OPTIMAL PARAMETERS FOR NEGATIVE STIFFNESS CONTROL BASED ON SKYHOOK CONTROL ANALOGY

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ABSTRACT :

Various vibration control methods, including the skyhook control and the negative stiffness control, have been proposed to achieve dynamic response reduction in terms of the absolute response. It is often observed that negative stiffness appears in the behavior of control devices following the skyhook control. In this study, negative stiffness appearing in the skyhook control method is theoretically evaluated with the use of equivalent control parameters in the negative stiffness control. Based on the result, relevance of the negative stiffness control to the skyhook control and the significance of negative stiffness are discussed, and the optimal design of the negative stiffness control is proposed.

KEYWORDS: vibration control, skyhook control, negative stiffness control

1. INTRODUCTION

Various vibration control methods have been proposed to achieve structural dynamic response reduction in terms of the absolute response. The negative stiffness control was proposed by Iemura *et al*. in 2002. The target load of the negative stiffness control includes not only viscosity but also negative stiffness. So this control method can make the natural period of the structure longer and reduce the absolute acceleration response. And all what are necessary to decide the target load are only relative displacement and relative velocity of the damper setting point, so the application to the real structure is easier than other complicated control algorithms.

Some problems on the parameter tuning and design of the negative stiffness control remain to be solved. Especially, the optimum value of the negative stiffness parameter, which plays the most important role in the negative stiffness control, is equal to the stiffness of the controlled structure in theory. It implies that the use of ideal negative stiffness control system requires a damper of a large load capacity in the case of the application of the system to large-scale structures including civil engineering structures. Although it would be often the case to use control devises with limited performance and capacity in actual practice, rational criteria of the selection of negative stiffness parameter and the relationship between the parameter value and the control performance have not been discussed enough.

On the other hand, the skyhook control was proposed by Karnopp *et al.* in 1974. The skyhook control can also achieve absolute response reduction. Various kinds of application and extension of the skyhook control have been studied in the mechanical engineering field, for example, Shinkan-sen in Japan. However, the skyhook control has the shortage that it needs the measurement of absolute velocity, so it is desirable to achieve skyhook control performance by simpler algorithms, such as the negative stiffness control.

In this study, the relationships between the skyhook control and the negative stiffness control are discussed. Theoretical and numerical evaluation of negative stiffness appearing in the skyhook control is described. Two kinds of equivalent parameters are defined and evaluated by focusing on their hysteretic loops and dynamic amplification factors resulting from the control schemes. Based on the result, design strategy for the negative stiffness control is proposed.

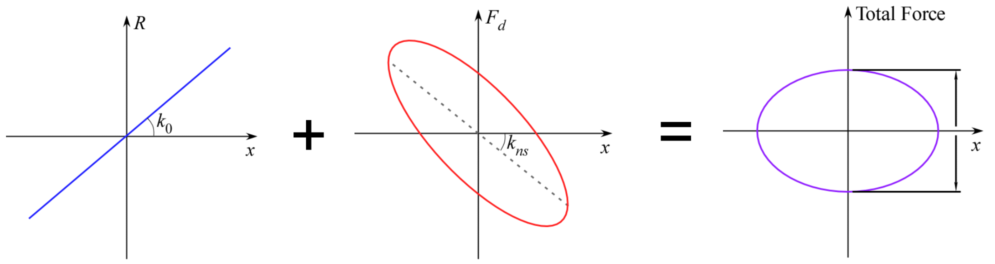
2. NEGATIVE STIFFNESS CONTROL

2.1. Introduction to Negative Stiffness Control

The negative stiffness control was proposed by Iemura *et al*. in 2002. The target load of the negative stiffness control is given by

$$F_d = -k_{ns}x + c_{ns}\dot{x} \tag{2.1}$$

where k_{ns} is the negative stiffness parameter, c_{ns} is the viscosity parameter, and x is the damper deformation. As shown in Eqn. 2.1, the target load includes not only viscosity but also negative stiffness, so the negative stiffness control can make the natural period longer and decrease the absolute acceleration response. Figure 1 shows the principle of the negative stiffness control. As shown in Figure 1c, the negative stiffness control can add damping without increasing the maximum base shear.



