SIMPLIFIED DESIGN METHODOLOGY FOR SYSTEMS EQUIPPED WITH NON-LINEAR VISCOUS DAMPERS

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ABSTRACT :

A simplified design method for SDF and MDF systems equipped with non-linear viscous dampers is proposed in this paper. It is known that the response of non-linear viscous dampers is proportional to a fractional power-law of the velocity, whose exponent ranges between 0.1 and 1. The response of these systems is usually investigated by evaluating the supplemental damping ratio due to the non-linear dampers under elastic conditions. The method proposed by seismic codes and guidelines is the equivalent energy approach. Following this approach the supplemental damping ratio is related to the maximum displacement, so that iterative procedures are required. In order to avoid these iterative procedures a direct method has been studied. A new dimensionless parameter, called damper index, not related to the maximum displacement of the system, has been introduced. The response of the whole system can be expressed as a function of the fundamental period of the structure, of the exponent of velocity and of the proposed parameter. The response of SDF and MDF systems has been calculated, numerically, considering harmonic external force and recorded ground motions. This numerical investigation has been performed in order to validate the proposed method. Finally a procedure for determining the spectrum of supplemental damping ratio related to the damper index from response spectra provided in seismic codes is proposed.

KEYWORDS: Non-Linear Viscous Damper, Supplemental Damping Ratio, Earthquake Response

1. INTRODUCTION

The simplified procedures proposed in literature to design structures equipped with non-linear viscous dampers are usually based on the assessment of the effective damping ratio ($\xi_{eff.}$). Operating in this way it becomes easy to predict the response of the system by using the damping reduction factor (B) tabled in FEMA 450 (BSSC, 2003) on the basis of recent works (Ramirez et al., 2002b). The effective damping ratio is the sum of three terms (Ramirez et al., 2002a): the inherent damping ratio (ξ_0), usually equal to 5%, the damping ratio related to the presence of viscous dampers (ξ_V) and the hysteretic damping ratio related to the non-linear behavior of the structure (ξ_H). It has been shown that ξ_V can be expressed as $\xi_{sd} f(\mu, \alpha)$, where ξ_{sd} is the supplemental damping ratio due to the presence of viscous dampers under elastic conditions and $f(\mu, \alpha)$ is a factor related to the non-linear behavior of the structure (μ is the displacement ductility demand) and to the exponent of velocity α. The main issue is to assess the supplemental damping ratio, which is the term really affected by the presence of the viscous dampers.The supplemental damping ratio can be obtained by imposing the equivalence between a non-linear damper and a linear one. The criteria developed in literature are expressed in terms of energy dissipated (Lin and Chopra, 2002) or power consumption (Peckan et al., 1999). In all these approaches the equivalent damping ratio is related to the maximum displacement of the system, so that iterative procedures have to be implemented. These criteria can be easily extended to the MDF case (Ramirez et al., 2000; Whittaker et al., 2003). In this work a procedure to assess the supplemental damping ratio directly, without any iterative process, is proposed.

2. SUPPLEMENTAL DAMPING RATIO

The supplemental damping ratio can be evaluated by imposing the equivalence between a non-linear viscous damper and a linear one, so that the supplemental damping ratio is equal to the damping ratio of the linear damper.

2.1. SDF system

Considering a cycle of harmonic motion $u(t)=u_0\sin\Omega t$, the supplemental damping ratio can be expressed as follows (Lin and Chopra, 2002):

$$\xi_{sd} = \frac{\lambda}{\pi}\frac{1}{2m\omega}\frac{c_\alpha}{\left(\Omega u_0\right)^{1-\alpha}} \tag{2.1}$$

where m is the mass of a SDF system undergoing the harmonic motion, ω is the natural frequency, c_α is the damper coefficient and α is the damper exponent. It should be noticed that in Eqn. 2.1 the term u_0, which is the maximum displacement, is related, in usual problems, to the response. The constant λ has been calculated (Ramirez et al., 2002a) and tabled in FEMA 450 (BSSC, 2003) for different values of α. For a linear damper (α= 1.00) λ is equal to π.

