# PROBABILISTIC ASSESSMENT OF SEISMIC RISK IN URBAN AREAS

**A.Aguilar[1], L. Pujades[2], A. Barbat[3] and M. Ordaz[4]**

[1] *PhD Student, Dept. of Geotechnical Engineering and Geosciences, Technical University of Catalonia, Barcelona. Spain*
[2] *Professor, Dept. of Geotechnical Engineering and Geosciences, Technical University of Catalonia, Barcelona. Spain*
[3] *Professor, Dept. of Structural Mechanics, Technical University of Catalonia, Barcelona. Spain*
[4] *Professor, Institute of Engineering, National Autonomous University of Mexico, Mexico City, Mexico.*
*E-mail: armaguilar@uv.mx, lluis.pujades@upc.edu, alex.barbat@upc.edu, mors@pumas.ii.unam.mx*

## ABSTRACT:

A probabilistic approach to estimate the expected seismic physical damage of existing buildings in urban areas is presented. The main steps of the procedure are seismic hazard, vulnerability and structural response. These three steps are undertaken with a fully probabilistic point of view. Seismic hazard is assessed by means of the annual rate of exceedance of a parameter quantifying the expected seismic action. Buildings may be characterized by a class in a building typology matrix and/or by a vulnerability index or, even, by its capacity curve, but in any case its seismic strength must be also defined in a probabilistic way. In the case of vulnerability index we can use beta or similar probability distributions. Finally, adequate empirical, semi-empirical or analytical damage functions or fragility curves may be used to estimate the expected damage. The method is enlightened by means of four cases, corresponding to two un-reinforced masonry buildings located in the Colima and Coatzacoalcos cities in México. The seismic risk computed for these cases is presented in terms of the annualized damage.

**KEYWORDS:** Seismic risk, vulnerability, damage, exceedance rate

## 1. INTRODUCTION

The growing of the urban areas in the world has been important in the last decades and will be significant in the future (Figure 1), especially in developing countries [United Nations, 2006]. Some of these urban areas where the population is growing are located in moderate to high seismicity zones. These places are an example of regions where the seismic risk assessment must be an essential task.

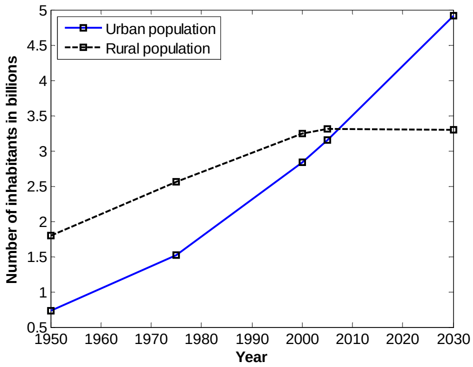


Figure 1. Urban and rural population in the world, 1950-2030 according to United Nations [2006]

This paper describes a probabilistic approach to estimate the expected physical damage of existing buildings in urban areas. The approach is applied to the LM1 method developed within the RISK-UE project [Milutinovic &

Trendafiloski, 2003]. With this modified LM1 method annual exceedance rate of the most likely damage grade can be determined.

## 2. FORMULATION OF THE DAMAGE ESTIMATION

The main elements to assess the seismic risk are showed in figure 2. These three steps, the Probabilistic Seismic Hazard Analysis, the Seismic Vulnerability Analysis and the Damage Functions are undertaken with a probabilistic point of view, in order to avaluate seismic risk.

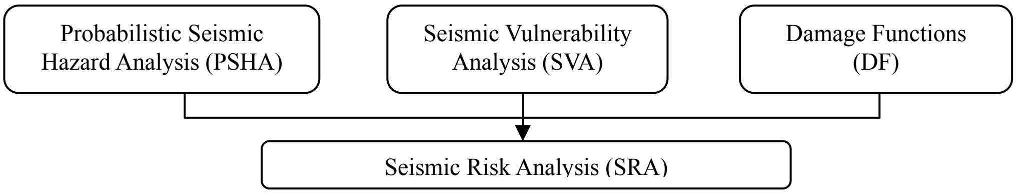


Figure 2. Basic elements of a Seismic Risk Analysis

Conceptually, the present approach is represented by Eqn 2.1 which can be used to estimate the annual probability that the Damage exceeds a certain level of earthquake damage *d*.

$$P[Damage > d] = \iint P[Damage > d \mid I, V]\gamma'[I]P[V]dVdI \tag{2.1}$$

where γ' [ *I* ] is the frequency of occurrence of the seismic intensity *I, P* [ *V* ] is the probability of occurrence of the seismic vulnerability *V* and *P* [*Damage > d* | *I, V*] is the probability of Damage exceeding *d* for a given seismic intensity *I* and a given building with vulnerability *V*. Equation 2.1 applies the total probability theorem, taking into account that the vulnerability *V* and the intensity *I* are independent random variables.