(a) Elastic Restoring Force (b) Negative Stiffness Control (c) Total Base Shear
Figure 1 Principle of Negative Stiffness Control

2.2. Analysis of SDOF Systems with Negative Stiffness Control under Sinusoidal Input

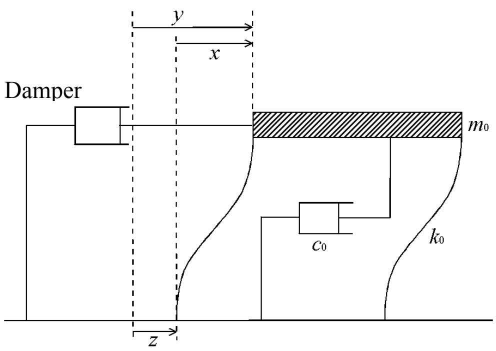


Figure 2 SDOF System with Negative Stiffness Control

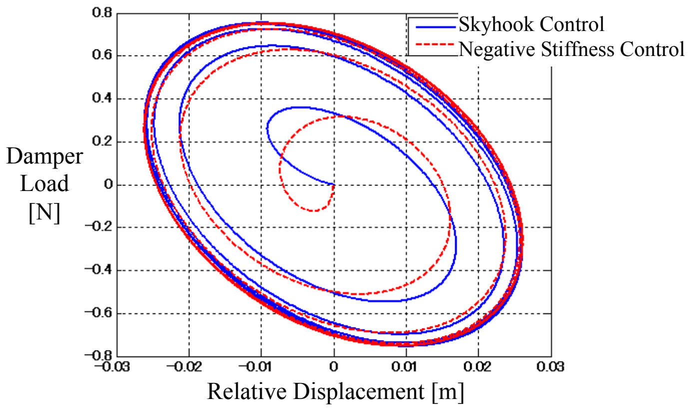


Figure 3 Hysteretic Loops of Negative Stiffness Control and Skyhook Control

Table 2.1 Parameters of SDOF System with Negative Stiffness Control

m_0	c_0	k_0	c_{ns}	k_{ns}
1.0[kg]	1.0[N.s/m]	100[N/m]	2.7[N.s/m]	10[N/m]

A typical numerical analysis of a SDOF (single degree-of-freedom) system with the negative stiffness control is shown in this section. Figure 2 shows the SDOF system with a damper which is controlled by the negative stiffness control algorithm. The symbol m_0 represents the mass, c_0 is the viscosity coefficient, k_0 is the stiffness of the structure, z is the ground motion, x is the relative displacement response, and y is the absolute displacement response. The parameters used in Figure 2 are shown in Table 2.1. The sinusoidal wave with the amplitude of 1.0[m/s^2] and the angular frequency of $\omega = (k_0 / m_0)^{1/2}$ is used as the ground motion. As can be seen in the hysteretic loop shown in figure 3, the hysteretic loop of the negative stiffness control has negative stiffness.

2.3. Theoretical Stationary Solution under Harmonic Input

The theoretical stationary solution of the SDOF system shown in Figure 2 is calculated in this section. The equation of motion of the SDOF system with the negative stiffness control shown in Figure 2 is

$$m_0\ddot{x} + c_0\dot{x} + k_0 x + \left(-k_{ns}x + c_{ns}\dot{x}\right) = -m_0\ddot{z} \tag{2.2}$$

$$\ddot{x} + \left(2h_0\omega_0 + 2h_{ns}\omega_0\right)\dot{x} + \frac{\left(k_0 - k_{ns}\right)}{m_0}x = -\ddot{z} \tag{2.3}$$

where
$$\omega_0 = \sqrt{\frac{k_0}{m_0}},\quad h_0 = \frac{c_0}{2m_0\omega_0},\quad h_{ns} = \frac{c_{ns}}{2m_0\omega_0} \tag{2.4}$$

