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Probabilistic Assessment of Buildings Damage Considering Aftershocks of Earthquakes

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ABSTRACT :

In Japan, buildings are always exposed to the risk of earthquakes. Various researches dealing with earthquake risk have been undertaken very actively in Japan. Most of the research, however, focuses on the risk from the main shock while research treating aftershock risks has been seldom paid attention to so far. Although it is obvious that the major building damage comes from the main shock, it has been reported that some serious building damage occurred due to a series of aftershocks of the Chuetsu earthquake on Oct.23, 2004 in Niigata area. That is why it is necessary to demonstrate how much the seismic risk can increase when taking into consideration of aftershocks.
It is assumed in this study that the relationship between the magnitude of aftershocks and the total number of aftershocks can be represented by the Gutenberg-Richter law, and the relationship between the elapsed time after the main shock and the occurrence rate of aftershock follows the modified Omori's law. Using the ground motion attenuation relation together with the above two laws, the ground motion caused by aftershocks can be calculated and then an aftershock hazard can be estimated.
In this paper, three damage states of buildings, corresponding to the three modes of an event-tree in the Markov chain, are taken: collapse, half collapse and no damage. Then the seismic risk of aftershocks along with the main-shocks can be evaluated theoretically by use of an event-tree and the Markov chain. The Markov chain is very appropriate because the occurrence rate of aftershocks varies depending upon the elapsed time, and the state of buildings might change due to consecutive occurrence of aftershocks. Some numerical examples show the important effect due to aftershocks.

KEYWORDS: *Earthquake, Aftershock, Loss, Disaster, Risk, Fragility, cumulative*

1. INTRODUCTION

Buildings are exposed to various natural disasters such as earthquakes, snows, and winds. We have received an economic loss, a functional loss, the collapse of the building, and the damage of the personal suffering etc. due to such a disaster. In a past catastrophe, disaster causes serious and longterm damages; and before revitalization is complete, yet another disaster strikes. For instance, the life of a lot of buildings and residents encountered damage by the occurrence of earthquakes in the Chuetsu earthquake that had occurred at 5:56PM, October 23rd, 2004. When the characteristics in the Chuetsu earthquake are reviewed, we see there were many aftershocks following the main shock. The Chuetsu earthquake was an earthquake whose number of aftershocks is more than those of general earthquakes.
The Chuetsu earthquake addresses two problems in a past disaster risk evaluation technique like the evaluation of the aftershock risk as mentioned above. The past research on the aftershock risk hardly includes the research about how much aftershock accelerate earthquake risk and effected overall damage and loss of earthquake. Most of past research focuses on the aftershock magnitude and the probability of occurrence. However, the possibility that a building has already been damaged by the main shock receives extensive damage by the aftershock can be critical in some earthquakes like the Chuetsu example. Therefore, the past aftershock research is insufficient.
In this research, the compounding of the main shock damage and the aftershock damage is examined. And, it aims at constructing an earthquake risk evaluation technique for considering not only the main shock but also the aftershock in this research to solve the problem clarified by the Chuetsu earthquake of the above-mentioned past research.

2. AFTERSHOCK HAZARD EVALUATION

2.1 Occurrence Frequency of Aftershocks

As for the aftershock hazard, the relationship between the main shock magnitude and the aftershock magnitude is studied actively. And the relationship between the elapsed time and the aftershock occurrence frequency when the main shock generation is made a starting point is also popular theme. However, it is insufficient only in information like aftershock magnitude distribution and the aftershock occurrence rate, because the final goal of this research is to treat the damage probability in the structures by the aftershock. Necessary information is distribution of the seismic ground motion by the aftershock in the evaluated site. Therefore, the seismic ground motion distribution on the evaluation site should be derived by using the relation of the occurrence frequency of the aftershock and magnitude of aftershock clarified by the past research.
When the aftershock hazard is described in this research, the following two expressions could be used.