## 3. APPLICATION

The approach proposed in the present document can be implemented in several existing methodologies to estimate seismic risk. For illustration purposes this approach was developed by using the LM1 method (vulnerability index) of the RISK-UE project [Milutinovic & Trendafiloski, 2003]. We call this new version of the method *modified LM1 (mLM1) method.* The LM1 method defines the seismic action with macro-seismic seismic Intensities, while the vulnerability is defined by means of a vulnerability index. Semi-empirical functions link Intensity, vulnerability to obtain expected damage. This mLM1 method is applied to estimate the annual exceedance rates of physical damage of un-reinforced masonry buildings located in two Mexican cities: Colima and Coatzacoalcos. Two buildings are considered here, mainly differing in the confidence level of our knowledge of the structural type and in the data available. We refer to these two buildings as building N. 1 and building N. 2. Table 1 summarizes the main characteristics of these buildings.

Table 1. Main characteristics of the buildings

| Building | Use | walls | floors | Structural type | Confidence in the structural type assignment (0-10) | Soil |
|---|---|---|---|---|---|---|
| 1 | Resi-dential | Un-reinforced Masonry | Reinforced Concrete slabs | Un-reinforced Masonry with reinforced concrete slabs | 9 | Firm ground |
| 2 | | | | | 7 | |

In this Table, the degree of uncertainty in the structural type is considered by means of a value representing the confidence level in the structural type assigned.

### *3.1. Vulnerability analysis (SVA)*

The seismic vulnerability is the predisposition of the exposed building to be affected or of being susceptible to suffer damage or loss as a result of the occurrence of a seismic intensity in the place where the building is located [Barbat *et al*, 2006]. In order to assess the total seismic vulnerability of a building all the factors that increase or reduce the seismic strength of the building must be considered. For this purpose, we introduce the concepts of *Intrinsic Vulnerability* and *Extrinsic Vulnerability*. Therefore, the total seismic vulnerability of a building can be represented in a conceptual manner with the Eqn. 3.1

$$\text{Total Seismic Vulnerability} = \text{Intrinsic vulnerability} + \text{Extrinsic vulnerability} \tag{3.1}$$

The first is defined as the seismic vulnerability depending only of the building and its own characteristics. Soft stories are examples of *Intrinsic Vulnerability*. *Extrinsic vulnerability* depends on the relation of the building with its environment. The pounding effect by neighbor buildings is an example of *Extrinsic Vulnerability*, which only accounts for external contributions to its vulnerability.

#### *3.1.1. probabilistic assessment of Vulnerability*

In order to allow an easy comparison, in the present study only the intrinsic vulnerability is considered. In the mLM1 method the intrinsic vulnerability is expressed by three curves, the mean, the upper and the lower vulnerability curves. The mean vulnerability curve that describes the variation of a vulnerability index is obtained for the buildings of Table 1 using the following procedure: a) A mean vulnerability index is computed for each studied building. This vulnerability index is a value between 0 and 1. The values close to 1 correspond to a high vulnerability and the values close to 0 correspond to low vulnerability b) A range of possible values of the vulnerability index for the studied building is estimated. c) The mean vulnerability index and the two extreme points, delimiting the range of possible values, are then fitted by means of a probability density function with a 95% confidence interval. The graph of this function is the mean vulnerability curve. Using a similar procedure the upper and lower vulnerability curves are computed.