2.2. MDF system

The supplemental damping ratio for a MDF system equipped with non linear viscous dampers can be evaluated by using the concept of equivalent linear viscous damping (Chopra, 2001). Operating in this way, the supplemental damping ratio for the first mode of vibration is obtained as follows:

$$\xi_{sd,1} = \frac{\sum_j \left(2\pi\right)^{\alpha_j} T_1^{2-\alpha_j} \lambda_j c_{\alpha j} f_j^{1+\alpha_j} D_{roof}^{\alpha_j -1} \phi_{rj}^{1+\alpha_j}}{8\pi^3 \sum_i \left(\frac{w_i}{g}\right)\phi_{i1}^2} \tag{2.2}$$

where D_{roof} is the amplitude of roof displacement, T_1 is the undamped first period of vibration, ϕ_{i1} is the first undamped mode shape (normalized so that ϕ_{i1}=1.00 for i corresponding to roof), the index j is referred to each damper, $c_{\alpha j}$ is the damper coefficient, f_j is the displacement magnification factor, $\phi_{rj} = \phi_{j1} - \phi_{(j-1)1}$ is the difference between the first modal ordinates associated with degrees of freedom j and (j -1) and α_j is the damper exponent.

3. EQUATIONS OF MOTION

By introducing the supplemental damping ratio, the equation of motion for a SDF system with mass m equipped with a non-linear viscous damper, and subjected to an harmonic external force $ma_0\sin\Omega t$, is:

$$\ddot{u} + 2\xi_0\omega\dot{u} + 2\xi_{sd}\omega\left(\lambda/\pi\right)^{-1}(\Omega u_0)^{1-\alpha}\,\mathrm{sgn}\left(\dot{u}\right)\left|\dot{u}\right|^{\alpha} + \omega^2 u = a_0 \sin\left(\Omega t\right) \tag{3.1}$$

It should be noticed that the term related to the response, u_0, is still included in the equation. In case of ground acceleration $\ddot{u}_g(t)$, in the expression of ξ_{sd} (Eqn. 2.1) the frequency of the external force, Ω, is taken equal to the natural frequency, ω, so that Eqn. 3.1 becomes:

$$\ddot{u}+2\xi_0\omega\dot{u}+2\xi_{sd}\omega\left(\lambda/\pi\right)^{-1}(\omega u_0)^{1-\alpha}\,\mathrm{sgn}\left(\dot{u}\right)\left|\dot{u}\right|^{\alpha}+\omega^2 u=-\ddot{u}_g\left(t\right) \tag{3.2}$$

It has been shown that the response of the Eqn. 3.2 is directly dependent on the amplitude of the ground acceleration $\ddot{u}_{g,0}$ (Lin and Chopra, 2002). This result is really significant because it allows to express the response in terms of displacement, by a dimensionless parameter: the deformation response factor R_d=$u_0\omega^2/\ddot{u}_{g,0}$. Furthermore we have pointed out that, referring to the Eqn. 3.1, the response is affected only by the ratio of the frequencies and not by their single values. By introducing the deformation response factor into Eqn. 3.1, the following equation of motion is derived:

$$\ddot{u}+2\xi_0\omega\dot{u}+2\xi_{sd}\frac{\Omega^{1-\alpha}}{\omega^{1-2\alpha}}\left(\lambda/\pi\right)^{-1}a_0^{1-\alpha}R_d^{\;1-\alpha}\,\mathrm{sgn}\left(\dot{u}\right)\left|\dot{u}\right|^{\alpha}+\omega^2 u=a_0\sin\left(\Omega t\right) \tag{3.3}$$

4. DAMPER INDEX

The damper index ε is defined as follows:

$$\varepsilon=\frac{\lambda}{\pi}\frac{c_\alpha}{2m}\frac{\omega^{1-2\alpha}}{\Omega^{1-\alpha}}\left(a_0\right)^{\alpha-1} \tag{4.1}$$

Operating in this way, the following relationship between ξ_{sd} and ε can be written:

$$\xi_{sd}=\varepsilon\left(R_d\right)^{\alpha-1} \tag{4.2}$$

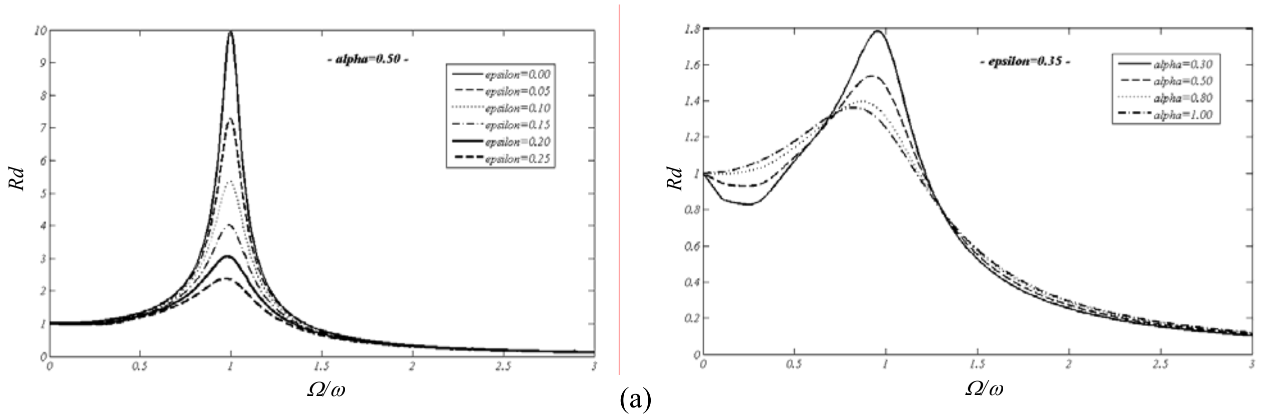
It can be noticed that for α=1.00 we find out ξ_{sd}=ε and the damper index does not depend on the response. The damper index is then introduced into Eqn. 3.1 and the following equation is obtained:

$$\ddot{u}+2\xi_0\omega\dot{u}+2\varepsilon\frac{\Omega^{1-\alpha}}{\omega^{1-2\alpha}}\left(\lambda/\pi\right)^{-1}a_0^{1-\alpha}\,\mathrm{sgn}\left(\dot{u}\right)\left|\dot{u}\right|^{\alpha}+\omega^2 u=a_0\sin\left(\Omega t\right) \tag{4.3}$$

The Eqn. 4.3 does not include any terms related to the response; all the terms included can be calculated once the characteristics of the system and the external force are known. In this way the response can be evaluated directly without any iterative procedure. The definition of the damper index (Eqn. 4.1) can be extended for the case of ground acceleration by substituting the frequency of the external force, Ω, with the natural frequency of the system, ω. In the same way the Eqn. 4.3 can be adapted.

4.1. Analysis of the response in terms of Damper Index

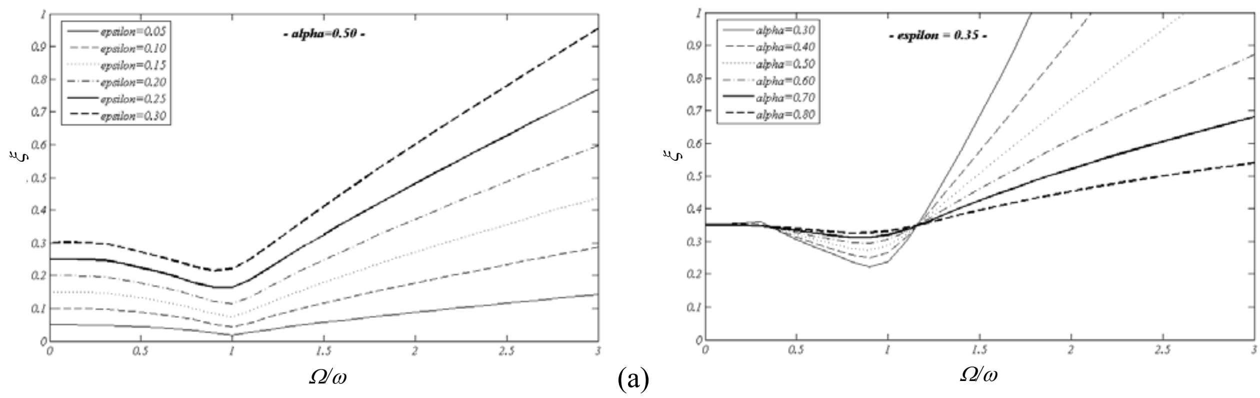
By solving numerically the Eqn. 4.3 it is possible to obtain the response. The graphs of the deformation response factor versus the ratio of the frequencies Ω/ω, for ξ_0 = 5%, are shown below for different values of ε and α=0.50 (Fig. 1a) and for different values of α and ε=0.35 (Fig 1b). Fig. 1a shows that the damper index affects the response significantly for values of the ratio Ω/ω around the resonance. Fig. 1b shows how much the exponent α affects the response for a fixed value of ε (that means fixed characteristics of the system): we can individuate a first zone for Ω/ω less than 0.70, where for increasing values of α the response increases, a second zone from Ω/ω equal to 0.70 to Ω/ω equal to 1.40 where this trend is inverted and a last zone for Ω/ω larger than 1.40 where the behavior is similar to that of first zone.