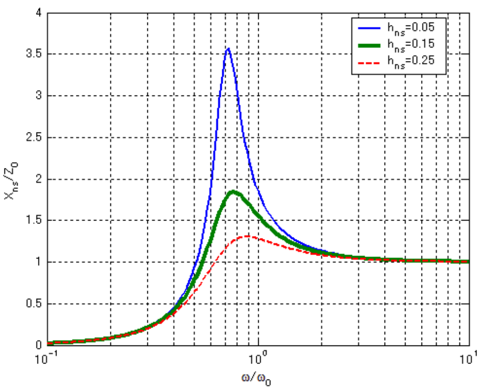
The stationary solution can be assumed as

$$\begin{cases} x = X_{sh}e^{i\omega t} \\ z = Ze^{i\omega t} \end{cases} \tag{2.5}$$

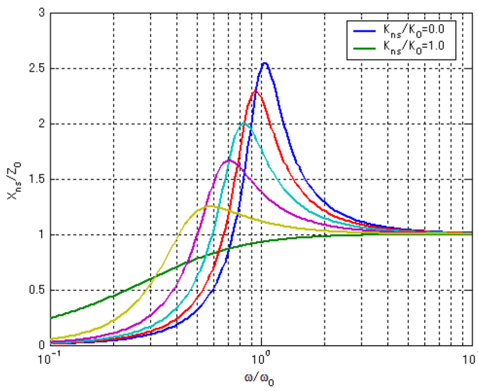
where ω is the angular frequency of the ground motion. Substituting Eqn. 2.5 for Eqn. 2.3, the following frequency response function is obtained.

$$\frac{X_{ns}}{Z} = \frac{r_\omega^2}{1 - \dfrac{k_{ns}}{k_0} - r_\omega^2 + 2i\left(h_0 + h_{ns}\right)r_\omega} \tag{2.6}$$

Where
$$r_\omega = \frac{\omega}{\omega_0} \tag{2.7}$$



(a) Influence by h_{ns}



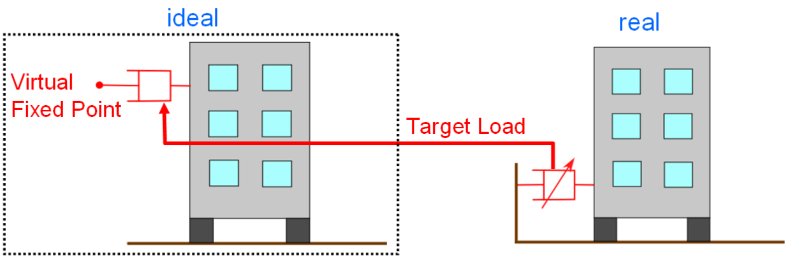
(b) Influence by k_{ns}

Figure 4 Amplification Factor of SDOF System with Negative Stiffness Control

Since Eqn. 2.6 is the frequency response function, the absolute value of Eqn. 2.6 is the dynamic amplification factor and the clockwise phase angle of Eqn. 2.6 corresponds to the phase lag of the relative displacement response from the ground motion. The dynamic amplification factor with $h_0 = 0.05$ is shown in Figure 4. Figure 4a shows the influence of h_{ns} under fixed $k_{ns}/k_0 = 0.5$, and Figure 4b shows the influence of k_{ns} under fixed $h_{ns} = 0.15$. As seen in Figure 4b, the negative stiffness k_{ns} decreases the resonance frequency and the peak value of the dynamic amplification factor. Figure 4 reveals that not only the viscosity parameter but also the negative stiffness parameter can decrease the peak value of the dynamic amplification factor.

3. SKYHOOK CONTROL

3.1. Introduction to Skyhook Control



(a) Ideal Condition (b) Real Condition

Figure 5 Skyhook Control

The skyhook control was proposed by Karnopp *et al*. in 1974. The basic concept of the skyhook control is shown in Figure 5. In the ideal condition of the skyhook control (Figure 5a), a structure is connected to a virtual fixed point through a dashpot. A system having the virtual fixed point is called a 'skyhook system.' The skyhook control is the method to control a groundhook damper (Figure 5b) to follow the target load that would be generated by the skyhook dashpot (Figure 5a). It is intuitively inferred from Figure 5 that the skyhook control can reduce absolute response. Various kinds of application and extension of the skyhook control have been studied.