##### *3.1.1.1 Mean vulnerability index*

A mean intrinsic vulnerability index $\overline{V_I}$ is evaluated with the following equation:

$$\overline{V_I} = V_I^* + \Delta V_R + \Delta V_m \tag{3.2}$$

where $V^*_I$ is a Typological Vulnerability Index, $\Delta V_R$ is a Regional Vulnerability Factor and $\Delta V_m$ represents an overall score taking into account additional factors modifying the intrinsic vulnerability index. In order to compute the overall score that modifies the typological vulnerability index, a number of possible modifiers ($N_{Vm}=N_{Vm(-)}+N_{Vm(0)}+N_{Vm(+)}$) are considered. the following equation is used.

$$\Delta V_m = \sum_{i}^{N_{Vm(-)}} V_{m(-)i} + \sum_{j}^{N_{Vm(0)}} V_{m(0)} + \left( \sum_{k}^{N_{Vm(+)}} V_{m(+)k} \right) FA \tag{3.3}$$

Where $V_{m(-)i}$ is the i-th modifier with a score lower than cero. $V_{m(0)j}$ is the j-th modifier which do not contributes to the vulnerability index of a specific building. $V_{m(+)k}$ is the k-th modifier with a score greater tan cero. FA is a factor depending on $N_{Vm(+)}$ allowing to increase the vulnerability when there are many factors which diminish the strength of the building when they act together.

Furthermore, the mean vulnerability index $\overline{V_I}$ computed with the Eqn. 3.2 is bounded by $V^{min}$ and $V^{max}$, proposed by Giovinazzi [2005]. These bounds are distinctive for each building type and, of course, the global vulnerability index is between 0 and 1. The following equation summarizes these properties:

$$Max(\overline{V_I};V_I^{\min};0) \le \overline{V_I} \le Min(\overline{V_I};V_I^{\max};1) \qquad (3.4)$$

The buildings No. 1 and No. 2 can be classified according to Giovinazzi [2005] as a M6 Typology then the Typological Vulnerability Index ( $V_I^*$ ) for these buildings is equal to 0.616 (Table 2). The additional characteristics of the buildings No.1 and No.2 that were taken into account to compute $\Delta V_m$ are summarized in table 3.

Table 2. Vulnerability indexes for Un-reinforced Masonry with reinforced Concrete Slabs Giovinazzi [2005]

| Typology | | Building type | Vulnerability indexes | | | | |
|---|---|---|---|---|---|---|---|
| | | | $V_I^{min}$ | $V_I^-$ | $V_I^*$ | $V_I^+$ | $V_I^{max}$ |
| Masonry | M6 | Unreinforced Masonry with reinforced concrete slabs. | 0.3 | 0.49 | 0.616 | 0.79 | 0.86 |

Table 3. Behavior intrinsic modifiers [Giovinazzi, 2005] and scores assigned to buildings analyzed

| Behavior modifier | Masonry | *Vm* | Data of buildings No. | | *Vm* for the buildings No. | | Confidence (0 - 10) in the data of the buildings No. | |
|---|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 1 | 2 | 1 | 2 |
| State of preservation | Good | -0.04 | Good | Good | -0.04 | -0.04 | 8 | 7 |
| | Bad | +0.04 | | | | | | |
| Number of floors | Low (1/2) | -0.08 | 3 | 3 | 0 | 0 | 10 | 10 |
| | Medium(3/5) | 0 | | | | | | |
| | High (≥ 6) | +0.08 | | | | | | |
| Structural system | Wall thickness Wall distance Wall connections | -0.04/+0.04 | Good structural system | Good structural system | -0.04 | -0.04 | 10 | 7 |
| Plan irregularity | Geometry Mass distribution | +0.04 | No | WD* | 0 | WD* | 9 | WD* |
| Vertical irregularity | Geometry Mass distribution | +0.04 | No | WD* | 0 | WD* | 8 | WD* |
| Superimposed floors | | +0.04 | No | WD* | 0 | WD* | 10 | WD* |
| Retrofitting Intervention | | -0.08/+0.08 | No | WD* | 0 | WD* | 10 | WD* |
| | | | | Sum | -0.08 | -0.08 | | |

* WD means without data

The degree of uncertainty in the structural type is considered by means of a value that represents the confidence in the structural type assigned (Table 1). For this purpose a scale between 0 and 10 can be used, where 0 corresponds to total lack of knowledge and 10 corresponds to absolute certainty in the data. With the same objective a confidence value must be given to each factor modifying the seismic behavior of the building (Table 3).

Table. 4 Vulnerability indexes for the buildings No.1 and No.2

| Building No. | Building type | $V_I^*$ | $\Delta V_m$ | $\overline{V_{ID}}$ | $\overline{V_{ID}}\_error$ |
|---|---|---|---|---|---|
| 1 | Unreinforced Masonry with reinforced concrete slabs. | 0.616 | -0.08 | 0.536 | ± 0.034 |
| 2 | | 0.616 | -0.08 | 0.536 | ± 0.099 |

The confidence values assigned to each building datum (Table 1 and Table 3) are used to compute a global error of the mean vulnerability index (Table 4).

