BASE-ISOLATION TECHNOLOGY FOR EARTHQUAKE PROTECTION OF ART OBJECTS

P.C. Roussis[1], E.A. Pavlou[2] and E.C. Pisiara[3]

[1] *Lecturer, Dept. of Civil & Environmental Engineering, University of Cyprus, Nicosia, Cyprus*
[2] *Research Associate, Dept. of Civil & Environmental Engineering, University of Cyprus, Nicosia, Cyprus*
[3] *Graduate Student, Dept. of Civil & Environmental Engineering, University of Cyprus, Nicosia, Cyprus*
Email: roussis@ucy.ac.cy

ABSTRACT :

In this paper the dynamic response of base-isolated block-like slender objects, such as statues, subjected to horizontal ground excitation is investigated. The structural model employed consists of a rigid block supported on a rigid base, beneath which the isolation system is accommodated. Assuming no sliding of the block relative to the supporting base, when subjected to ground excitation the system may exhibit two possible patterns of motion, namely pure translation, in which the system in its entirety oscillates horizontally (1 degree-of-freedom response), and rocking, in which the rigid block pivots on its edges with respect to the horizontally-moving base (2 degree-of-freedom response). The dynamic response of the system is strongly affected by the occurrence of impact between the block and the horizontally-moving base, as impact can modify not only the energy but also the degrees of freedom of the system by virtue of the discontinuity introduced in the response. Therefore, the critical role of impact in the dynamics of the system necessitates a rigorous formulation of the impact problem. In this study, a model governing impact from the rocking mode is derived from first principles using classical impact theory. Numerical results are obtained via an ad hoc computational scheme developed to determine the response of the system under horizontal ground excitation.

KEYWORDS: base isolation, art objects, earthquake, rigid block

1. INTRODUCTION

There is a wealth of elements of cultural heritage worldwide and thus a widespread challenge to preserve this important legacy. From irreplaceable museum artifacts and statues to intricate decorative art objects, the range and complexity of materials, forms and geometry makes protecting this heritage against earthquakes a formidable task. Yet despite the need for conservation efforts, only recently research attempts have been pursued in this direction. In this paper the seismic performance of base-isolated block-like objects, such as statues, is investigated. The seismic behavior of such objects can be analyzed within the context of rigid-body dynamics.

In the literature there is a wealth of research papers on the seismic behavior of block-like structures. Housner's landmark study has provided the basic understanding on the rocking response of a slender rigid block and sparked modern scientific interest. His model is based on the assumption of perfectly-inelastic impact and sufficient friction to prevent sliding during impact. Following Housner’s fundamental work, numerous studies [e.g. Yim et al. (1980), Ishiyama (1982), Spanos and Koh (1984), Shenton and Jones (1991), Makris and Roussos (2000)] have been reported in the literature dealing with various aspects of the complex dynamics of the single rigid block. Moreover, the dynamic behavior of two-block structures has been studied by Psycharis (1990) and Spanos et al. (2001), who analyzed the non-linear dynamic response of systems consisting of two blocks, one placed on top of the other, free to rock without sliding. Such a configuration can be thought of as a model of ancient statue placed on top of a block-like base.

Only recently has the seismic protection of objects of cultural heritage become a subject of interest of many researchers. Agbabian et al. (1988) were perhaps the first to explicitly deal with the seismic protection of art objects. Their work aimed at the development of analytical and experimental procedures for the evaluation of the seismic mitigation of various museum contents, at the Jean Paul Getty Museum in Malibu, California. Subsequently, Augusti et al. (1992) studied the seismic response of art objects and proposed some simple rules for the design of displays in order to mitigate the seismic risk of valuable exhibits. Aiming at the protection of cultural heritage through the application of base isolation, Vestroni and Di Cinto (2000) performed a parametric study on the response of an isolated statue modeled as a single-degree-of-freedom system, with the isolator characterized by a hysteretic force-displacement law. Moreover, Myslimaj et al. (2003) proposed the installation of Tuned Configuration Rail (TCR), a rolling type base-isolation system, underneath showcases, preservation racks, shelves and statues to control their seismic response. More recently, Caliò and Marletta (2003) examined the vibrations of art objects modeled as rigid blocks isolated with viscoelastic devices and performed numerical investigations under impulsive and seismic excitations.