In the skyhook control, the target load is proportional to the absolute velocity response, say,

$$F_d = c_{sh}\left(\dot{x} + \dot{z}\right) \tag{3.1}$$

where c_{sh} is the viscosity coefficient of the skyhook dashpot. Eqn. 3.1 indicates that absolute velocity, which is difficult to directly measure, is needed in calculating the target load. This is one of the disadvantages of the skyhook control. In order to overcome this difficulty, the simple algorithm for realizing the skyhook control performance is proposed.

3.2. Analysis of SDOF Systems with Skyhook Control under Sinusoidal Input

A typical numerical analysis of the SDOF system with the skyhook control is shown in this section. Figure 6 shows the SDOF system with the skyhook dashpot. The parameters used in Figure 6 are shown in Table 3.1. The sinusoidal wave with the amplitude of 1.0[m/s^2] and the angular frequency of $\omega = (k_0 / m_0)^{1/2}$ is used as the ground motion. The result of the hysteretic loop is shown in Figure 3. As seen in Figure 3, the hysteretic loop of the skyhook control is similar to that of the negative stiffness control. In other words, negative stiffness appears in the skyhook control. This characteristic will be proved in the following section. Figure 3 indicates that the skyhook control can be represented by negative stiffness and viscosity.

Table 3.1 Parameters of SDOF System with Skyhook Dashpot

m_0	c_0	k_0	c_{sh}
1.0[kg]	1.0[N.s/m]	100[N/m]	3.0[N.s/m]

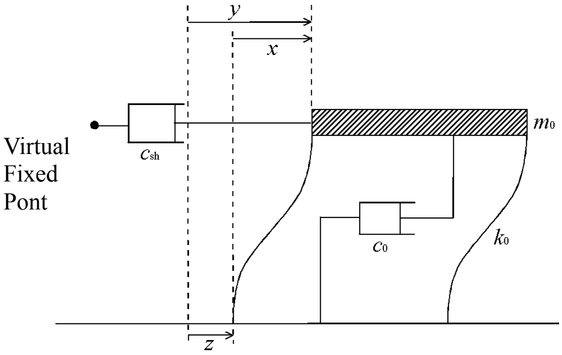


Figure 6 SDOF System with Skyhook Dashpot

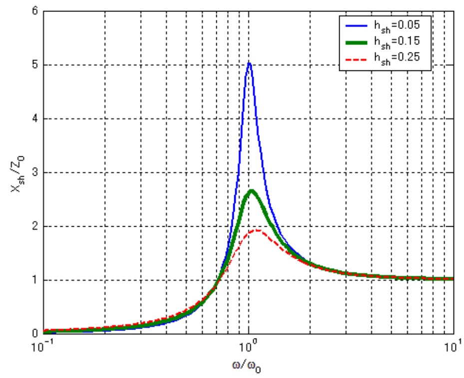


Figure 7 Amplification Factor of SDOF System with Skyhook Control

3.3. Theoretical Stationary Solution under Harmonic Input

The theoretical stationary solution of the SDOF system with the skyhook dashpot shown in Figure 6 is calculated in this section. The equation of motion of the SDOF system shown in Figure 6 is

$$m_0\ddot{x} + c_0\dot{x} + k_0 x + c_{sh}(\dot{x}+\dot{z}) = -m_0\ddot{z} \tag{3.2}$$

$$\ddot{x} + (2h_0\omega_0 + 2h_{sh}\omega_0)\dot{x} + \omega_0^2 x = -\ddot{z} - 2h_{sh}\omega_0\dot{z} \tag{3.3}$$

where
$$h_{sh} = \frac{c_{sh}}{2m_0\omega_0} \tag{3.4}$$

The frequency response function is obtained as follows.

$$\begin{aligned}\frac{X_{sh}}{Z} &= \frac{r_\omega^2 - 2ih_{sh}r_\omega}{1 - r_\omega^2 + 2i(h_0+h_{sh})r_\omega}\\ &= \frac{r_\omega^2(1-r_\omega^2) - 4h_{sh}(h_0+h_{sh})r_\omega^2}{(1-r_\omega^2)^2 + 4(h_0+h_{sh})^2 r_\omega^2} + i\frac{-2h_0 r_\omega^3 - 2h_{sh}r_\omega}{(1-r_\omega^2)^2 + 4(h_0+h_{sh})^2 r_\omega^2}\end{aligned} \tag{3.5}$$

The dynamic amplification factor represented by the absolute values of Eqn. 3.5 with $h_0 = 0.05$ and typical h_{sh} is shown in Figure 7. As seen in Figure 7, the skyhook control parameter h_{sh} can decrease the peak value of the dynamic amplification factor.