(a) (b)

Figure 1a Deformation Response Factor for ε equal to 0.00, 0.05, 0.10, 0.15, 0.20, 0.25 and $\alpha = 0.50$
Figure 1b Deformation Response Factor for α equal to 0.30, 0.50, 0.80, 1.00 and $\varepsilon = 0.35$

Once the response has been found by solving numerically the Eqn. 4.3, we can assess the deformation response factor, R_d and, through the Eqn. 4.2, the supplemental damping ratio, ξ_{sd}. The graphs of ξ_{sd} versus the ratio of the frequencies Ω/ω are illustrated below for different values of ε and α=0.50 (Fig. 2a) and for different values of α and ε=0.35 (Fig 2b).



(a) (b)

Figure 2a Supplemental Damping Ratio for ε equal to 0.05, 0.10, 0.15, 0.20, 0.25, 0.30 and $\alpha = 0.50$
Figure 2b Supplemental Damping Ratio for α equal to 0.30, 0.40, 0.50, 0.60, 0.70, 0.80 and $\varepsilon = 0.30$

Fig. 2b shows that the spectrum of the supplemental damping ratio can be divided into three zones, as it has been noticed in previous paragraph for the deformation response factor. In the two lateral zones, for a fixed value of ε, the supplemental damping ratio increases for decreasing values of α: that means that for small values of α the system is able to dissipate a larger quantity of energy and to reduce more efficiently the response. This result is in accordance with previous observations about Fig. 1b. This trend changes for the values of the ratio Ω/ω close to the resonance.

4.2 Damper index for MDF

In order to define the damper index, the relationship of Eqn. 4.2 can be used as follows:

$$\varepsilon_1 = \xi_{sd,1}\left(R_{D,1}\right)^{1-\alpha} \tag{4.4}$$

where $\xi_{sd,1}$ is expressed in Eqn. 2.2, $R_{D,1}$ is the deformation response factor for the first mode of vibration referred to the displacement of the top level, D_{roof}, and ε_1 is the damper index for the first mode of vibration. We can express $R_{D,1}$ as follows:

$$R_{D,1} = \frac{D_{roof}}{\Gamma_1 (PGA)} \omega_1^2 \tag{4.5}$$

where PGA is the peak ground acceleration, ω_1 is the frequency of the first mode of vibration and Γ_1 is the first mode participation factor. If we put the Eqn. 2.2 and the Eqn. 4.5 into the Eqn. 4.4 we finally obtain the subsequent relationship, considering α constant for all the dampers:

$$\varepsilon_1 = \frac{T_1^{\alpha}}{(2\pi)^{1+\alpha}} \frac{\lambda}{(\Gamma_1 PGA)^{1-\alpha}} \frac{\sum_j c_{\alpha j} f_j^{1+\alpha_j} \phi_{rj}^{1+\alpha_j}}{\sum_i \left(\frac{w_i}{g}\right) \phi_{i1}^2} \tag{4.6}$$

Once again, in the Eqn. 4.6 any term related to the response does not appear.

5. CASES OF STUDY

In order to validate the procedure, four cases of study have been solved by using the iterative procedure proposed in literature, based on the equivalent energy approach, and the direct procedure proposed in this paper, based on the calculation of the damper index. These results have been compared with the ones obtained by solving numerically, using the Fast Non Linear Analysis implemented in the computer program SAP 2000NL, the same four cases. Two steel moment resisting frames (Frame A and Frame B) equipped with dampers with exponent α = 0.50 have been studied. The two frames are characterized by the same resisting elements (beams and columns). They have dampers with different coefficients ($c_{\alpha j}$) and masses at each level that have been varied proportionally according to a factor equal to 0.354.

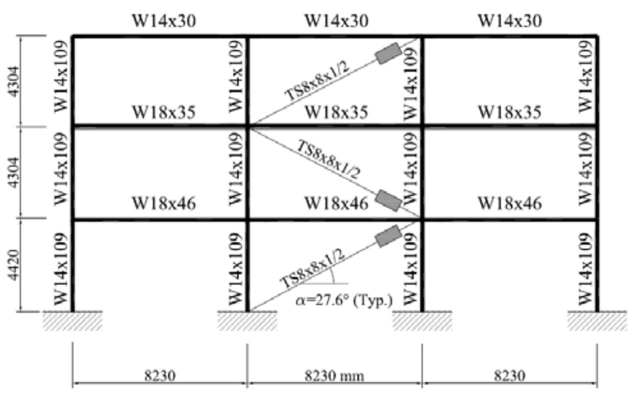


Figure 3 Steel Moment Resisting Frame

Operating in this way the two frames have same mode shapes and same participation factors. In the Table 5.1 the characteristics of the two frames are shown. Two recorded ground accelerations have been applied to the two structures so that four cases have been investigated. The selected ground motions are El Centro and Newhall, and they have been scaled in order that PGA is equal to 0.25g.

