

## OPTIMAL SEISMIC PERFORMANCE OF FRICTION ENERGY DISSIPATING DEVICES

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### ABSTRACT :

Friction-based passive energy dissipation devices (or dampers) are known to be very effective for controlling the seismic response of multistory buildings. The preliminary design of these devices is based on numerical simulations wherein sliding friction is modeled with Coulomb friction having a constant coefficient of friction. However, the basic laws for sliding materials and experimental investigations show strongly non-linear relationship between dry friction and sliding velocity, which includes stiction and Stribeck effect. This paper investigates energy dissipation devices in which the friction model considers the stiction and Stribeck effects. The optimal seismic performance of friction devices to reduce the response behavior of frame buildings has been numerically investigated.

**KEYWORDS:** Earthquake-Resistant Structures, Energy Dissipation, Friction Damper, Optimization, Passive Control, Performance Evaluation.

### 1. INTRODUCTION

The design parameters that control the influence of friction energy-dissipating devices used for aseismic design of structures are: (1) Damper locations in the building frame, (2) Slip loads at device level, and (3) Stiffness of the braces in which the devices are installed. Filiatrault and Cherry (1990) have developed a specialized algorithm to obtain the optimum slip-load distribution for the friction devices modeled as Coulomb's friction by minimizing a relative performance index (RPI) derived from energy concepts. They also developed slip-load spectrum for quick evaluation of optimum slip load. The spectrum takes into account the properties of the structure and of the ground motion anticipated at the site. From this study an important conclusion was drawn that the optimum slip load depends on the frequency and amplitude of the ground motions and is not strictly the structural property. Moreschi (2000) and Asahina et al. (2004) have followed a genetic algorithm approach to obtain optimum slip-load at device level and optimal configuration within the frames. The available procedure to get optimum slip-load provides acceptable response reduction to the frame. However, the effect of different performance indices (from different response parameters) has not been addressed in these studies. The optimum slip-load may be different for different performance indices. These and other past studies have prescribed procedures for determining optimal slip load and distribution of friction devices within frame structures based on Coulomb friction model. Realistic dry friction models, which consider the stiction and Stribeck effects, can improve the present practice by providing more accurate assessment of the performance of these devices. Earthquakes are also random phenomena with both uncertain intensities and frequency contents. So ground motion characterization also needs to be taken into consideration for robust design of energy dissipation devices.

In this paper, the response of four-storey example frame building with friction devices has been investigated. The paper discusses the following aspects: (i) Evaluation of various dimensionless performance indices to characterize the effectiveness of friction devices, and (ii) Evaluation of optimal seismic performance of friction devices considering stiction and Stribeck effect into realistic friction model.

### 2. REALISTIC DRY FRICTION MODEL

The following realistic friction model has been considered in the investigation of response behavior of frame structure with friction device (Fig. 1), where  $F$  is the friction force and  $\dot{u}$  is the relative velocity. It has been

observed through various experimental studies that friction force does not remain constant. The frictional resistance of dry friction modeled based on experimental and theoretical investigations by Wang and Shieh (1991) shows that the friction force during sliding,  $F$ , obeys the following exponential law:

$$F = \left[ F_d + (F_s - F_d) \exp\left(-\left|\frac{\dot{u}}{\dot{u}_s}\right|\right) \right] \text{sgn}(\dot{u}) \quad (1)$$

$$F_d = \mu_d F_N, \quad F_s = \mu_s F_N$$

where  $F_N$  is the normal force (may be controllable if applied through prestress force) on the sliding surface,  $F_d$  is the lower bound limit of the sliding frictional resistance and  $F_s$  is its upper bound limit,  $\mu_d$  is the coefficient of sliding friction at relatively large velocity, and  $\mu_s$  is the coefficient of sliding friction at the point of zero velocity. The frictional resistance at sliding stage,  $F$ , varies from the upper bound limit ( $F_s$ ) to the lower bound limit ( $F_d$ ). The variation of friction resistance is a function of the sliding velocity ( $\dot{u}$ ) and the Stribeck velocity ( $\dot{u}_s$ ). The Stribeck velocity ( $\dot{u}_s$ ) can be regarded as the decay rate of the sliding friction coefficient. The typical friction force variation for realistic friction model is shown in Fig. 1(b).

