathrm{Var}(\delta) = \mathrm{Var}\left[\frac{F_{max,e}}{F_{max,t}}\right] = 0.12 \tag{3.4}$$

The basic random variables E_f, f_c and f_{ct} are considered as statistically independent among them. As for the coefficients of variation, for f_c and f_{ct} a value of V_{f_c} = $V_{f_{ct}}$ = 0.2 has been assumed, while for E_f a value of V_{E_f} = 0.0 has been taken.

3.5. Estimation of mean and variance of the capacity model

According to equation (2.8), the mean value of the debonding strength is:

$$f_{fdm} = \sqrt{\frac{2k_G k_b \cdot E_{fm} \sqrt{f_{cm} \cdot f_{ctm}}}{t_f}} \tag{3.5}$$

Therefore the variance of f_{f_d}, expressed in a simplified form, is given by:

$$\operatorname{Var}\left(f_{fd}\right)=\left[k_1\cdot E_{fm}^{1/2}\cdot\left(f_{cm}\cdot f_{ctm}\right)^{1/4}\right]^2\cdot\left[\frac{1}{4}\frac{\operatorname{Var}\left(E_f\right)}{E_{fm}^2}+\frac{1}{16}\frac{\operatorname{Var}\left(f_c\right)}{f_{cm}^2}+\frac{1}{16}\frac{\operatorname{Var}\left(f_{ct}\right)}{f_{ctm}^2}+\operatorname{Var}(\delta)\right] \tag{3.6}$$

3.6. Estimation of the characteristic value of the capacity model

By applying equation (2.11), the characteristic value of f_{fd}, with the assumptions made above on the coefficients of variation and the value computed in the equation (3.5), becomes:

$$f_{fdk}=0.4\sqrt{\frac{2k_Gk_b\cdot E_{fm}\sqrt{f_{cm}\cdot f_{ctm}}}{t_f}} \tag{3.7}$$

Note that: a) the equation yielding the 'true' characteristic value of the capacity is now expressed in terms of the mean values of the basic variables, b) their variability is contained within the external coefficient (0.4), which also includes the model error, c) the coefficient k_G has the meaning of a fine-tuning coefficient.

3.7. Check of the error properties

1) Check of the normality hypothesis of error (normality of residuals):
1.a) Construction of the normal probability graph: the data for the model error δ, obtained from the literature and reported in Alessandri et al., are arranged in increasing order and are plotted at the cumulative probability $m/(n+1)$. The resulting graph of data points (Fig. 2) shows a linear trend that fit the straight line passing through the points $(0.5,\delta_m=1)$ and $(0.84,\delta_{.84}=1.346)$; δ_m is the mean value of the model error δ, and $\delta_{.84}=\delta_m+\sigma_\delta=1.346$ is the value with probability p = 0.84; the normal distribution is therefore applicable to the model error.

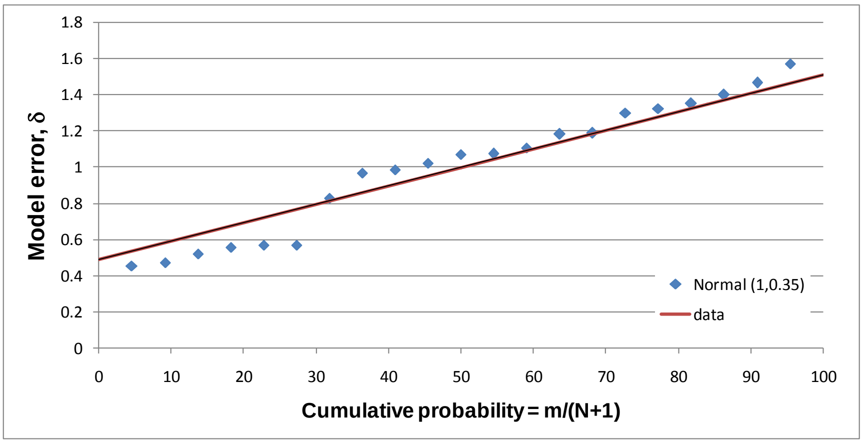

Fig. 2. Model error plotted on a probability paper.

