EVALUATION OF CURRENT NONLINEAR STATIC PROCEDURES FOR REINFORCED CONCRETE BUILDINGS

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ABSTRACT :

This paper evaluates the nonlinear static procedures specified in the FEMA-356, ASCE/SEI 41-06, ATC-40, and FEMA-440 documents for seismic analysis and evaluation of building structures using strong-motion records of reinforced concrete buildings. For this purpose, maximum roof displacement predicted from the nonlinear static procedure is compared with the value "derived" directly from recorded motions. It is shown that: (1) the nonlinear static procedures either overestimate or underestimate the peak roof displacement for several of the buildings considered in this investigation; (2) the ASCE/SEI 41-06 Coefficient Method (CM), which is based on recent improvements to the FEMA-356 CM suggested in the FEMA-440 document, does not necessarily provide better estimate of the roof displacement; and (3) the improved FEMA-440 Capacity Spectrum Method (CSM) generally provides better estimates of the roof displacement compared to the ATC-40 CSM. However, there is no conclusive evidence of either the CM procedures (FEMA-356 or ASCE/SEI 41-06) or the CSM procedure (ATC-40 or FEMA-440) leading to better estimate of the peak roof displacement when compared with the value derived from recorded motions.

KEYWORDS: Buildings, Earthquake Engineering Nonlinear Static Analysis, Pushover Analysis

1. INTRODUCTION

Buildings are expected to be deformed beyond their linearly-elastic range during design level earthquake. Therefore, accurate estimation of seismic demands requires explicit consideration of inelastic behavior of the structure. While nonlinear response history analysis (RHA) is the most rigorous procedure to compute seismic demands, current structural engineering practice prefers to use the nonlinear static procedure (NSP) or pushover analysis. The two key steps in estimating seismic demands in the NSP are: (1) estimation of the target node (or roof) displacement; and (2) pushover analysis of the structure subjected to monotonically increasing lateral forces with specified height-wise distribution until the target displacement is reached. Both the force distribution and target displacement are typically based on the assumption that the response is controlled by the fundamental mode and that the mode shape remains unchanged after the structure yields.

The two widely used procedures to estimate the target displacement are: (1) the Coefficient Method (CM) defined in the FEMA-356 document (ASCE, 2000); and (2) the Capacity Spectrum Method (CSM) specified in ATC-40 document (ATC-40, 1997). The CM utilizes a displacement modification procedure in which several empirically derived factors are used to modify the response of a linearly-elastic, single-degree-of-freedom (SDOF) model of the structure. The CSM is a form of equivalent linearization. This technique uses empirically derived relationships for the effective period and damping as a function of ductility to estimate the response of an equivalent nonlinear SDOF oscillator.

Various researchers have found that the CM and CSM may provide substantially different estimates of target displacement for the same ground motion and the same building (Aschheim et al., 1998; Akkar and Metin, 2007; Chopra and Goel, 2000; Goel, 2007; Miranda and Ruiz-Garcia, 2002) and have proposed improved procedures for estimating the target displacement. Recently, FEMA-440 document (ATC-55, 2003) re-examined the existing NSPs and proposed improvements to both the CM and CSM; recommendations in the FEMA-440 document have been adopted in the ASCE/SEI 41-06 standard (ASCE, 2007).

Development and evaluation of the NSPs have mostly been based on computer simulation studies; a comprehensive list of previous investigations is available in the FEMA-440 document. Recorded motions of strongly shaken buildings, especially those deformed into the inelastic range, provide a unique opportunity to evaluate such procedures. Therefore, the principal objective of this investigation is to evaluate the current NSPs – Coefficient Method in the FEMA-356 document; Capacity Spectrum Method in the ATC-40 report; improved Coefficient Method in the ASCE/SEI 41-06 document; and improved Capacity Spectrum Method proposed in the FEMA-440 document – for seismic analysis and evaluation of building structures using strong-motion records of reinforced-concrete buildings. The accuracy of these NSPs is evaluated by comparing the peak roof (or target node) displacement computed from various NSPs with that derived directly from recorded motions. The work presented in this paper is a summary of a comprehensive study reported elsewhere (Goel and Chadwell, 2007; Goel, 2007).