In order to simplify the equations, following symbols are defined.

$$\begin{cases} a_{sh}(\omega) = \mathrm{Re}\left(\dfrac{X_{sh}}{Z}\right) = \dfrac{r_\omega^2(1-r_\omega^2) - 4h_{sh}(h_0+h_{sh})r_\omega^2}{(1-r_\omega^2)^2 + 4(h_0+h_{sh})^2 r_\omega^2} \\ b_{sh}(\omega) = \mathrm{Im}\left(\dfrac{X_{sh}}{Z}\right) = \dfrac{-2h_0 r_\omega^3 - 2h_{sh}r_\omega}{(1-r_\omega^2)^2 + 4(h_0+h_{sh})^2 r_\omega^2} \end{cases} \tag{3.6}$$

It should be noted that $b_{sh} \le 0$. Therefore, the frequency response function of the load of the skyhook dashpot is

$$\begin{aligned}\frac{F_{sh}}{Z} &= c_{sh}\frac{\dot{Y}_{sh}}{Z} = c_{sh}\frac{i\omega Y_{sh}}{Z} = c_{sh}\frac{i\omega(X_{sh}+Z)}{Z}\\ &= -\omega c_{sh}b_{sh}(\omega) + i\omega c_{sh}(a_{sh}(\omega)+1)\end{aligned} \tag{3.7}$$

The time history of the relative displacement response and the damper load can be obtained as follows.

$$x(t)=\mathrm{Re}\left(X_{sh}e^{i\omega t}\right)=\left|X_{sh}\right|\mathrm{Re}\left(e^{i\omega t+i\varphi_x}\right) \tag{3.8}$$

$$\begin{aligned} f(t)&=\mathrm{Re}\left(F_{sh}e^{i\omega t}\right)\\ &=\omega c_{sh}\left|X_{sh}\right|\mathrm{Re}\left(\left\{\frac{b_{sh}(\omega)}{a_{sh}^2(\omega)+b_{sh}^2(\omega)}+i\left(1+\frac{a_{sh}(\omega)}{a_{sh}^2(\omega)+b_{sh}^2(\omega)}\right)\right\}e^{i\omega t+i\varphi_x}\right) \end{aligned} \tag{3.9}$$

where φ_x is the phase angle of X_{sh}. When $\omega t+\varphi_x=0$, the relative displacement response is maximum and the damper load is

$$f(t)=\omega c_{sh}\left|X_{sh}\right|\frac{b_{sh}(\omega)}{a_{sh}^2(\omega)+b_{sh}^2(\omega)}<0 \tag{3.10}$$

Eqn. 3.10 indicates that the value of the damper load of the skyhook damper is negative when the relative displacement response reaches the maximum value. Hence, the hypothesis that the skyhook control has the negative stiffness is proved. This fact suggests that the skyhook control can be represented by the negative stiffness control.

4. EQUIVALENT PARAMETER IN TERMS OF HYSTERETIC LOOPS

4.1. Definition of Equivalent Negative Stiffness Control

The equivalent negative stiffness control is introduced to represent the hysteretic loop of the skyhook control. As already described, negative stiffness appears in the skyhook control, implying that it is possible to let the negative stiffness control have the same hysteretic loop as the skyhook control by setting proper control parameters. The equivalent negative stiffness control is defined as the negative stiffness control that has the same hysteretic loop as that of a specified skyhook control.

The equivalent negative stiffness control is represented by the negative stiffness parameter k_{ens} and the viscosity parameter c_{ens}. If these equivalent parameters are obtained, the negative stiffness control, which requires only relative displacement and relative velocity, can have the same performance as the skyhook control, which needs absolute velocity measurement.