In this paper the dynamic response of base-isolated block-like slender objects, such as statues (Figure 1), subjected to seismic excitation is investigated. The structural model employed consists of a rigid block supported on a rigid base, beneath which the isolation system is placed. Assuming sufficient friction to prevent sliding of the block relative to the supporting base, when subjected to ground excitation, the system may exhibit two possible patterns of motion, namely *pure translation*, in which the system as a whole realizes only horizontal displacement $u(t)$, and *rocking*, in which the rigid block experiences rotation $\theta(t)$ with respect to the horizontally-moving base. Thus, the active degrees of freedom of the system considered are dependent on the nature of the response, with the system possessing 2 degrees of freedom for motion realized in the rocking mode and 1 degree of freedom for motion realized in the pure-translation mode. The formulation of the problem involves derivation of the nonlinear equations of motion, transition from one mode to the other, and a rigorous formulation of the impact model based on classical impact theory.

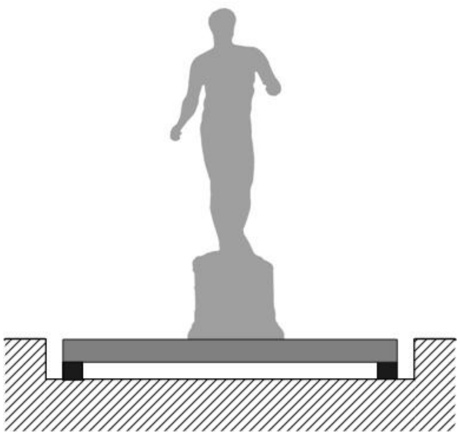


Figure 1: Schematic of a base-isolated statue

2. ANALYTICAL MODEL

2.1. Model Considered

The system considered consists of a symmetric rigid block of mass m and centroid mass moment of inertia I, supported on a rigid base of mass m_b (Figure 2a). The entire system is base-isolated with a linear isolation system composed of a linear spring k and a linear viscous dashpot c. The rigid block of height $H = 2h$ and width $B = 2b$ is assumed to rotate about the corners O and O'. The distance between one corner of its base and the mass centre is denoted by r and the angle measured between r and the vertical when the body is at rest is denoted by α, where $\alpha = \tan^{-1}(b/h)$.

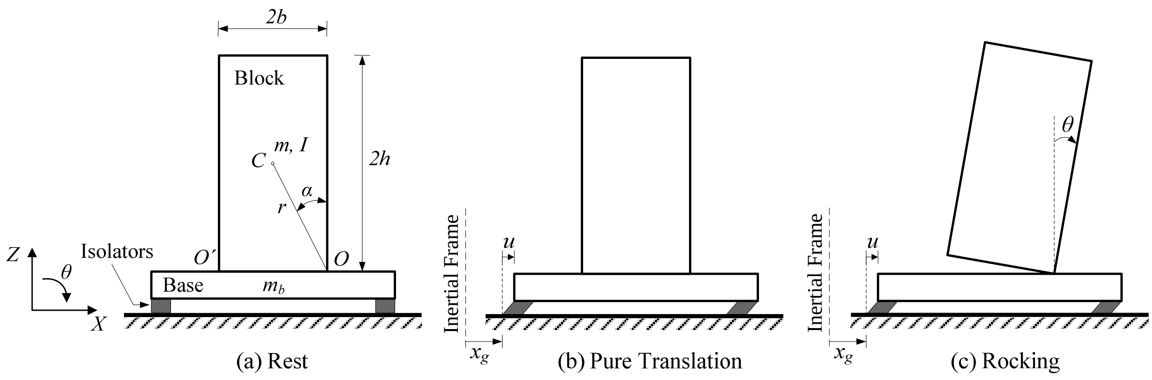


Figure 2: Model at rest and oscillation patterns

The horizontal and vertical displacements of the mass center of the block relative to an inertial frame of reference are denoted by $X(t)$ and $Z(t)$ respectively, while the corresponding displacements relative to the base are denoted by $x(t)$ and $z(t)$. The angular rotation of the block is denoted by $\theta(t)$, positive in the clockwise direction, and the horizontal displacement of the base relative to the foundation is denoted by $u(t)$.