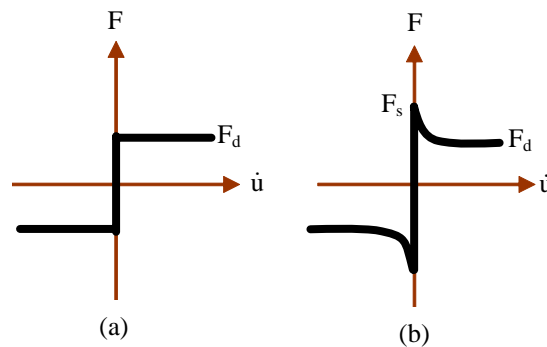


Figure 1. Dry Friction Models (a) Coulomb Friction Model, (b) Realistic Friction Model

### 3. MATHEMATICAL FORMULATION OF EQUATIONS OF MOTION

The mathematical formulation of multi-degree-of-freedom (MDOF) frame structure with friction slider mounted on Chevron brace (Fig. 2) has been presented herein. The structure is considered as a two-dimensional (2-D) shear building. Two degrees-of-freedom are present on each floor, corresponding to the horizontal displacement of the storey and the brace, respectively, relative to the ground, as shown in Fig. 2(a). Simple friction energy-dissipation devices with slotted bolted connection (SBC) has been considered, where the sliding plate within the vertical plane is connected to the centerline of beam soffit as shown in Fig. 2(b). It may be noted that the beam weight and its loading does not have any effect to the normal load applied on the sliding surface. The sliding plate having slotted holes is sandwiched between two clamping plates. The clamping plates are rigidly mounted on the Chevron brace and connected to the sliding plate through prestressed bolts. The slotted holes facilitate the sliding of the sliding plate over the frictional interface at a constant controllable prestress force. The placement of sliders in vertical plane of the beam ensures that only the prestress force controls the normal load on the sliding surface. The presence of two friction interfaces for each bolt doubles the frictional resistance. In the formulation of the MDOF frame, the structure degrees-of-freedom is denoted with subscript  $f$  and the brace with device degrees-of-freedom with subscript  $d$ . Two lumped mass models, one for the free frame structure and another for the brace with device, are required to idealize the dynamic behavior of the structure. Through the entire solution process, the equations of motion are split into two subsets with sub-indices  $st$  representing the stick phase (non-sliding phase) and  $sl$

representing the sliding phase respectively. The motion of any storey of the structure consists of either of two phases: (1) non-sliding or stick phase wherein the frictional resistance ( $F_{st}$ ) between the floor and the device has not been overcome, and (2) sliding or slip phase in which sliding frictional resistance ( $F_{sl}$ ) exceeds and the friction force, and acts opposite to the direction of the relative velocity between the floor and friction device. Linear behavior of the structure with friction devices is assumed, and verified, at both stick and sliding stage of response. The overall response for each storey consists of series of non-sliding and sliding phases. The number of active degree of freedom ranges between  $N$  (all the devices in non-sliding phase) and  $2N$  (all devices are in sliding phase). If the total number of non-sliding floors are denoted by  $n_{st}$  and total number of sliding floors  $n_{sl}$ , then the total number of degrees of freedom at any instant of time is equal to  $n_{st} + 2 \times n_{sl}$ .