4.2. Equivalent Negative Stiffness Control Parameters

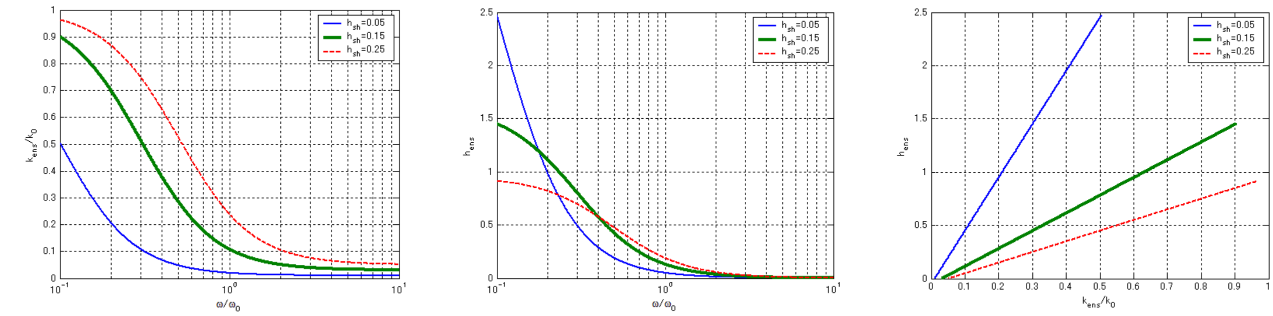
The theoretical solution of the stationary response of the SDOF system with the skyhook control shown in Figure 6 is given in the previous chapter. Eqn. 3.9 and 3.10 show the solution of the relative displacement response and the damper load of the skyhook control. From these results, equivalent negative stiffness control parameters can be obtained from a viewpoint of the negative stiffness control to trace the hysteretic loop of the skyhook control.

$$\begin{cases} \dfrac{k_{ens}}{k_0}=2\dfrac{-b_{sh}(\omega)}{a_{sh}^2(\omega)+b_{sh}^2(\omega)}r_\omega h_{sh}\\ h_{ens}=\left(1+\dfrac{a_{sh}(\omega)}{a_{sh}^2(\omega)+b_{sh}^2(\omega)}\right)h_{sh} \end{cases} \tag{4.1}$$

Eqn. 4.1 indicates that equivalent negative stiffness control parameters (k_{ens} and c_{ens}) are obtained from the skyhook control parameter h_{sh}, structural parameters, and excitation frequency.

4.3. Implication of Equivalent Negative Stiffness Control

As mentioned in the previous section, the equivalent negative stiffness control parameters depend on the excitation frequency. Figure 8 shows the frequency dependence of the equivalent negative stiffness control parameters.



(a) Negative Stiffness vs. Frequency (b) Damping vs. Input Frequency (c) Damping vs. Negative Stiffness

Figure 8 Equivalent Negative Stiffness Control Parameters

As seen in Figure 8a, higher negative stiffness appears as the skyhook control parameter h_{sh} increases. This implies the importance of negative stiffness. Figure 8c shows the relationship between the equivalent negative stiffness parameter and the equivalent damping parameter. As seen in Figure 8c, higher negative stiffness is needed for higher parameter of the skyhook control and the same damping parameter. Hence Figure 8c also indicates the importance of negative stiffness. This result shown in Figure 8c can be interpreted as equivalent parameters in terms of the hysteretic loops.

5. EQUIVALENT PARAMETER IN TERMS OF DYNAMIC AMPLIFICATION FACTORS

The equivalent skyhook control is introduced to represent the peak value of the dynamic amplification factor of the SDOF system with negative stiffness control. As mentioned in section 2.3, there are two parameters to decrease the peak value of the dynamic amplification factor in the negative stiffness control. Therefore, the negative stiffness control parameter sets corresponding to one equivalent skyhook control parameter are not unique.