5.1 Assessment of the Supplemental Damping Ratio

The supplemental damping ratio has been assessed by using an iterative procedure proposed in literature (Lin and Chopra, 2002). The calculation of the supplemental damping ratio by the direct procedure requires the definition of the spectra of supplemental damping ratio versus period in terms of ε and α, for each ground acceleration. The following procedure has been adopted: 1) ε_1 is calculated using Eqn. 4.6; 2) Eqn. 4.3 is solved numerically for each ground acceleration by varying the natural period of the system in order to obtain $R_{D,1}$; 3) the supplemental damping ratio $\xi_{sd,1}$ is calculated for each period using Eqn. 4.2. Once the spectrum is made it is possible to assess the supplemental damping ratio for known values of ε_1 and α. The spectrum for the case of Frame A and El Centro ground acceleration is shown in the following figure. In this case ε_1= 0.2424. In the Table 5.2 the results for the four cases are illustrated. It can be noticed that the difference between the two procedures in assessing the supplemental damping ratio is very low for all the cases.

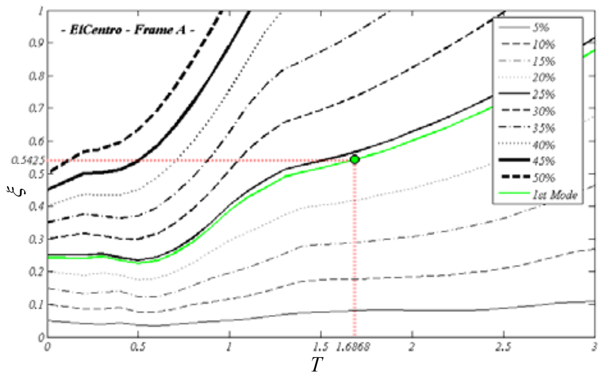


Figure 4 Spectrum of the Supplemental Damping Ratio for different values of ε and α=0.50 for El Centro ground acceleration (green line is the spectrum for the case of Frame A)

Table 5.1 Properties of the two considered frames

	Level	Mass	c_{aj}	Mode	*T*
Frame A	1	2900 kN	32.00 kN/(sec/mm)$^{0.5}$	1	1.6868 sec
	2	2900 kN	32.00 kN/(sec/mm)$^{0.5}$	2	0.5388 sec
	3	1567 kN	32.00 kN/(sec/mm)$^{0.5}$	3	0.2873 sec
	Level	**Mass**	c_{aj}	**Mode**	***T***
Frame B	1	1028 kN	8.00 kN/(sec/mm)$^{0.5}$	1	1.000 sec
	2	1028 kN	8.00 kN/(sec/mm)$^{0.5}$	2	0.3194 sec
	3	555 kN	8.00 kN/(sec/mm)$^{0.5}$	3	0.1710 sec

Table 5.2 Supplemental Damping Ratio

Ground acceleration	Iterative Procedure	Direct Procedure	Δ
El Centro	0.5355	0.5425	1.31%
Newhall	0.4096	0.4005	2.22%
Ground acceleration	**Iterative Procedure**	**Direct Procedure**	**Δ**
El Centro	0.1631	0.1645	0.86%
Newhall	0.1056	0.1076	1.89%

5.2 Results

The response of the frames, without dampers, has been calculated through a modal linear analysis. The damping ratio of the first mode has been evaluated by using the iterative procedure and the direct procedure described before. The damping ratio of the higher modes has been evaluated by using a method proposed in literature (Ramirez et al., 2000). In the Table 5.3 the maximum displacements of the top level are shown. They are compared with the ones obtained by using the Fast Non Linear Analysis implemented in SAP2000 NL, where the frames still have an elastic behavior, but they are equipped with non-linear dampers.