*3.1.1.2 Bounds and probability density functions (pdf) for the intrinsic vulnerability index*

The extreme values delimiting the range of possible values of the vulnerability indices are called $V_{ID\_LOW}$ and $V_{ID\_HIGH}$. These bounds depend on $V_I^{min}$, $V_I^-$, $V_I^+$ and $V_I^{max}$ of each structural type [Giovinazzi 2005] and on the quantity and quality of the data of each building. The mean value $\overline{V_I}$ and the extreme values $V_{ID\_LOW}$ and $V_{ID\_HIGH}$ estimated for each building are then fitted by a pdf describing the probabilistic intrinsic vulnerability index. A similar procedure to that used by ATC-13 [1985] is used here, by assuming that $V_{ID\_LOW}$ and $V_{ID\_HIGH}$ are 95 % confidence limits. In this case a Beta function is assumed but other pdf's may be taken. The parameters *a* and *b* defining the beta distributions that represent the mean vulnerability curves of the buildings No.1 and No.2 are showed in Table 5. The beta distribution was selected because there is a direct relation between vulnerability and damage, and because some studies consider that the Beta pdf is reasonably adequate to describe relations between damage and intensity [ATC-13, 1985; McGuire, 2004]. Using an analogous procedure the Beta functions that represent the upper and lower vulnerability curves are computed (Table 5). Figure 3 shows these pdf's describing the seismic vulnerability of the buildings.

Table 5. Values of the parameters *a* and *b* of beta functions that represents the vulnerability curves of the buildings No.1 and No.2

| Vulnerability curves | Beta function | | | |
|---|---|---|---|---|
| | a | b | a | b |
| | Building No.1 | | Building No.2 | |
| Lower | 27.76 | 27.5 | 13.97 | 18 |
| Mean | 20.79 | 18 | 6.35 | 5.5 |
| Upper | 22.5 | 17 | 6.79 | 3.9 |

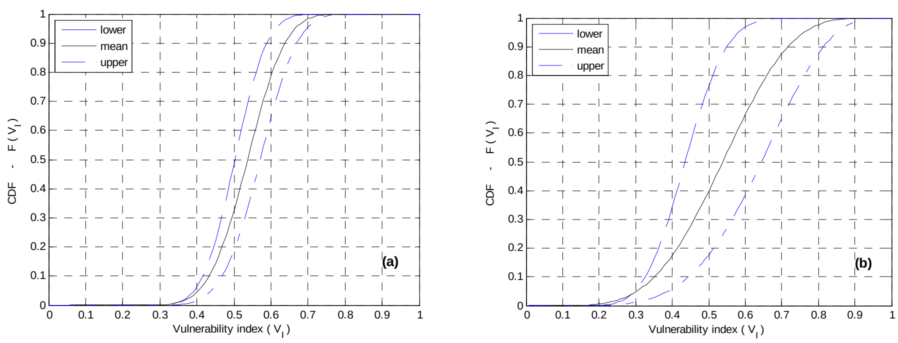


Figure 3. Seismic vulnerability curves, representing the intrinsic vulnerability of the Buildings No.1 (a) and No. 2 (b), represented as cumulative probability functions (cdf) as a function of the intrinsic vulnerability index ($V_I$).

### *3.2. The seismic hazard (PSHA) in Coatzacoalcos and Colima*

Coatzacoalcos is one of the port cities located in the Gulf of Mexico. This city is in the southern part of the Mexican state of Veracruz. As per the CFE [1993], Mexico has been divided into 4 seismic zones. The zone A corresponds to the lowest seismic zone and the zone D corresponds to the highest seismic zone. According to these seismic zones, Coatzacoalcos is located in the seismic zone B, whereas Colima, a city located near of the Pacific coast in the southwest of Mexico, is inside of the seismic zone D [CFE, 1993]. The vibration periods of the structural response of the buildings No. 1 and No. 2 were estimated and the value obtained was 0.15 s. Therefore, seismic hazard curves in the firm soil of Coatzacoalcos and Colima for a structural period of 0.15 s were used (Figure 4). These seismic hazard curves are expressed in terms of spectral pseudo-acceleration and they were obtained with the PSM software [Ordaz, *et al*, 1996]. PSM means *"Peligro Sísmico en México"*