2.2. Initiation of Rocking

When subjected to ground acceleration $\ddot{x}_g$, the supporting base will oscillate in the horizontal direction with a displacement $u(t)$ relative to the foundation. The rigid block will be set into rocking on top of the moving base when the overturning moment due to external loads, $M_{over} = m(\ddot{u}+\ddot{x}_g)h$, exceeds the available resisting moment due to gravity, $M_{res} = mgb$, yielding

$$\left|\ddot{u}+\ddot{x}_g\right| > \frac{b}{h}g \tag{2.1}$$

2.3. Equations of Motion

Assuming no sliding of the block relative to the supporting base, when subjected to ground excitation the system may exhibit two possible patterns of motion: (a) pure translation, in which the system in its entirety oscillates horizontally with displacement $u(t)$ (1 degree-of-freedom response), and (b) rocking, in which the rigid block pivots on its edges with rotation $\theta(t)$ as the supporting base translates horizontally with $u(t)$ (2 degree-of-freedom response). The governing equations for each pattern of motion are herein formulated by means of the Lagrange method.

2.3.1 Pure-translation mode

The equation of motion of the system in the pure-translation mode is

$$(m+m_b)\ddot{u}+c\dot{u}+ku = -(m+m_b)\ddot{x}_g \tag{2.2}$$

which is the classical linear second-order differential equation governing the response of a single-degree-of-freedom system to ground excitation.

2.3.2 Rocking mode

In the rocking mode the system possesses two degrees of freedom. Using as generalized coordinates $q_1 \equiv u$, the horizontal translation of the base relative to the ground, and $q_2 \equiv \theta$, the rotation angle of the object about a bottom corner, Lagrange's equations take the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}}\right)-\frac{\partial T}{\partial u}+\frac{\partial V}{\partial u}=Q_u \quad \text{and} \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right)-\frac{\partial T}{\partial \theta}+\frac{\partial V}{\partial \theta}=Q_\theta \tag{2.3}$$

in which T denotes the kinetic energy of the system, V the potential energy of the system, and Q_k the generalized nonconservative forces.

The kinetic energy of the system is obtained as

$$T=\frac{1}{2}m_b\left(\dot{u}+\dot{x}_g\right)^2+\frac{1}{2}m\left[\left(\dot{u}+\dot{x}_g+h\dot{\theta}\cos\theta+b\dot{\theta}\sin\theta\right)^2+\left(b\dot{\theta}\cos\theta-h\dot{\theta}\sin\theta\right)^2\right]+\frac{1}{2}I\dot{\theta}^2 \tag{2.4}$$

in which the first term is associated with pure translation of the base, and the second and third term are associated with general planar motion of the block.

The potential energy of the system is obtained as

$$V=\frac{1}{2}ku^2+mg\left[b\sin\theta-h\left(1-\cos\theta\right)\right] \tag{2.5}$$

in which the first term is associated with the potential energy due to elastic deformation of the spring and the second term is associated with the potential energy due to gravity.