2. SELECTED BUILDINGS

Recorded motions of buildings that were strongly shaken and potentially deformed beyond the yield limit during the earthquake are required for this investigation. For this purpose, five concrete buildings, ranging from low-rise to high-rise, have been selected (Table 1). The strong-motion data used in this investigation are also identified in Table 1 for each building. These data are available from the US National Center for Engineering Strong Motion Data (NCESMD) (http://www.strongmotioncenter.org).

Table 1. Five concrete buildings selected.

Buildings name	CSMIP Station	Number of Stories	Strong-Motion Data from earthquake
Imperial County Services Building, El Centro	01260	6/0	1979 Imperial Valley
13-Story Commercial Building, Sherman Oaks	24322	13/2	1994 Northridge
20-Story Hotel, North Hollywood	24464	20/1	1994 Northridge
4-Story Commercial Building, Watsonville	47459	4/0	1989 Loma Prieta
3-Story UCSB Office Building, Santa Barbara	25213	3/0	1978 Santa Barbara

3. CURRENT NONLINEAR STATIC PROCEDURES

The nonlinear static procedures in the FEMA-356, ATC-40, FEMA-440, and ASCE/SEI 41-06 documents require development of a pushover curve which is defined as the relationship between the base shear and lateral displacement of a control node. The height-wise distributions of lateral loads for pushover analysis is typically selected from: (1) Equivalent lateral force (ELF) distribution: $s_j^* = m_j h_j^k$ (the floor number) $j = 1, 2 \ldots N$ where s_j^* is the lateral force and m_j the mass at jth floor, h_j is the height of the jth floor above the base, and the exponent $k = 1$ for fundament period $T_1 \le 0.5\,\text{sec}$, $k = 2$ for $T_1 \ge 2.5\,\text{sec}$; and varies linearly in between; (2) Fundamental mode distribution: $s_j^* = m_j \phi_{j1}$ where ϕ_{j1} is the fundamental mode shape component at the jth floor; and (3) Response Spectrum Analysis (RSA) distribution: the vector of lateral forces $\mathbf{s}^*$ is defined by the lateral forces back-calculated from the story shears determined by linear response spectrum analysis of the structure including sufficient number of modes to capture 90% of the total mass; and (4) "Uniform" distribution: $s_j^* = m_j$. The FEMA-356 NSP requires development of the pushover curve for two height-wise distributions of lateral forces: one selected from the first three of the aforementioned distributions and the second selected as the "Uniform" distribution. The ATC-40, FEMA-440, and ASCE/SEI 41-06 NSPs require development of the pushover curve only for the fundamental mode distribution.

The structure is pushed statically to a target displacement at the control node to check for the acceptable structural performance. The NSPs in the FEMA-356, FEMA-440, ATC-40, and ASCE/SEI 41-06 documents differ primarily in computation of this target displacement. These methods are summarized next.

3.1. FEMA-356 Coefficient Method

The target displacement in the FEMA-356 CM is computed from

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2} g \tag{3.1}$$

where S_a = Response spectrum acceleration at the effective fundamental vibration period and damping ratio of the building under consideration; g = Acceleration due to gravity; T_e = Effective fundamental period of the building in the direction under consideration computed by modifying the fundamental vibration period from elastic dynamic analysis, e.g., eigen-value analysis, T_i, by:

$$T_e = T_i \sqrt{\frac{K_i}{K_e}} \tag{3.2}$$

in which K_i is the elastic stiffness of the building and K_e is the effective stiffness of the building obtained by idealizing the pushover curve as a bilinear relationship; C_0 = Modification factor that relates the elastic response of a single-degree-of-freedom (SDF) system to the elastic displacement of the multi-degree-of-freedom (MDF) building at the control node taken as the first mode participation factor or selected from tabulated values in FEMA-356; C_1 = Modification factor that relates the maximum inelastic and elastic displacement of the SDF system computed from