Table 5.3 Maximum Displacements (mm)

	Ground acceleration	FNA Analysis	Iterative Procedure	Direct Procedure	Δ Iterative	Δ Direct
Frame A	El Centro	47.2780	50.5483	49.9261	6.92%	5.60%
	Newhall	101.0035	86.1975	86.6950	14.66%	14.17%
Frame B	**Ground acceleration**	**FNA Analysis**	**Iterative Procedure**	**Direct Procedure**	**Δ Iterative**	**Δ Direct**
	El Centro	56.7627	55.8673	55.6251	1.58%	2.00%
	Newhall	132.5149	134.5471	133.8828	1.53%	1.03%

It can be noticed that the errors are smaller for Frame B. It can be easily shown that a part of the total error illustrated in Table 5.3 is due to the linearization of the problem. This part becomes more relevant for some periods and nearly zero for other periods. These two behaviors can be found for El Centro ground acceleration at a period equal to 1.68 sec (period of first mode of Frame A) and at period equal to 1.00 sec (period of first mode of Frame B).

6. SPECTRA OF THE SUPPLEMENTAL DAMPING RATIO

As previously illustrated the direct procedure requires to define the spectrum of the supplemental damping ratio. In order to achieve this purpose on the basis of spectra provided by international seismic codes, the following method is proposed. The damper index can be written in terms of pseudo-acceleration, which can be derived from response spectrum provided by seismic codes:

$$\varepsilon = \xi_{sd}\left(\overline{S}_a\left(T,\xi_{sd}\right)\right)^{1-\alpha} \tag{6.1}$$

where $\overline{S}_a$ is the pseudo-acceleration normalized to PGA. Usually the relationship between $\overline{S}_a$ and the damping ratio is provided by codes, so that once the damping ratio is known, the pseudo-acceleration is known.

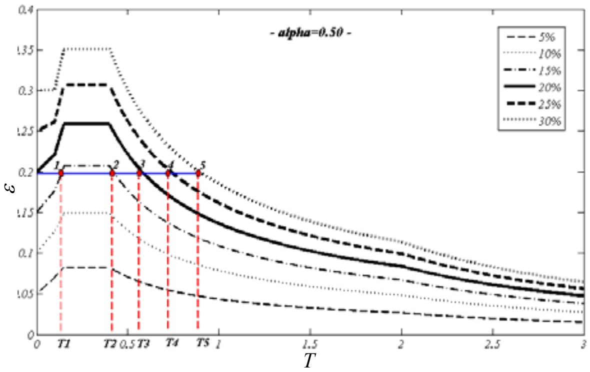


Figure 5 Spectrum of Damper Index

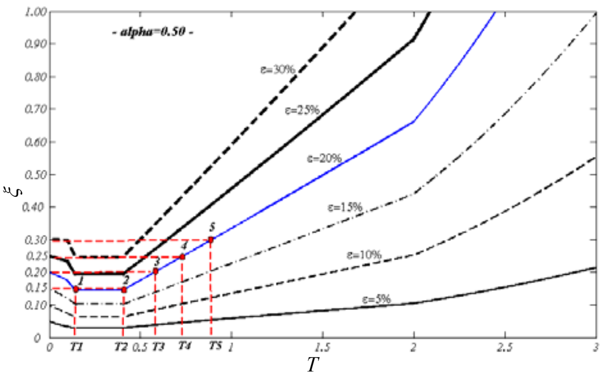


Figure 6 Spectrum of Supplemental Damping Ratio

Moving from this consideration, Eqn. 6.1 can be used to define the spectra of the damper index, for a fixed value of the exponent α. These spectra are useful because if a horizontal line for a constant value of the damper index (such as 20%) is drawn, this line intersects the spectral curves associated to different values of the supplemental damping ratio. Therefore it is possible to find some points that can be plotted in a graph to create the spectra of the supplemental damping ratio. This procedure is shown in the Figures 5 and 6, that have been realized by using design spectrum of the Italian Seismic Code (2003) for a category of soil A and an exponent α equal to 0.50.

7. CONCLUSIONS

The method introduced in this paper shows how the supplemental damping ratio, due to the presence of the non-linear viscous dampers, can be assessed by a direct procedure. To this purpose a new dimensionless parameter, ε, has been proposed. The cases of study have shown that the direct procedure proposed leads to results very close to the ones obtained by the iterative procedure reported in literature. In order to get the direct procedure easily usable by structural designers, a method for defining the spectra of supplemental damping ratio for known values of α and ε on the basis of spectra provided by seismic codes has been proposed.

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