(Seismic Hazard in México). This software is a geographical information system containing the seismic hazard results for México that were computed by using the CRISIS95 software. The CRISIS95 software and later versions including the CRISIS2007, compute the seismic hazard using a probabilistic method based in the standard Esteva-Cornell approach [see Esteva, 1970 and Ordaz, *et al*, 2007].The LM1 considers the seismic action in terms of EMS-98 [Grüntal, 1998] macro-seismic intensities which roughly are equivalent to MMI (Modified Mercalli Intensity). MMI scale is well described in ATC-13 [1985]. For these reasons, the exceedance rate curves of the Sa (Figure 4) and the empirical relationship between MMI grades and response spectra proposed by Atkinson and Sonley [2000], were used to computed exceedance rates for MMI grades.

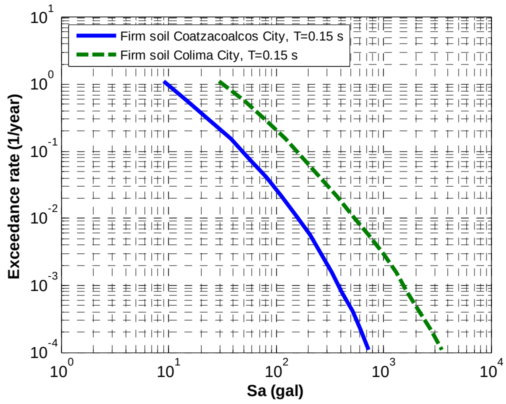


Figure 4. Seismic hazard curves for a structural period of 0.15 s in the firm soil of Coatzacoalcos and Colima, presented as annual probability of exceedance as a function of spectral pseudo-acceleration (Sa).

### *3.3. The damage function (DF)*

In the present study a known semi-empirical intensity-vulnerability-damage function was used to estimate damage. This relationship, which is applied in the LM1 method, requires an EMS-98 intensity grade and a vulnerability index in order to estimate the mean damage grade [Milutinovic & Trendafiloski, 2003]. The variation in the mean damage grade is represented by a binomial-equivalent beta probability density function. This beta distribution can be computed by means of the expressions given below [Giovinazzi, 2005]:

$$a = t(0.007\mu_D^3 - 0.0525\mu_D^2 + 0.2875\mu_D) \quad (3.5)$$

$$\mu_D = 2.5\left[1 + \tanh\left(\frac{I + 6.25V_I - 13.1}{2.3}\right)\right] \quad (3.6)$$

$$t = 8 \text{ and } b = t - a \quad (3.7)$$

where $a$ and $b$ are canonical parameters defining the Beta distribution, $\mu_D$ is the mean damage grade, $t$ is equal to 8, I is the macro-seismic EMS-98 intensity and $V_I$ is the vulnerability index. Then, for instance, the probability that the damage will be less or equal to a $k$-damage grade is computed with the following equation:

$$P\,[\,D < k \mid I, V_I] = CDF\_Beta\,((k+1)/6,\, a,\, b) \quad (3.8)$$

where *CDF_Beta (x; a, b*) is the Cumulative Distribution Function Beta, $k$ is the damage state and (k+1)/6 is the normalized damage state, being 6 the number of considered damage states See MatLab 7.5 [2007]. $a$ and $b$ are the canonical parameters of the Beta distribution of $k$ that can be computed with the Eqn. 3.5 and 3.7, respectively.

### *3.4. Results*

The annual exceedance rate curves for the damage grades, *k,* computed with the approach summarized by Eqn.2.1, for the buildings No.1 and No.2 when located in Coatzacoalcos are showed in the Figure 5. According to these results the annual exceedance rate for the moderate damage grade ($k$=2) for the building No. 1 is between 0.00038 and 0.0006, with a mean value of 0.0005 (Figure 5a). On the other hand, there is a probability between 1.9 % and 3.1 % that the building No.1 in Coatzacoalcos reaches or exceed the damage grade 2 in the next 50 years, whereas for the building No.2 there is a probability between 1.3 % and 5.5 % that this building reaches or exceed the damage grade 2 in the next 50 years (Figure 5b).