The generalized forces Q_k are derived via the virtual work of the nonconservative forces as

$$Q_u=-c\dot{u}, \quad Q_\theta=0 \tag{2.6}$$

Substituting Equations (2.4) through (2.6) into Equations (2.3) yields the governing equations of motion for rotation about O ($\theta>0$). The governing equations of motion for rotation about O' ($\theta<0$) can be derived in a similar manner. Combining the equations for rotation about O and O', leads to a compact set of equations governing the rocking mode of the object on top of the moving base:

$$\left(m+m_b\right)\ddot{u}+c\dot{u}+ku+m\left[h\cos\theta+\operatorname{sgn}\theta\left(b\sin\theta\right)\right]\ddot{\theta}+m\left[\operatorname{sgn}\theta\left(b\cos\theta\right)-h\sin\theta\right]\dot{\theta}^2=-\left(m+m_b\right)\ddot{x}_g \tag{2.7}$$

$$\left(mr^2+I\right)\ddot{\theta}+m\ddot{u}\left[h\cos\theta+\operatorname{sgn}\theta\left(b\sin\theta\right)\right]+mg\left[\operatorname{sgn}\theta\left(b\cos\theta\right)-h\sin\theta\right]=-m\left[h\cos\theta+\operatorname{sgn}\theta\left(b\sin\theta\right)\right]\ddot{x}_g \tag{2.8}$$

where $\operatorname{sgn}\theta$ denotes the signum function in θ. Note that equations (2.7) and (2.8) hold only in the absence of impact ($\theta \neq 0$). At that instant, both corner points O and O' are in contact with the base, rendering the above formulation invalid. The impact problem is addressed separately in the following section.

3. IMPACT MODEL

The dynamic response of the system is strongly affected by the occurrence of impact(s) between the block and the horizontally-moving base. In fact, impact affects the system response on many different levels. On one level, it renders the problem nonlinear (aside from the nonlinear nature of the equations themselves) by virtue of the discontinuity introduced in the response (i.e. the governing equations of motion are not valid for $\theta = 0$). That is, impact causes the system to switch from one oscillation pattern to another (potentially modifying the degrees of freedom), each one governed by a different set of differential equations. Further, the integration of equations of motion governing the post-impact pattern must account for the ensuing instantaneous change of the system's velocity regime. On another level, the effect of impact on the dynamic response is also evident in the energy loss of the system manifested through the reduction of post-impact velocities.

Therefore, the critical role of impact in the dynamics of the system necessitates a rigorous formulation of the impact problem. In this paper a model governing impact is derived from first principles using classical impact theory. According to the principle of impulse and momentum, the duration of impact is assumed short and the impulsive forces are assumed large relative to other forces in the system. Changes in position and orientation are neglected, and changes in velocity are considered instantaneous. Moreover, this model assumes a point-impact, zero coefficient of restitution (perfectly inelastic impact), impulses acting only at the impacting corner (impulses at the rotating corner are small compared to those at the impacting corner and are neglected), and sufficient friction to prevent sliding of the block during impact.

Under the assumption of perfectly inelastic impact, there are only two possible response mechanisms following impact: (a) rocking about the impacting corner when the block re-uplifts (no bouncing), or (b) pure translation when the block's rocking motion ceases after impact. The formulation of impact is divided into three phases: pre-impact, impact, and post-impact as illustrated schematically in Figure 3. In the following, a superscript "-" refers to a pre-impact quantity and a superscript "+" to a post-impact quantity.