1) ***Gutenberg-Richter(GR) law*** (scale dependencies of number of aftershocks generation)
The aftershock number n(M) whose magnitude range from M+dM from M could be expressed as follows. a and b are constants, and a is a parameter that shows the aftershock activity.

$$n(M) = 10^{(a-bM)} \tag{2-1}$$

2) ***the modified Omori(MO) law*** (time changes of number of generation of aftershocks)
The number of aftershocks decreases as time passes. ν(t) is aftershock occurrence number per time when t has been passing since generation of main shock, and then ν(t) follows the next expression .

$$\nu(t) = K(t+c)^{-p} \tag{2-2}$$

K, c, and p are constants of each aftershock.
$N(T_1, T_2)$ expected value of the aftershock occurrence frequency whose magnitude is more than M from $t=T_1$ to $t=T_2$ was expressed as eq.(2-3) at time when the above two formulae were combined. Please refer to the research of Hosono for more details.

$$\begin{aligned} N(T_1,T_2) &= K\exp\left(\beta(M-M_{th})\right)A(T_1,T_2) \\ &= 10^{\alpha+b(M_m-M)}A(T_1,T_2) \end{aligned} \tag{2-3}$$

$$A(T_1,T_2) = \begin{cases} \dfrac{(T_2+c)^{1-p}-(T_1+c)^{1-p}}{1-p} & (p \neq 1) \\ \ln(T_2+c)-\ln(T_1+c) & (p=1) \end{cases} \tag{2-4}$$

$$\alpha \equiv \log K - b(M_m - M_H) \tag{2-5}$$

And, the probability that the earthquake more than magnitude M occurs more than once between T2 and T1 can be shown as follows by the use of the idea of Poisson process.

$$P = 1-\exp(10^{\alpha+b(M_m-M)}A(T_1,T_2)) \tag{2-6}$$

2.2 Parameter

α , c, p, and b were peculiar value to the aftershock group. However, those values can be determined only after an aftershock does happen. Therefore it can be said that it is the most realistic method to treat an average value of the parameter as prior information on the aftershock generation.
A central value of the parameter in the past earthquakes that is proposed by Utsu (1969) is used in this research. The value is indicated in Table 1.

Table 1 Parameter

α	b	c	p
−1.83	0.85	0.3	1.3

2.2 Aftershock Area

The area where the hypocenters of the aftershock are distributed is called an aftershock area. It is known that the aftershock area expands by growing of the main shock magnitude. The aftershock area is assumed to be a circular region in which centers on the main shock epicenter in this research for easiness.
The following formula is known about area A(km2) of area where aftershock occurs when earthquake of main shock magnitude *Mm* occurs)

$$\log_{10} A = M_m - 3.7 \tag{2-7}$$

2.4. Earthquake hazard by aftershock

Step A

Next, the occurrence frequency of the aftershock of magnitude M is derived in the aftershock area. Γ mean occurrence rate of aftershocks whose magnitude M range from m to $m+dm$ during one day (from T to $T+1$) is expressed as below.

$$\begin{aligned}\gamma[m < M < m+dm, T] &= N(T, T+1, m+dm) - N(T, T+1, m) \\ &= \frac{dN}{dm}dm\end{aligned} \tag{2-8}$$

When the occurrence of the aftershock is assumed to be uniform in the aftershock, the probability that the aftershock occurs in minute area dA=dxdy of the point where is distant R from the evaluation site in the ground level is expressed as follows.

$$P[\textit{aftershock occurs in } dA] = \frac{dA}{A} \tag{2-9}$$

Step B

Probability P[$PGV>pgv$ |M at R] that PGV exceeds certain value pgv in the evaluation site (ground level) should be derived under the condition that the value of distance R(km) from the epicenter to the evaluation site and aftershock magnitude M are fixed. PGV_{v600} peak ground motion at engineering bedrock is shown as Eq.10 by Midorikawa's (1999) attenuation relation. R expresses the epicentral distance.

$$\begin{aligned}\log_{10} PGV_{v600} &= 0.58M + 0.0038D + d - 1.29 - \log_{10}(R(x,y) \\ &+ 0.0028 \times 10^{0.50M}) - 0.002R(x,y) + \varepsilon\end{aligned} \tag{2-10}$$