$$C_1 = \begin{cases} 1.0; & T_e \geq T_s \\ \dfrac{1.0 + (R-1) T_s / T_e}{R}; & T_e < T_s \\ 1.5; & T_e < 0.1s \end{cases} \tag{3.3}$$

in which R is the ratio of elastic and yield strengths and T_s is the corner period where the response spectrum transitions from constant pseudo-acceleration to constant pseudo-velocity; C_2 = Modification factor to represent the effects of pinched hysteretic shape, stiffness degradation, and strength deterioration selected either from tabulated values depending on the framing system (see FEMA-356 for details of various framing systems) and the performance level or taken as one for nonlinear analysis; and C_3 = Modification factor to represent increased displacement due to P-delta effects computed from

$$C_3 = \begin{cases} 1.0; & \alpha \geq 0 \\ 1.0 + \dfrac{|\alpha| (R-1)^{3/2}}{T_e}; & \alpha < 0 \end{cases} \tag{3.4}$$

in which α is the ratio of the post-yield stiffness to effective elastic stiffness.

3.2. ATC-40 Capacity Spectrum Method

The target displacement in the ATC-40 CSM is computed as the maximum displacement of a linearly-elastic SDF system with equivalent period, T_{eq}, and equivalent damping ratio, ζ_{eq} given by:

$$T_{eq} = T_o \sqrt{\frac{\mu}{1+\alpha\mu-\alpha}}; \; \zeta_{eq} = \zeta_o + \kappa \frac{1}{\pi} \frac{(\mu-1)(1-\alpha)}{\mu(1+\alpha\mu-\alpha)} \tag{3.5}$$

in which T_o is the initial period of vibration of the nonlinear system, α is the post-yield stiffness ratio, μ is the maximum displacement ductility ratio, and κ is the adjustment factor to approximately account for changes in hysteretic behavior of reinforced concrete structure. The ATC-40 document defines three types of hysteretic behaviors – Type A with stable, reasonably full hysteretic loops; Type C with severely pinched and/or degraded loops; and Type B between Types A and C – and provides equations for computing κ for each of the three types of hysteretic behavior.

Since the equivalent linearization procedure requires prior knowledge of the displacement ductility ratio (see Eqn. 3.5), ATC-40 document describes three iterative procedures: Procedures A, B, and C. Procedures A and B are the most transparent and convenient for programming, whereas Procedure C is purely a graphical method that is not suitable for programming. Details of these procedures are available in ATC-40 document and are not presented here for brevity.

3.3. ASCE/SEI 41-06 Coefficient Method

The ASCE/SEI 41-06 CM is based on the improvements to the FEMA-356 CM (Eqn. 3.1) proposed in the FEMA-440 document. In the ASCE/SEI 41-06 CM, the coefficient C_1 is given by

$$C_1 = \begin{cases} 1.0; & T_e > 1.0\,\text{s} \\ 1.0 + \dfrac{R-1}{aT_e^2}; & 0.2\,\text{s} < T_e \le 1.0\,\text{s} \\ 1.0 + \dfrac{R-1}{0.04a}; & T_e \le 0.2\,\text{s} \end{cases} \tag{3.6}$$

in which a is equal to 130 for site class A and B, 90 for site class C, and 60 for site classes D, E, and F (see ASCE/SEI 41-06 for details of various site classes), respectively. The coefficient C_2 is given by

$$C_2 = \begin{cases} 1.0; & T_e > 0.7\,\text{s} \\ 1 + \dfrac{1}{800}\left(\dfrac{R-1}{T_e}\right)^2; & T_e \le 0.7\,\text{s} \end{cases} \tag{3.7}$$