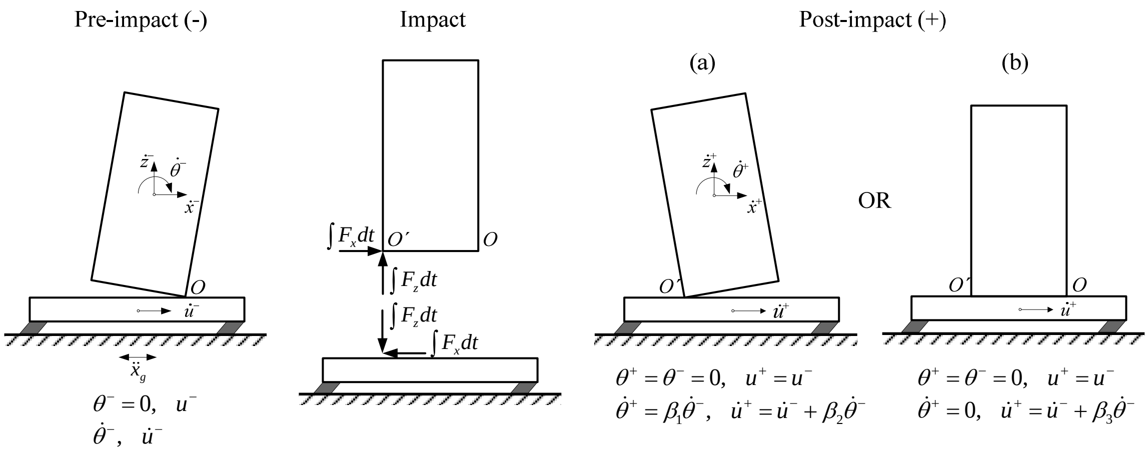


Figure 3: Impact from rocking about O followed by (a) re-uplift about O' and (b) termination of rocking

3.1. Rocking continues after impact

Consider the system at the instant when the block hits the moving base from rocking about O and re-uplifts pivoting about the impacting corner, O' (Figure 3a). As mentioned before, impact is accompanied by an instantaneous change in velocities, with the system displacements being unchanged. Therefore, the impact analysis is reduced to the computation of the initial conditions for the post-impact motion, $\dot{u}^+$ and $\dot{\theta}^+$, given the position and the pre-impact velocities, $\dot{u}^-$ and $\dot{\theta}^-$.

With regard to the block, the principle of linear impulse and momentum in the x and z direction states that

$$\int F_x dt = m\dot{X}^{+} - m\dot{X}^{-} = m\dot{u}^{+} + m\dot{x}^{+} - m\dot{u}^{-} - m\dot{x}^{-} \tag{3.1}$$

$$\int F_z dt = m\dot{Z}^{+} - m\dot{Z}^{-} = m\dot{z}^{+} - m\dot{z}^{-} \tag{3.2}$$

in which $\int F_x dt$ and $\int F_z dt$ are the horizontal and vertical impulses (assumed to act at O'); $\dot{X}^{-} = \dot{u}^{-} + \dot{x}_g + \dot{x}^{-}$, $\dot{X}^{+} = \dot{u}^{+} + \dot{x}_g + \dot{x}^{+}$ and $\dot{Z}^{-} = \dot{z}^{-}$, $\dot{Z}^{+} = \dot{z}^{+}$ are the absolute pre- and post-impact horizontal and vertical velocities of the mass center of the block, respectively.

In addition, the principle of angular impulse and momentum states that

$$b\left(\int F_z dt\right) - h\left(\int F_x dt\right) = I\dot{\theta}^{+} - I\dot{\theta}^{-} \tag{3.3}$$

In Equations (3.1) and (3.2), the pre- and post-impact horizontal and vertical components of the relative translational velocity of the mass center can be expressed in terms of the angular velocity of the block as

$$\dot{x}^{-} = h\dot{\theta}^{-}, \quad \dot{z}^{-} = b\dot{\theta}^{-}, \quad \dot{x}^{+} = h\dot{\theta}^{+}, \quad \dot{z}^{+} = -b\dot{\theta}^{+} \tag{3.4}$$

Substituting Equations (3.4) into Equations (3.1) and (3.2) yields

$$\int F_x dt = m\dot{u}^{+} + mh\dot{\theta}^{+} - m\dot{u}^{-} - mh\dot{\theta}^{-} \tag{3.5}$$

$$\int F_z dt = -mb\dot{\theta}^{+} - mb\dot{\theta}^{-} \tag{3.6}$$

Equations (3.3), (3.5) and (3.6) constitute a set of three equations in four unknowns. Equivalently, these equations can be combined in one equation (by eliminating the two impulses) in two unknowns ($\dot{\theta}^{+}$, $\dot{u}^{+}$):

$$\left(4b^2 + 4h^2\right)\dot{\theta}^{+} + 3h\dot{u}^{+} = \left(4h^2 - 2b^2\right)\dot{\theta}^{-} + 3h\dot{u}^{-} \tag{3.7}$$

in which for rectangular block the centroid mass moment of inertia was taken as $I = m\left(b^2 + h^2\right)/3$.