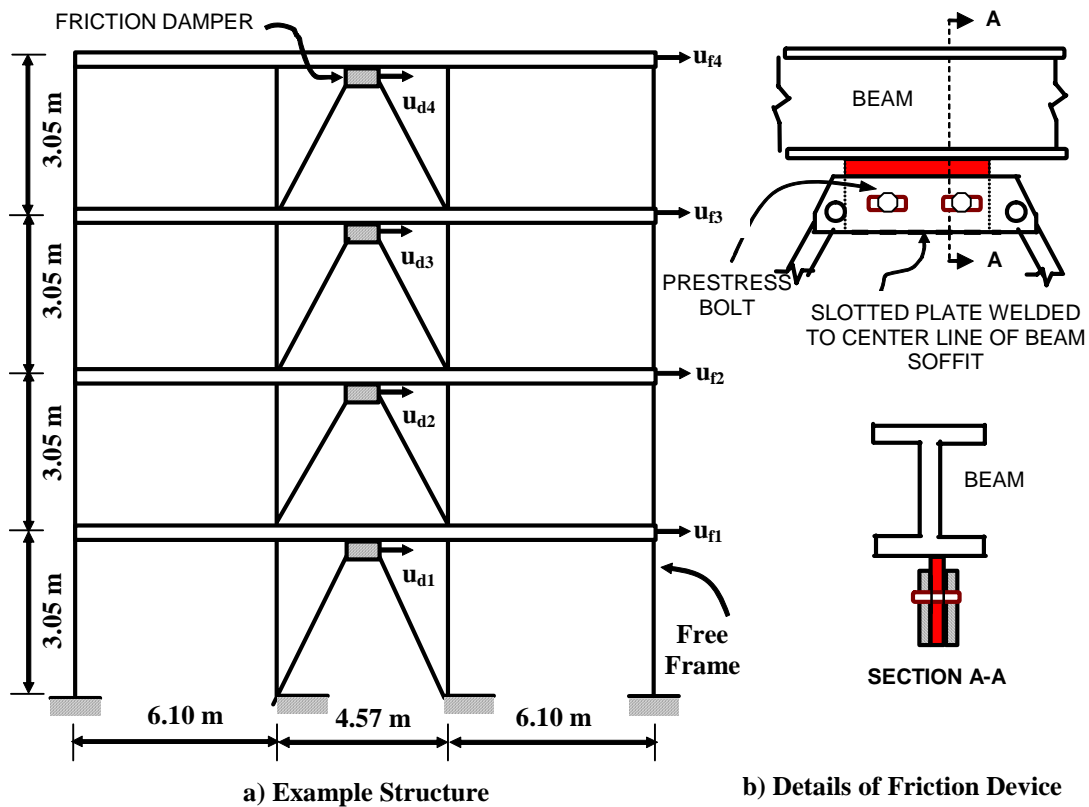


Figure 2. Schematic diagram of four-storey building with friction devices (Dimova et al. 1995)

The generalized governing equations of motion in matrix form can be given as:

$$\mathbf{M}\ddot{\mathbf{u}}_{st+sl} + \mathbf{C}\dot{\mathbf{u}}_{st+sl} + \mathbf{K}\mathbf{u}_{st+sl} = -\mathbf{M}\mathbf{r}\ddot{u}_g - \mathbf{F}_{f+sl} \quad (2)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$ , and  $\mathbf{K}$  are the mass, damping and stiffness matrices, respectively,  $\mathbf{r}$  is the force-influence vector,  $\mathbf{u}$  represents the displacement degrees of freedom relative to the base of the structure and  $u_g$  is the ground displacement. The over dot represent derivatives with respect to time. The friction force vector is represented as  $\mathbf{F}$ , and the matrices are given as:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_f & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_d \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{C}_f + \mathbf{C}_{d2} & \mathbf{C}_{d3} \\ (\mathbf{C}_{d3})^T & \mathbf{C}_{d1} \end{bmatrix}, \mathbf{K} = \begin{bmatrix} \mathbf{K}_f + \mathbf{K}_{d2} & \mathbf{K}_{d3} \\ (\mathbf{K}_{d3})^T & \mathbf{K}_{d1} \end{bmatrix}, \quad (3)$$

$$\mathbf{u}_{st+sl} = \begin{Bmatrix} \mathbf{u}_{f,st+sl} \\ \mathbf{u}_{d,st+sl} \end{Bmatrix}, \mathbf{r} = \begin{Bmatrix} \mathbf{r}_f \\ \mathbf{r}_d \end{Bmatrix}, \mathbf{F} = \begin{Bmatrix} +\mathbf{F}_{f+sl} \\ -\mathbf{F}_{f+sl} \end{Bmatrix}, \mathbf{r}_f = \mathbf{1}, \mathbf{r}_d = \mathbf{1}$$