Finally, ASCE/SEI 41-06 CM has dropped the coefficient C_3 but imposes a limitation on strength to avoid dynamic instability. This limitation on strength is specified by imposing a maximum limit on R given by

$$R_{\max} = \frac{\Delta_d}{\Delta_y} + \frac{|\alpha_e|^{-h}}{4}; \quad h = 1.0 + 0.15\ln(T_e) \tag{3.8}$$

in which Δ_d is the deformation corresponding to peak strength, Δ_y is the yield deformation, and α_e is the effective negative post-yield slope given by

$$\alpha_e = \alpha_{P-\Delta} + \lambda\left(\alpha_2 - \alpha_{P-\Delta}\right) \tag{3.9}$$

where α_2 is the negative post-yield slope ratio defined in Figure 1, $\alpha_{P-\Delta}$ is the negative slope ratio caused by $P-\Delta$ effects, and λ is the near-field effect factor given as 0.8 for $S_1 \ge 0.6$ and 0.2 for $S_1 < 0.6$ (S_1 is defined as the 1-second spectral acceleration for the Maximum Considered Earthquake). The α_2 slope includes $P-\Delta$ effects, in-cycle degradation, and cyclic degradation.

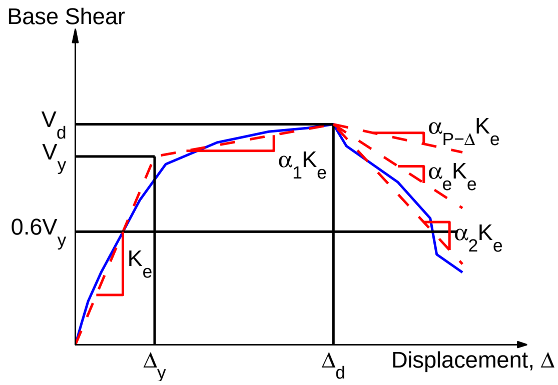

Figure 1. Idealized force-deformation curve in ASCE/SEI 41-06.

3.4. FEMA-440 Capacity Spectrum Method

The improved FEMA-440 CSM includes new expressions to determine the effective period and effective damping developed by Guyader and Iwan (2006). Consistent with the original ATC-40 procedure, three iterative procedures for estimating the target displacement are also outlined. Finally, a limitation on the strength is imposed to avoid dynamic instability (Eqn. 3.7).

The improved formulas for effective period and damping ratio in the FEMA-440 CSM are:

$$T_{eff} = \begin{cases} \left[0.2(\mu-1)^2 - 0.038(\mu-1)^3 + 1\right]T_o; & \mu < 4.0 \\ \left[0.28 + 0.13(\mu-1) + 1\right]T_o; & 4.0 \le \mu \le 6.5 \\ \left[0.89\left(\sqrt{\dfrac{(\mu\text{-}1)}{1+0.05(\mu-2)}} - 1\right) + 1\right]T_o; & \mu > 6.5 \end{cases} \tag{3.10a}$$

$$\zeta_{eff} = \begin{cases} 4.9(\mu-1)^2 - 1.1(\mu-1)^3 + \zeta_o; & \mu < 4.0 \\ 14.0 + 0.32(\mu-1) + \zeta_o; & 4.0 \le \mu \le 6.5 \\ 19\left[\dfrac{0.64(\mu-1)-1}{0.64(\mu-1)^2}\right]\left(\dfrac{T_{eq}}{T_o}\right)^2 + \zeta_o; & \mu > 6.5 \end{cases} \tag{3.10b}$$

These formulas apply for periods in the range of 0.2 and 2.0s. The FEMA-440 document also provides formulas with constants A to L that are specified depending on the force-deformation relationships (bilinear, stiffness-degrading, strength-degrading) and the post-yield stiffness ratio, α; these formulas are not included here brevity.

4. ANALYTICAL MODEL

Needed for evaluating a NSP is the pushover curve of the building. For this purpose, three-dimensional analytical models of the selected buildings were developed using the structural analysis software Open System for Earthquakes Engineering Simulation (*OpenSees*) (McKenna and Fenves, 2001). Two models were developed for each building: linearly-elastic model for computing the mode shapes and frequencies (or vibration periods), and a nonlinear model for pushover analysis. The beams, columns, and shear walls were modeled using *elasticBeamColumn* element in *OpenSees* with effective section properties as per the FEMA-356 recommendations (FEMA-356, 2000). The beams, columns, and shear walls in the nonlinear model were modeled with *nonlinearBeamColumn* element with fiber section in *OpenSees*. Further details of linear and nonlinear models are available in Goel and Chadwell (2007); they are not included here for brevity.