One additional equation is therefore required to uniquely determine the post-impact velocities $\dot{\theta}^{+}$ and $\dot{u}^{+}$. By considering the system in its entirety during the impact, it can be stated that the horizontal impulse on the system is zero, resulting in the conservation of the system's linear momentum in the horizontal direction. That is,

$$\left(m_b + m\right)\dot{u}^{+} = \left(m_b + m\right)\dot{u}^{-} - mh\dot{\theta}^{+} + mh\dot{\theta}^{-} \tag{3.8}$$

Combining Equations (3.7) and (3.8) gives the post-impact velocities as

$$\dot{\theta}^{+} = \frac{\lambda^2\left(\overline{m}+4\right) - 2\left(\overline{m}+1\right)}{\lambda^2\left(\overline{m}+4\right) + 4\left(\overline{m}+1\right)}\dot{\theta}^{-} \equiv \beta_1\dot{\theta}^{-} \tag{3.9}$$

and

$$\dot{u}^{+} = \dot{u}^{-} + \frac{6\overline{m}h}{\lambda^2\left(\overline{m}+4\right) + 4\left(\overline{m}+1\right)}\dot{\theta}^{-} \equiv \dot{u}^{-} + \beta_2\dot{\theta}^{-} \tag{3.10}$$

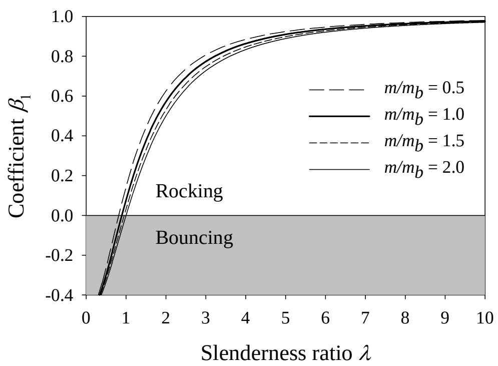
in which $\lambda = h/b$ is the geometric aspect ratio and $\overline{m} = m/m_b$ is the mass ratio.

Equations (3.9) and (3.10) give the post-impact velocities for impact from rocking about O (realized when $\dot{\theta} < 0$). Identical expressions are derived for the case of impact from rocking about O' (realized when $\dot{\theta} > 0$).

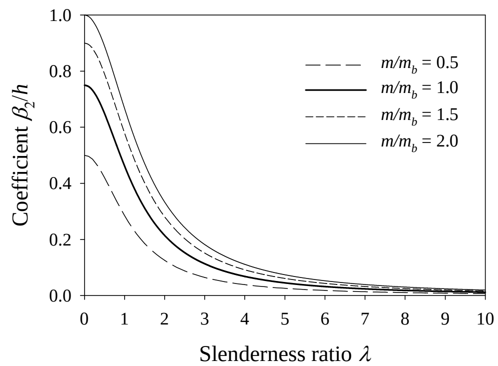
It is worth noting that the coefficient of restitution e as defined in classical impact theory, relates pre- to post-impact *translational* velocities normal to the impact surface, and hence it must not be confused with the coefficient of "angular restitution" β_1 defined in Equation (3.9), which relates the pre- to post-impact *angular* velocities of the body. In the derivation presented herein, the coefficient of restitution e enters in the expression $\dot{z}_{O'}^{+} = -e\dot{z}_{O'}^{-}$ which relates pre- to post-impact vertical relative velocities of the impacting corner (O'). The assumption of perfectly inelastic impact is then justified by considering $e = 0$.