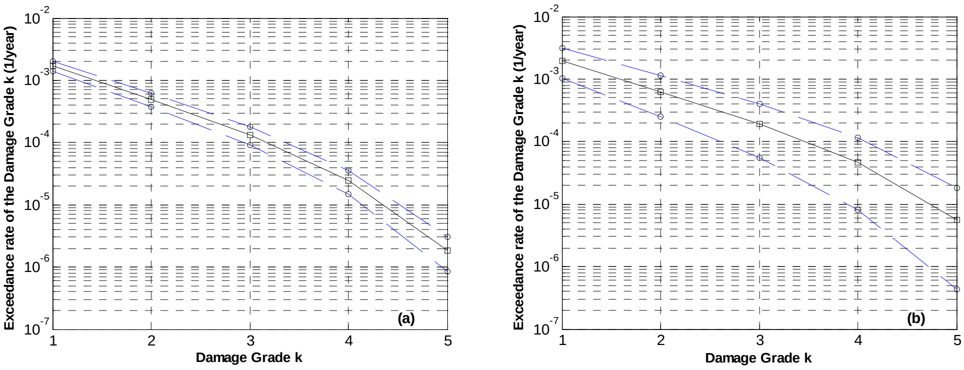


Figure 5. Damage curves for the buildings No.1 (a) and No.2 (b) when located in Coatzacoalcos, presented as annual probability of exceedance as a function of Damage grade k.

For the building No. 1,when located in Colima, the annual exceedance rate for the moderate damage grade (k=2) is between 0.0076 and 0.0106, with a mean value of 0.009 (Figure 6a). In other words, we can expect that when the building No.1 is located in Colima moderate damage grade will occur in average one time in the next 94 to 132 years, with a mean value of 110 years (Figure 6 b). On the other hand, there is a probability between 31.5% and 41.1% that the building No.1, located in Colima, reaches or exceeds the damage grade 2 in the next 50 years, whereas for the building No. 2 in the same place there is a probability between 24.8 % and 55.17 % of reaching or exceeding the damage grade 2 in the next 50 years.

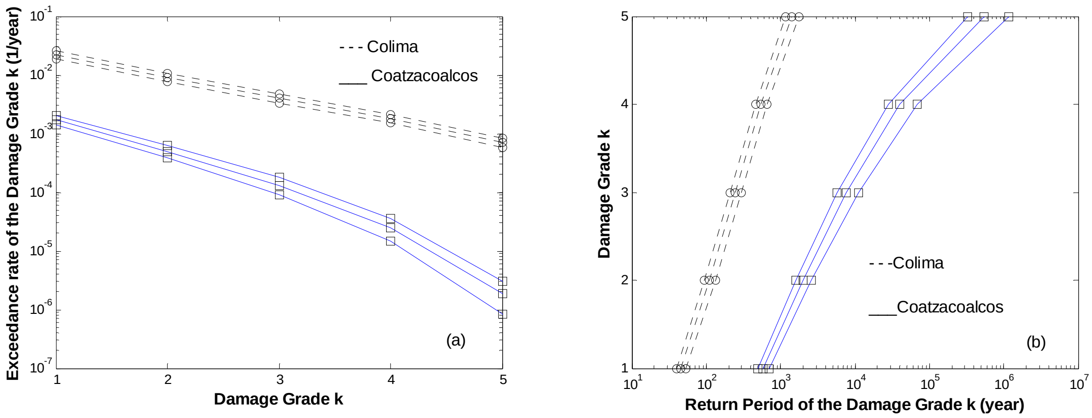


Figure 6. Damage curves for the building No.1 when located in Coatzacoalcos and Colima, presented as both annual probability of exceedance as a function of Damage grade k (a) and Damage grade k as a function of return period (b)

## 4. CONCLUSIONS

The probabilistic approach proposed in this study to compute the seismic risk, can be used by conveniently modifying existing risk estimation methodologies. Herein we have used the modified LM1 method to illustrate the possibilities of this fully probabilistic approach. The modified LM1 method allows obtaining reasonable and representative values of the seismic risk of buildings in urban areas which are usually understood as average expected values for credible earthquake scenarios. The modified LM1 method has the following advantages: a) basic data from each studied building are used to estimate reasonable pdf for seismic vulnerability, b) a number of elements modifying the seismic vulnerability can be considered c) the seismic vulnerability of each studied building is represented by pdf's, d) The probabilistic seismic hazard can be chosen in different ways; Here we use that seismic hazard linked to the structural response period the building, e) A semi-empirical intensity-vulnerability-damage function that it is represented by probability density functions is used, f) The vulnerability, the hazard and the damage functions are defined from a probabilistic point of view, g) Finally, the seismic risk is expressed in terms of annual exceedance rate of the damage grade. Other existing methodologies are included in this proposed expansion as particular cases.

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