where

$$\mathbf{M}_d = \begin{bmatrix} m_{d1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & m_{dN} \end{bmatrix},$$

$$\mathbf{C}_{d1} = \begin{bmatrix} c_{d1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & c_{dN} \end{bmatrix}, \mathbf{C}_{d2} = \begin{bmatrix} c_{d2} & 0 & 0 \\ 0 & \ddots & c_{dN} \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{C}_{d3} = \begin{bmatrix} 0 & -c_{d2} & 0 \\ 0 & 0 & \ddots & -c_{dN} \\ 0 & 0 & 0 \end{bmatrix}, \quad (4)$$

$$\mathbf{K}_{d1} = \begin{bmatrix} k_{d1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & k_{dN} \end{bmatrix}, \mathbf{K}_{d2} = \begin{bmatrix} k_{d2} & 0 & 0 \\ 0 & \ddots & k_{dN} \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{K}_{d3} = \begin{bmatrix} 0 & -k_{d2} & 0 \\ 0 & 0 & \ddots & -k_{dN} \\ 0 & 0 & 0 \end{bmatrix}$$

In the above equations,  $\mathbf{M}_f$ ,  $\mathbf{C}_f$ , and  $\mathbf{K}_f$  are the  $N \times N$  mass, damping and stiffness matrices of the structure excluding the bracing members,  $\mathbf{M}_d$ ,  $\mathbf{C}_{d1}$ ,  $\mathbf{C}_{d2}$ ,  $\mathbf{C}_{d3}$ ,  $\mathbf{K}_{d1}$ ,  $\mathbf{K}_{d2}$  and  $\mathbf{K}_{d3}$  are  $N \times N$  mass, damping and stiffness matrices of the brace with friction device, respectively. The damping property of the free frame (excluding the brace with device) structure may be different from the same of the brace with device. So the complete structure is non-classical damped system. The non-classical damping matrix  $[\mathbf{C}]$  for the structure is obtained by first evaluating the classical damping matrix for the free frame,  $[\mathbf{C}_f]$ , based on the damping ratios appropriate for the structure (Chopra 2001).

The structure and the brace degree of freedoms (DOFs) at any storey satisfy the following conditions during the stick phase:

$$\begin{aligned} \ddot{\mathbf{u}}_{f,st} &= \ddot{\mathbf{u}}_{d,st} \\ \dot{\mathbf{u}}_{f,st} &= \dot{\mathbf{u}}_{d,st} \\ \mathbf{u}_{f,st} - \mathbf{u}_{d,st} &= \text{constant} \end{aligned} \quad (5)$$

In Eq. (2), stick or non-sliding phase of a particular floor requires that the corresponding friction force satisfy the equation,

$$|F_{f,st}| < F_{st} \quad (6)$$

The friction force vector consisting of the friction force in all the devices is given by:

$$\begin{aligned} \mathbf{F}_{f,st} &= \mathbf{M}_{f,st} \ddot{\mathbf{u}}_{f,st+sl} + \mathbf{M}_{f,st} \mathbf{r}_f \ddot{\mathbf{u}}_g + (\mathbf{C}_{f,st} + \mathbf{C}_{d2,st}) \dot{\mathbf{u}}_{f,st+sl} + \\ &\quad \mathbf{C}_{d3,st} \dot{\mathbf{u}}_{d,st+sl} + (\mathbf{K}_{f,st} + \mathbf{K}_{d2,st}) \mathbf{u}_{f,st+sl} + \mathbf{K}_{d3,st} \mathbf{u}_{d,st+sl} \end{aligned} \quad (7)$$

In Eq. (7),  $\mathbf{F}_{f,sl}$  is the vector of frictional resistance of all friction devices at stick stage. When the condition in Eq. (6) is not satisfied for a particular floor, the damper on that floor enters into the sliding phase. Then the corresponding brace degree-of-freedom at the floor level also becomes active in the equations of motion. The maximum frictional resistance in stick stage ( $F_{st}$ ) and frictional resistance in sliding stage ( $F_{sl}$ ) for a friction device considering realistic friction model is given by Eq. (1). The direction of sliding of a brace degree of freedom can be found from the following relationship:

$$\text{sgn}(\dot{u}_f - \dot{u}_d) = -\frac{F_{f,sl\max}}{|F_{f,sl\max}|} \quad (8)$$