PGV_{v600} peak ground velocity (cm/s) : at soil whose S-wave velocity is 600m/s
M Moment Magnitude
D earthquake focal depth (km)
d : Coefficient of according to earthquake type crustal earthquake d=0
interpolate earthquake d= 0.12
ε variation standard deviation(log) 0.53

PGV_{v600} is a seismic ground motion observed on an engineering bedrock right under the evaluation site, it is necessary to consider the effect of the ground amplification in the evaluation. The next expression proposed by Midorikawa is used for ground amplification.

$$\log_{10} R_V = 1.83 - 0.66 \log_{10}(AVS_{30}) \tag{2-11}$$

R_V shows the amplification rate of PGV on engineering bedrock and AVS_{30} shows the average S wave velocity of the ground up to 30m in depth. The value of the ground amplification in Tokyo becomes 1.36 by using this expression. The relation between PGV in the ground level and PGV_{v600} is expressed by the following expressions.

$$PGV = 1.36 PGV_{v600} \tag{2-12}$$

$P[PGV>pgv \mid M \ at \ R]$ probability that PGV exceeds pgv at evaluation site could be expressed as Eq.(2-13), when $f_{PGV600}(v)$ means PDF of variation ε by attenuation relation.

$$P\left[PGV > pgv \middle| M \ at \ R\right] = \int_{pgv/1.36}^{+\infty} f_{pgv600}(v) dv \tag{2-13}$$

Step C

Frequency that earthquake wave whose PGV exceed pgv is observed at evaluation site in one day (from T-th days to (T+1)-th day) is derived as follows by use of McGuire method.

$$\begin{aligned} &\gamma(PGV(T) > pgv) \\ &= \iiint \left|dN/dM\right| P\left[PGV(T) \ge pgv \middle| M \ at \ R(x,y)\right] \frac{1}{A} dxdydM \end{aligned} \tag{2-14}$$

3. FRAGILITY OF STRUCTURE

3.1 Fragility

There are assumed to be three levels in structural damage state; major damage, middle damage and no damage. To carry out risk evaluation of structural damage, fragility data of structure is necessary as well as aftershock hazard. Murao's fragility curve is used for fragility curve that express probability from undamaged to major damage and from undamaged to middle damage.

However, the aftershock generation after the main shock should be evaluated in this research. For main purpose of this research is to construct probabilistic model in which aftershock causes serious damage on structures that have been damaged by main shocks. Therefore it is necessary to acquire the state transition probability that minor damaged structure becomes major damaged by another earthquake ground motion.

It is shown that there is relation of Table 2 between the Cumulative Plastic Ductility Ratio η and the damage level according to past research[12)]. That research divides damage level into four; no damage, minor, middle, major damage. In this research, minor damage is included to "no damage" and "no damage" means the structures whose Cumulative Plastic Ductility Ratio is from 0 to 10.

table2 relationship between cumulative plastic ductility ratio damage level

Damage level	cumulative plastic ductility ratio η
Major	$20<\eta$
Middle	$10<\eta<20$
Minor	$5<\eta<10$
No damage	$\eta<5$

3.2 Evaluation of State transition probability of loss

Taking single-mass system we can consider the changes in structural damage through elasto-plastic analysis. Given eleven integers from a sequence of 10 to 20 as the middle damage of a cumulative plastic deformation rate η, when we input the 315 waves whose PGV value is set to be integrated into that model, we can get a response through a bilinear model elasto-plastic analysis. In the 315 waves, we count the number of seismic waves which made η of single-mass systems exceed 20 (major damage condition), and the value divided by that figure's 315 is the middle- damage rate. By performing this method, we display with Eq. (3-1) and Fig. 1. The fragility curve leading from middle to major damages.

$$P_E(pgv) = \Phi((\ln pgv - 3.17)/0.65) \tag{3-1}$$

4. **Structural damage Assessment of Buildings Damage Considering Aftershocks of Earthquakes**