For two of the five selected buildings – Watsonville Commercial Building and Santa Barbara Office Building – the foundation flexibility was expected to significantly influence the response during strong ground shaking because both of these low-rise buildings contained longitudinal and transverse shear walls. The foundation flexibility was included in analytical models of these buildings by attaching six linear springs – three along the x-, y-, and z-translation, two about the x- and y- rocking, and one about the z-torsion – at the base as per the FEMA-356 recommendations for foundation flexibility modeling (ASCE, 2000).

5. EVALUATION OF CURRENT NONLINEAR STATIC PROCEDURES

Current NSPs are evaluated next by comparing the estimates of peak roof (or target node) displacement from the four NSPs – FEMA-356 CM, ASCE/SEI 41-06 CM, ATC-40 CSM, and FEMA-440 CSM – with the value derived from recorded motions of the selected buildings. The procedure to compute derived roof displacement from recorded motions is available elsewhere (Goel, 2005).

It must be noted that the FEMA-356 CM, ASCE/SEI 41-06 CM, ATC-40 CSM, and FEMA-440 CSM are typically restricted to buildings that respond primarily in the fundamental mode. In this investigation, however, these NSPs were applied to buildings that may have significant contributions form higher modes, e.g., Imperial County Services Building, 13-Story Commercial Building in Sherman oaks, and 20-Story Hotel in North Hollywood. Furthermore, the peak roof displacement in the FEMA-356 and ASCE/SEI 41-06 NSP CM was computed from the 5%-damped elastic response spectrum at vibration period T_e. Similarly, the peak roof displacement is estimated from the damped elastic response spectrum for ζ_{eq} and T_{eq} for the ATC-40 CSM, and for ζ_{eff} and T_{eff} for the FEMA-440 CSM. For each case, the elastic response spectrum is developed for the acceleration recorded at the base of the building in the appropriate direction.

The error in the peak roof displacement from a selected NSP, compared to the peak roof displacements derived from recorded motions, is computed as

$$E = 100 \times \frac{u_c - u_t}{u_t} \tag{5.1}$$

in which u_c is the peak roof (or target node) displacement computed form the NSP, and u_t is the corresponding value derived from recorded motions. Note that the peak roof (or target node) displacement derived from recorded motions is considered to be the exact value in computing the error.

Figure 2 shows the percent error in the target (or roof) displacement from the four procedures when compared to the value derived from recorded motions. The results are presented for Imperial County Services Building in the transverse direction (IC-NS), Sherman Oaks Commercial Building in the longitudinal and transverse directions (SO-EW and SO-NS), North Hollywood Hotel in the longitudinal and transverse directions (NH-EW and NH-NS), Watsonville Commercial Building in the longitudinal and transverse directions (WT-EW and WT-NS), and Santa Barbara Office Building in the longitudinal and transverse directions (SB-EW and SB-NS).

The presented results indicate that the current procedures may lead to significantly different estimate of the target displacement. Furthermore, these procedure lead to significant errors in the estimate of peak roof displacement: the errors range from about 50% underestimation, e.g., as is the case for FEMA-356 CM and ASCE/SEI 41-06 CM for the Santa Barbara Office Building in the longitudinal direction (see SB-EW in Figure 2), to about 60% overestimation, e.g., ATC-40 CSM and FEMA-440 CSM for the Watsonville Commercial Building in the transverse direction (see WT-NS in Figure 2).

Among the two CM procedures, the ASCE/SEI 41-06 CM, which is based on the improvements suggested recently in the FEMA-440 document, does not necessarily provide improved estimates for the selected buildings. For example, the ASCE/SEI 41-06 CM leads to larger overestimation for the Imperial County Services Building (see IC-NS in Figure 2) and larger underestimation for the Santa Barbara Office Building (see SB-EW and SB-NS in Figure 2) when compared to the results from the FEMA-356 CM.