The response of the structure always starts in the stick phase. This phase of response continues until the unbalanced frictional resistance of any floor exceeds the maximum frictional resistance of the brace with device at that floor. It is important to note that the number of stories experiencing stick and sliding conditions varies continuously through the entire response phase. When the relative sliding velocity ( $\dot{u}_f - \dot{u}_d$ ) at any floor becomes zero or changes its sign during motion, then the brace with device at that storey may or may not enter the stick phase. It may reverse its direction of sliding or have a momentary halt and continue in the same direction. The status of motion during transition phase can be evaluated from Eq. (6). The equations of motion corresponding to the floor is changed to the appropriate stick or sliding equations before evaluating response during the next time-step.

#### 4. PERFORMANCE INDICES

To characterize the seismic efficiency of friction devices, six dimensionless performance indices have been considered (Patro 2006). All these indices are defined as the ratios between the maximum values of a certain response quantity (displacements, acceleration, base shear, strain energy, input energy, and dissipated energy) of the frame with friction devices, and the maximum value of same responses of the bare or braced frame structure. The response quantities in the indices are determined from full time-history of response and among all the floors. Consequently, these indices are dimensionless and always positive with their value range usually between 0 and 1. Values close to zero indicate excellent performance of the friction devices in reducing the response while values close to 1 or higher indicate ineffectiveness of the friction devices. A number of different indices have been considered for the performance evaluation as given below. The performance indices are determined by normalizing the structural response when using the friction devices to the corresponding response of the free frame structure or the braced frame structure. Normalization with both free frame and braced frame structural response has been presented in this study to clearly demonstrate the difference in behavior of the structure with friction devices from both these configurations.

#### 5. EXAMPLE SYSTEM

A four-storey steel frame building with friction devices (Fig. 2) has been considered for evaluating the seismic performance of friction devices. The braces with damping devices exhibit highly non-linear behavior. The effect of energy dissipation due to viscous damping in the brace members is normally very small compared to the work done by friction sliding. So the viscous damping in the brace has been neglected. The structural damping ratios of the free-frame have taken as 2% of its critical damping. The example building has the following member properties (Dimova et al. 1995):

Floor masses:  $m_{f1} = m_{f2} = m_{f3} = 41610.0\text{kg}$ , and  $m_{f4} = 40820.0\text{kg}$

Moment of Inertia of ground and first floor columns:  $8740.8\text{ cm}^4$

Moment of Inertia of second and third floor columns:  $7117.5\text{ cm}^4$

Moment of Inertia of roof girders:  $12486.0\text{ cm}^4$

Moment of Inertia of floor girders:  $15567.0\text{ cm}^4$

Moment of Inertia of bracing members:  $77.8\text{ cm}^4$

Cross sectional area of bracing members:  $7.57 \text{ cm}^2$

Mass of bracings and friction dampers:  $m_{d1} = m_{d2} = m_{d3} = m_{d3} = 23.0 \text{ kg}$

Stiffness of bracing members:  $k_{d1} = k_{d2} = k_{d3} = k_{d4} = 28.6 \text{ MN/m}$

The fundamental time period of the free frame and braced frame buildings are 1.00 s and 0.56 s, respectively. The total normal load ( $F_N$ ) on the sliding surface is equal to  $2n_b F_{Ni}$ , where  $n_b$  is the number of prestress bolts, and  $F_{Ni}$  is the prestress force in a single bolt. All the bolts in a particular friction device are assumed to have the same prestress force. The value of coefficient of friction has been considered for steel on steel slider as reported by Bilkay and Anlagan (2004). Based on their investigations the following friction parameters have been used for friction models: (1) Minimum coefficient of sliding friction (sliding stage,  $\mu_d$ ) = 0.16, (2) Maximum coefficient of sliding friction (stick stage,  $\mu_s$ ) = 0.29, and (3) Stribeck velocity  $\dot{u}_s = 0.025 \text{ m/s}$ .