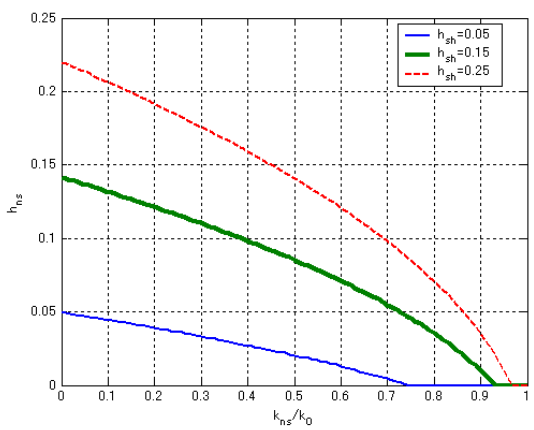


Figure 9 Equivalent Skyhook Control

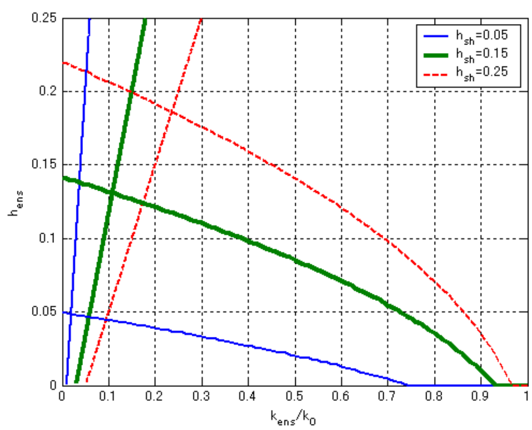


Figure 10 Equivalent Parameters

Figure 9 shows the result of the numerical search of the sets of negative stiffness control parameters corresponding to typical three equivalent skyhook controls. The vertical axis of the plot in Figure 9 is the damping parameter of the negative stiffness control and the horizontal axis is the negative stiffness parameter of the negative stiffness control. The plotted lines in Figure 9 can also be interpreted as the contour lines of the performance of the negative stiffness control parameters corresponding to equivalent skyhook controls in the negative stiffness control design space. This result shown in Figure 9 can be interpreted as equivalent parameters in terms of the dynamic amplification factors.

6. PROPOSAL OF DESIGN POINT

Figure 10 shows the two kinds of equivalent parameters, equivalent parameters in terms of the hysteretic loops (Figure 8) and equivalent parameters in terms of the dynamic amplification factors (Figure 9), in the same negative stiffness control design space.

For each skyhook damping parameter h_{sh}, an intersecting point of the two curves is found in Figure 10. The skyhook control and the negative stiffness control on the intersecting point have the same peak value of the dynamic amplification factor and have the same hysteretic loop in the excitation of a certain frequency near the natural frequency. In this study, the use of the intersecting points as the optimal design point of the negative stiffness control based on the skyhook control is proposed.

7. CONCLUSIONS

In this study, the relationships between the skyhook control and the negative stiffness control are discussed. It is shown that the hysteretic loop of the skyhook control can be approximated to that of the negative stiffness control. The two kinds of the equivalent parameters between the skyhook control and the negative stiffness control are obtained by focusing on the hysteretic loops and the dynamic amplification factors. Through these equivalent parameters, it is proved that negative stiffness appears in the skyhook control, and importance of negative stiffness is shown. The optimal parameter for the negative stiffness control based on the skyhook control is proposed.

ACKNOWLEDGEMENT

This research was supported by the Grant-in-Aid for Scientific Research (A). This support is highly appreciated.

REFERENCES

Iemura, H. and Pradono, M. H. (1995). Passive and Semi-Active Seismic Response Control of a Cable-Stayed Bridge. *Earthquake Engineering and Structural Dynamics* **vol.9**, 189-204.
Iemura, H. and Pradono, M. H. (2002). Negative Stiffness Dampers for Seismic Retrofit of a Cable-Stayed Bridge. *Proceedings of Passive Structural Control Symposium*, 7-16.
Iemura, H. and Pradono, M. H. (2003). Application of pseudo-negative stiffness control to the benchmark cable bridge. *Journal of Structural Control*, **10**, 187-203.
Karnopp, D., Crosby, M. J. and Harwood, R. A. (1974). Vibration Control Using Semi-Active Force Generators. *Journal of Engineering for Industry*, ASME J. Eng., **Vol96, No.2**, 619-626.
Igarashi, A., Higuchi, M. and Iemura, H. (2006). Performance Evaluation of the Pseudo Negative Stiffness Control Based on a Skyhook System Approach, *Proc. 4th World Conference on Structural Control and Monitoring*, Paper No.294 (8pp.).