The FEMA-440 CSM generally provides better estimated of the peak roof displacement compared to the ATC-40 CSM several buildings (see SO-EW, WT-EW, and SB-NS in Figure 2). For a few other buildings, the FEMA-440 CSM provides estimate that is only slightly worse compared to the ATC-40 CSM (see WT-NS and SB-EW in Figure 2). This indicates that the improvements to the CSM procedure suggested in the FEMA-440 document are likely to lead to better estimated of peak roof displacement.

Finally, there is no clear evidence of whether the CM procedure (FEMA-356 or ASCE/SEI 41-06) or the CSM procedure (ATC-40 or FEMA-440) provides better estimate of peak roof displacement when compared with the value derived from recorded motions. The CSM procedure lead to better estimates for some building (see IC-NS and SB-EW in Figure 2) but worse estimates for other (see SO-EW and WT-NS in Figure 2) compared to the CM procedure. For other buildings, the two procedures lead to essentially similar level of accuracy (see SO-NS, NH-EW, and NH-NS in Figure 2).

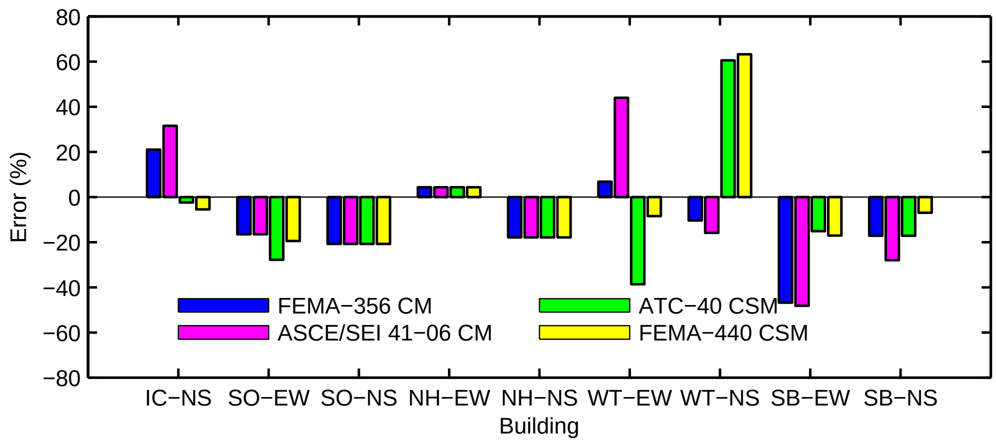

Figure 2. Percent error in peak roof displacements from the FEMA-356 CM, ASCE/SEI 41-06 CM, ATC-40 CSM, and FEMA-440 CSM.

6. CONCLUSIONS

This investigation on evaluation of the FEMA-356 CM, ASCE/SEI 41-06 CM, ATC-40 CSM, and FEMA-440 CSM using strong-motion records of five reinforced-concrete building have led to the following conclusions:

1. The various NSPs may lead to either significant overestimation or underestimation of the peak roof displacement.
2. The ASCE/SEI 41-06 CM, which is based on recent improvements to the FEMA-356 CM suggested in FEMA-440 document, does not necessarily provide better estimate of roof displacement for the buildings considered in this investigation.
3. The improved FEMA-440 CSM generally provides better estimates of peak roof displacements compared to the ATC-40 CSM.
4. There is no conclusive evidence that the CM procedures (FEMA-356 or ASCE/SEI 41-06) lead to better estimates of the peak roof displacement compared to the CSM procedure (ATC-40 or FEMA-440) or vice-versa.

It must be emphasized that the NSPs are typically designed to be used with smooth spectrum. Ideally, these procedures must be evaluated using a suite of design spectrum compatible ground motions, a wide range of buildings, and statistical analysis of results. Although, the evaluation of various NSPs in this investigation is conducted based on limited data – five buildings and one set of strong motion records for each building – and this investigation has led to some useful observations, it is still not possible to draw definitive conclusions about all aspects of various NSPs. More definitive conclusions may be drawn as additional data becomes available in future.

7. ACKNOWLEDGMENT

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