4.1 application of Markov

As mentioned above, the state of structure which could have three damages level changes by an occurrence of earthquake. So it is possible to construct probabilistic model with markov chain. The state of no damage is named “state 1”, the state of middle damage is named “state 2” and the state of major damage is named “state 3”. MO law shows that frequency of aftershock decrease by time elapsed since main shock. Therefore the event that one day passes should be regarded as one process in Markov chain. In this research, elements of transition-probability-matrix $p^n_{k,j}$ express the probability that the state of one structure on evaluation site change from k to j. Recovery of structure is ignored in this research. $p^n_{2,1}, p^n_{3,1}$ and $p^n_{3,2}$ are 0 and $p^n_{3,3}$ is 1. transition-probability-matrix can be expressed as Eq.(4-1)

$$P_{transition}(T) = \begin{bmatrix} p^T_{1,1} & p^T_{1,2} & p^T_{1,3} \\ 0 & p^T_{2,2} & p^T_{2,3} \\ 0 & 0 & 1 \end{bmatrix} \tag{4-1}$$

The state probability vector $\vec{P}(n)$ can express the state after n times transition of the system as Eq.(4-2). $p_k(n)$ expresses probability that structure becomes “state k” after n times transition of the system.

$$\vec{P}(n) = \left[p_1(n) \quad p_2(n) \quad p_3(n) \right] \tag{4-2}$$

$\vec{P}(n)$ is expressed as follows by use of the transition probability matrix and $\vec{P}(0)$ which indicate initial state of the system.

$$\vec{P}(n) = \vec{P}(0) P_{transition}(1) P_{transition}(2) \cdots\cdots P_{transition}(n) \tag{4-3}$$

The transition of the damage probability by elapsed time in the building since the main shock is derived by above expression.

5. Consideration concerning effect of aftershock

5.1 transition of damage

The left figure of fig.2 shows the graph in which the value of the damage probability (major damage) 30days later after main shock is shown in Z axis, epicentral distance *R* is shown in X axis and *Mm* is shown in Y axis. The right graph shows the damage probability (major damage) which doesn’t consider influence of aftershock. As the graph shows, the damage probability considering aftershock is higher than the damage probability without considering aftershocks. This graph cannot shows the area where the damage probability is relatively higher; large magnitude and small epicentral distance clearly. Therefore the graph in which Z axis is expressed in linear scale is shown in fig.3.

In the area where the damage probability (major damage) is over 10%, the minimum of aftershock effect is 1.3($M_{m=}$9, R=0km) and maximum of aftershock effect is 2.8 times ($M_{m=}$8.5, R=100km). The effect of aftershock is quite striking.