1.b) Kolmogorov-Smirnov test for the normality of residuals: the validity of the assumed Normal distribution is validated also by the Kolmogorov-Smirnov "goodness-of-fit test". The maximum difference is computed between $S_n(\delta)$ and the theoretical cumulative distribution function (CDF) $F(\delta)=N(\delta_m,\sigma_\delta)$ (Fig. 3).

$$D_n=\max_{\delta}\left|F(\delta)-S_n(\delta)\right|=0.177 \tag{3.8}$$

The observed value D_n is compared to the critical value D_n^α, which is that value by which:

$$P\left(D_n\leq D_n^\alpha\right)=1-\alpha \tag{3.9}$$

The test is here performed at the 5% significance level ($\alpha=0.05$). The critical value of $D_n^\alpha=0.286$ is evaluated by numerical interpolation; since it is verified that the maximum discrepancy is less than $D_n^{\alpha}=0.286$, the Normal distribution hypothesis is verified at the 5% significance level.

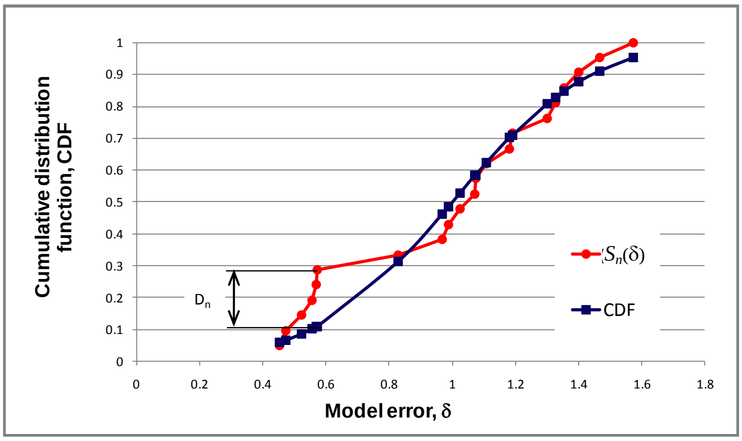

Fig. 3. Cumulative distribution of the model error δ

2) Check of the hypothesis of omoschedasticity of error: to verify the hypothesis of omoschedasticity of error, the residuals are plotted vs. the theoretical maximum force $F_{max,t}$ (Fig. 4); the points on the graph cover an homogeneous area around the horizontal line at $\delta_m=1$; this means that the variance of the residuals does not vary with respect to the independent variate and thus the model is well specified.

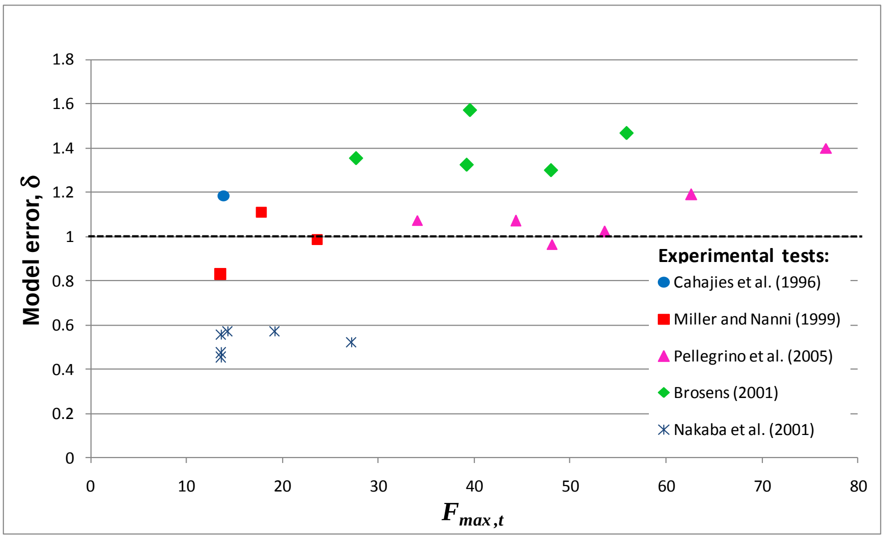

Fig. 4. Check of the hypothesis of omoschedasticity of the residuals

In the study, a comparison with the capacity model of the debonding strength f_{fd} described in CNR DT-200/2004, has been performed. The parameter k_G is calibrated on the basis of experimental tests in order to obtain the characteristic value of f_{fd}. The statistical analysis of tests results had provided an mean value $k_{G_m}=0.064$ and a standard deviation $\sigma_{k_G}=0.023$; the 5th percentile of the statistical distribution had been found as $k_{G_k}=0.026$. The characteristic value of the debonding strength was then obtained by plugging in the equation the characteristic values of both the concrete compression strength, f_{ck}, and the parameter k_G (therein rounded to 0.03), while the other variables were given as mean values:

$$f_{fdk,CNR} = \sqrt{\frac{2E_f k_{Gk} k_b \sqrt{f_{ck} \cdot f_{ctm}}}{t_f}} \tag{3.10}$$

The comparison between this formula and that (3.7) obtained with the proposed procedure shows that:

$$\frac{f_{f_{dk}}}{f_{f_{dk,CNR}}} = 0.84 \tag{3.11}$$

that is, the equation obtained with proposed rigorous procedure yields capacity values that are approximately 15% lower than those obtained with the formula appearing in CNR DT-200/2004, thus showing that there are cases where the approximations introduced as highlighted in the introduction to this paper in some formulations, often lead to non-conservative estimates of the design values.

5 CONCLUSIONS

It has been proposed a procedure to develop, in a consistent manner, design equations that allow to compute the capacity of resisting mechanisms with controlled reliability. Also, the procedure shows how experimental tests should be treated for fine-tuning the model and for arriving at the 'true' characteristic value of the analytical capacity models. The paper deals, in philosophical terms, with how theoretical models should be developed, with how experimental tests should be performed, especially regarding the parameters to measure, and with how to include the values of the basic variables in the equation of the theoretical model. The validity of the model should then be checked by means of a statistical interpretation of all available test data. The formulation here proposed then includes a new variable for the model error to be evaluated from experimental results. Once the statistical parameters of the model error are known, it is possible to define the parameters of the capacity model and to evaluate its characteristic value for application in design. The proposed procedure is applied to the development of a capacity formula for the debonding strength of FRP, starting from a formulation proposed in the Italian CNR Guidelines. The comparison with the latter for the evaluation of the characteristic value of the debonding strength shows that a non-rigorous procedure can yield non-conservative values of the capacity. The proposed procedure is currently being applied to a large series of capacity equations, in order to check for their consistency and reliability.

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REFERENCES

EN-1990, "EC0 (Eurocode 0) (1990) – Basis of structural design".

Ang A.H-S., Tang W.H., (1975). Probability Concepts in Engineering Planning and Decision, Vol. I, Basic Principles. John Wiley and Sons: New York, 1975.

CNR DT-200/2004, "Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Existing Structures". CNR Rome, July 13th, 2004. (www.cnr.it).

Chajes M.J., Finch W.W. jr, Januska T.F. and Thomson T.A. jr., (1996). "Bond and force transfer of composite material plates bonded to concrete". *ACI Structural Journal*, Vol. 93, pp. 208-217, 1996.

Miller, B., and Nanni, A., "Bond between CFRP sheets and concrete". (1999). Proceedings, ASCE 5th Materials Congress, Cincinnati, OH, L.C. Bank, Editor, May 10-12, 240-247, 1999.

Pellegrino, C., Boschetto, G., Tinazzi, D. and Modena, C., (2005). "Progress on understanding bond behaviour in RC elements strengthened with FRP". Proceedings of the International Symposium on Bond Behaviour of FRP in Structures, 2005.

Brosens, K., (2001). "Anchorage of externally bonded steel plates and CFRP laminates for strengthening concrete elements". Doctoral Thesis, Katholieke Universiteit Leuven, May, 2001.

Nakaba, K., Kanakubo, T., Furuta, T., and Yoshizawa, H.,(2001) "Bond Behavior between Fiber-Reinforced Polymer Laminates and Concrete". *ACI Structural Journa*l, Vol. 98, No.3, May-June 2001.

Alessandri S., Monti G., Santini S., "Design by testing: A Procedure for the Statistical Determination of Capacity Models". *J. of Construction and Building Materials, Special Issue on FRP Composites*, Elsevier (submitted).