In this investigation, the time-history responses of the example system have been evaluated for an ensemble of nine earthquake records (Patro 2006). The ground motions chosen in this study cover wide variety of earthquakes having different peak ground accelerations (PGA), frequency content and duration. Three time histories have been selected for each of soft soil (FSR1-3), alluvium soil (FMR1-3), and hard soil (FHR1-3).

Table 1. Optimal performance indices normalized with free frame for example structure considering realistic friction model

|                              | Ground Motion |       |       |       |       |       |       |       |       |
|------------------------------|---------------|-------|-------|-------|-------|-------|-------|-------|-------|
|                              | FSR1          | FSR2  | FSR3  | FMR1  | FMR2  | FMR3  | FHR1  | FHR2  | FHR3  |
| <b><math>F_N</math> (kN)</b> | 536.2         | 731.2 | 698.7 | 828.7 | 731.2 | 536.2 | 146.2 | 211.2 | 633.7 |
| <b>IDRF</b>                  | 0.298         | 0.244 | 0.452 | 0.345 | 0.569 | 0.491 | 0.564 | 0.364 | 0.571 |
| <b><math>F_N</math> (kN)</b> | 178.7         | 276.2 | 16.25 | 503.7 | 81.25 | 146.2 | 276.2 | 146.2 | 178.7 |
| <b>AARF</b>                  | 0.576         | 0.682 | 0.901 | 0.545 | 0.828 | 0.416 | 0.656 | 0.573 | 0.720 |
| <b><math>F_N</math> (kN)</b> | 276.2         | 471.2 | 16.25 | 796.2 | 731.2 | 146.2 | 146.2 | 211.2 | 536.2 |
| <b>DARF</b>                  | 0.494         | 0.538 | 0.932 | 0.543 | 0.739 | 0.473 | 0.619 | 0.498 | 0.696 |
| <b><math>F_N</math> (kN)</b> | 341.2         | 438.7 | 16.25 | 796.2 | 601.2 | 113.7 | 113.7 | 146.2 | 243.7 |
| <b>BSRF</b>                  | 0.447         | 0.565 | 0.955 | 0.498 | 0.616 | 0.613 | 0.714 | 0.562 | 0.819 |
| <b><math>F_N</math> (kN)</b> | 536.2         | 731.2 | 698.7 | 828.7 | 731.2 | 536.2 | 146.2 | 211.2 | 633.7 |
| <b>RPIF</b>                  | 0.074         | 0.074 | 0.226 | 0.132 | 0.387 | 0.301 | 0.284 | 0.158 | 0.240 |
| <b><math>F_N</math> (kN)</b> | 308.7         | 438.7 | 406.2 | 828.7 | 828.7 | 308.7 | 211.2 | 146.2 | 601.2 |
| <b>IWD</b>                   | 0.177         | 0.174 | 0.199 | 0.171 | 0.161 | 0.170 | 0.182 | 0.180 | 0.173 |

## 6. EVALUATION OF OPTIMAL PERFORMANCE

The performance of the friction devices for response reduction of buildings has been evaluated for a range of prestress forces. The prestress force has been varied from 1% ( $F_N = 16 \text{ kN}$ ) to 51% ( $F_N = 825 \text{ kN}$ ) of total floor weight (1625 kN) of the 4-storey example building subjected to an ensemble of base excitations corresponding to ground motions recorded under different soil conditions. The prestress force in damping devices is assumed to be same at all floor levels. The performance indices used to evaluate the seismic performance of friction devices must represent the overall response of the building. The various performance indices, which are discussed earlier, have been evaluated for realistic friction model for the example building subjected to all nine ground motions, and their results are shown in Fig. 3(a-i). In Fig. 3(a-i) performance indices are normalized with fixed braced frame response. It has been observed that the optimum prestress force varies between indices.