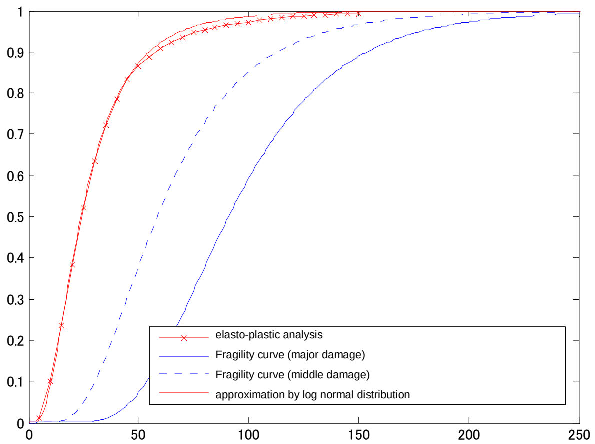

Fig.1 fragility curve of middle damage

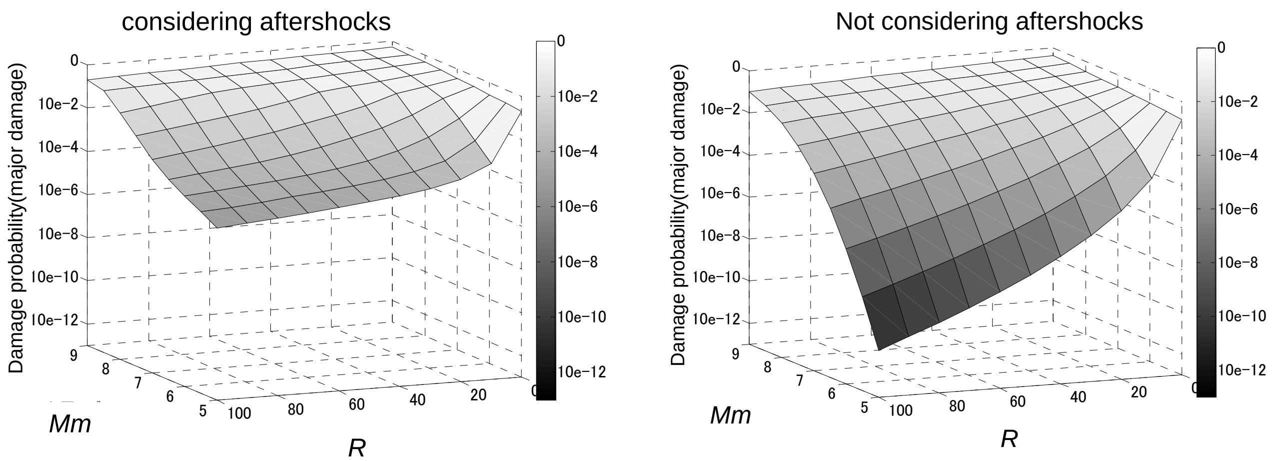

Fig.2 damage probability (log scale)

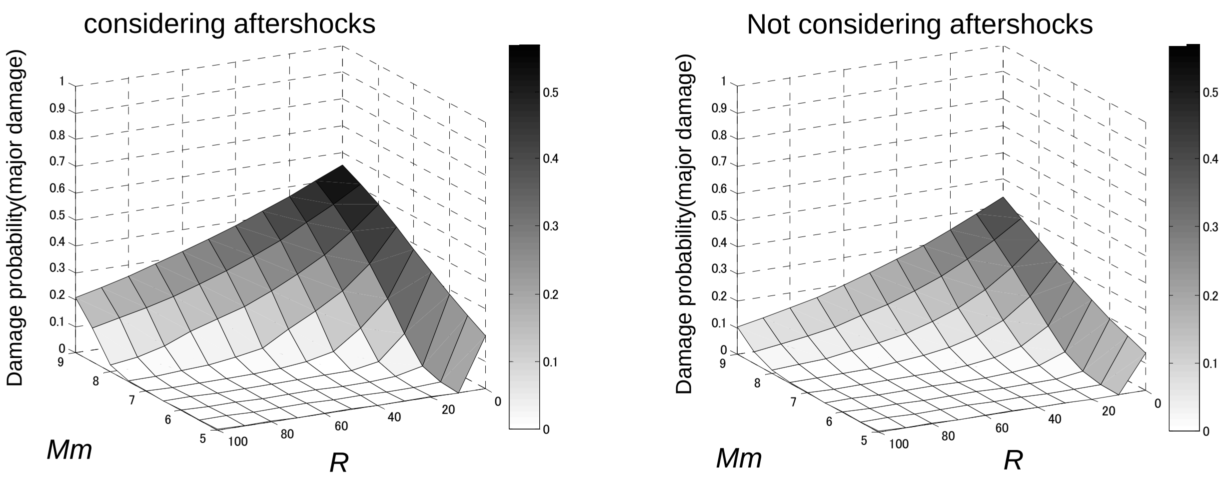

Fig.3 damage probability (linear scale)

6. Conclusion

This research paid attention to what influence the compound of hazard gave to the disaster risk evaluation, examined a basic technique for the event of compound of the main shock damage and the aftershock damage, and proposed a theoretical formula. As a result, the following findings were obtained.

1) How the damage probability changed by elapsed time from main shock could be clarified
2) It was possible to compare damage probability considering aftershocks and without considering aftershocks.

As for this theoretic model, a new kind of thinking of these treatments is introduced, and a concrete examination incorporating other study results is planned in the future to be performed with the theoretic model's generalization.

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