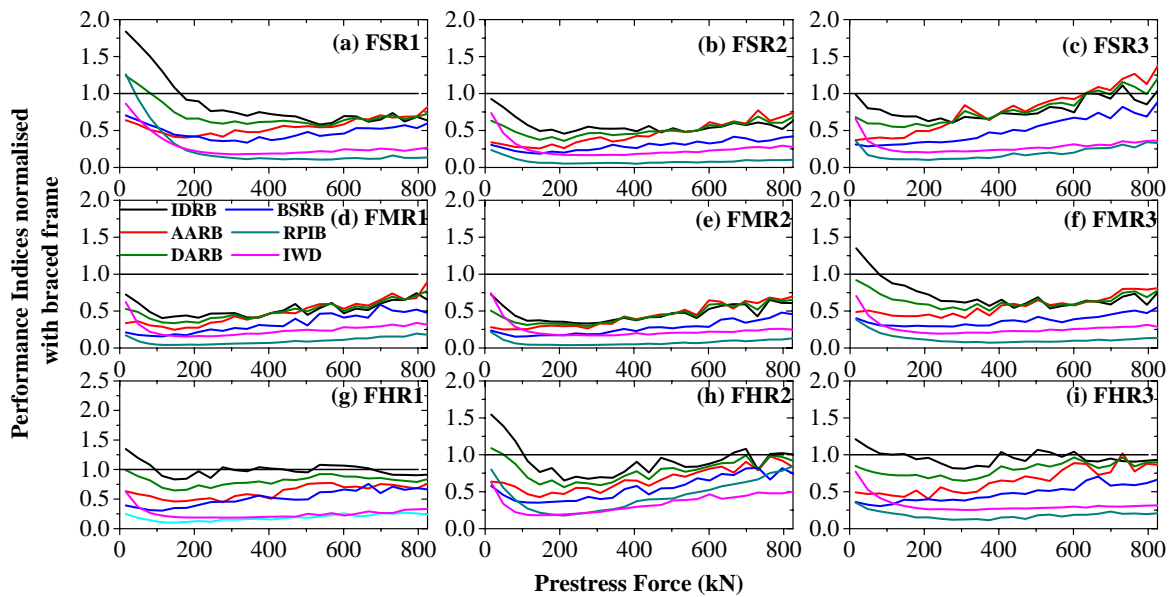


Figure 3. Performance indices, normalized with braced frame structure response, for different ground motions

To determine the optimal reduction of different response quantities, responses are estimated for the whole range of prestress force, and the minimum value of the performance indices and the corresponding prestress force are presented in Table 1. In Table 1 performance indices are normalized with free frame response. It is seen in Fig. 3(a-i) and Table 1 that optimum prestress force at indices using absolute floor acceleration differs significantly from those using floor displacements or inter-storey drift. The widely used Relative Performance Index (RPI) closely matches the pattern of drift response index (IDR). It should also be noted that the uses of friction devices reduce the structural responses through a combination of energy dissipation and increase in lateral stiffness. It can be seen in Table 1 that different performance indices are minimized for very different prestress forces. The optimum structural performance can be obtained for prestress force between 500 kN – 800 kN if the IDR or the RPI are to be minimized, and between 150 kN – 300 kN if the AAR are to be minimized. It is therefore not possible to determine a unique prestress force resulting in optimal reduction of all response quantities.

## 7. DISCUSSIONS AND CONCLUSIONS

The paper presents the aseismic response behavior of buildings with friction devices. The friction force has been modeled using the realistic dry friction model that includes both stiction and Stribeck effects. An ensemble of nine ground motions recorded on different soil conditions has been considered to evaluate the effectiveness of friction devices for vibration control. It is found that the prestress force (normal load on sliding surface) is the most important parameter for the design of the friction devices. It is also observed that optimal prestressing force varies over different response parameters.

Based on the investigations presented in this paper, the following main conclusions can be drawn:

- Most structures with friction-based energy dissipation systems are designed based on Coulomb friction; however behavior of actual friction is more complex and includes the stiction and Stribeck effects. The friction model should represent the actual behavior.
- Realistic friction model, which considers the effects of stiction and Stribeck, may alter the effectiveness of present practice.
- The optimum prestress force may be very different for different response quantities of interest such as

relative floor displacement, storey drift or absolute floor acceleration

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