From Equation (3.9), it can be seen that the coefficient of angular restitution, β_1, depends both on the geometric aspect ratio λ and the mass ratio $\overline{m}$. The variation of coefficient β_1 with the slenderness ratio λ is shown in Figure 4a for different values of the mass ratio $\overline{m}$. The dependency of coefficient β_1 on the mass ratio $\overline{m}$ is seen to be weak, and practically diminishes for very slender blocks (e.g. for $\lambda > 6$). The value $\beta_1 = 1$, implying preservation of the magnitude of the angular velocity after impact, presents an upper bound for the coefficient of angular restitution. Evidently, the more slender a block, the larger the associated coefficient β_1 is. For the assumption of *no-bouncing* to be satisfied, the coefficient of angular restitution β_1 should have a positive value. In such a case, the angular velocity of the block will maintain sign upon impact, implying switching pole of rotation from one corner to the other. This requires that $\lambda > \sqrt{2(\overline{m}+1)/(\overline{m}+4)}$.

The coefficient associated with the reduction of the linear velocity of base, β_2, depends not only on the parameters λ and $\overline{m}$, but also on the absolute size of the block (in terms of its height). The normalized coefficient $\overline{\beta}_2 \equiv \beta_2/h$ is plotted against the slenderness ratio λ for different values of the mass ratio $\overline{m}$ in Figure 4b. Observe that the value of the coefficient $\overline{\beta}_2$ decays rapidly with the slenderness ratio λ. Moreover, the influence of the mass ratio $\overline{m}$ on the coefficient $\overline{\beta}_2$ is much greater than that on the coefficient β_1.



(a)



(b)

Figure 4: Variation of coefficients β_1 and $\overline{\beta}_2$ with slenderness ratio

3.2. Rocking ceases after impact

When rocking of the block on top of the moving base ceases, the system will attain a pure-translation mode (Figure 3b). In this case, the impact analysis is reduced to the computation of the post-impact translational velocity of the system, $\dot{u}^{+}$, given the position and the pre-impact velocities, $\dot{u}^{-}$ and $\dot{\theta}^{-}$.

By considering the system as a whole during impact, it can be stated that the horizontal impulse on the system is zero, resulting in the conservation of the system's linear momentum in the horizontal direction. That is,

$$m_b\left(\dot{u}^- + \dot{x}_g\right) + m\left(\dot{u}^- + \dot{x}_g + h\dot{\theta}^-\right) = m_b\left(\dot{u}^+ + \dot{x}_g\right) + m\left(\dot{u}^+ + \dot{x}_g\right) \tag{3.11}$$

which upon rearranging terms becomes

$$\dot{u}^+ = \dot{u}^- + \frac{\overline{m}h}{\left(1+\overline{m}\right)}\dot{\theta}^- \equiv \dot{u}^- + \beta_3\dot{\theta}^- \tag{3.12}$$

4. NUMERICAL EXAMPLE

Numerical results are obtained through an ad hoc computational scheme developed to determine the response of the system under horizontal ground excitation. The numerical integration of the equations of motion is pursued in MATLAB through a state-space formulation (MATLAB 2006). In each time step, close attention is paid to the eventuality of transition from one pattern of motion to another and to the accurate evaluation of the initial conditions for the next pattern of oscillation, on the basis of the developed impact model.

As an example, the response of the system to the N-S component of 1995 Kobe, Japan earthquake was computed. Results are presented here for a system of a homogeneous block with height $H = 2.0$ m, base width $B = 0.7$ m and mass $m = 3800$ kg, and a rigid base of mass $m_b = 3800$ kg (i.e. $\overline{m} = 1$). The linear isolation system considered has period $T = 1.5$ s and viscous damping ratio $\xi = 0.20$. For the sake of comparison, results are also generated for the case of a non-isolated rigid block. Figure 5 depicts comparison of the computed response, in terms of the rotation and angular-velocity histories of the rigid block and the horizontal-displacement history of the base. It can be observed that the rocking response of the isolated block, in terms of the rotation amplitude, the number of impacts and the overall duration of oscillation, is reduced. In comparison with the non-isolated block, which is on the verge of overturning ($|\theta/\alpha|_{max} = 0.95$), the isolated block exhibits a maximum rotation amplitude $|\theta/\alpha|_{max} = 0.64$ and as many as 5 times less impacts.

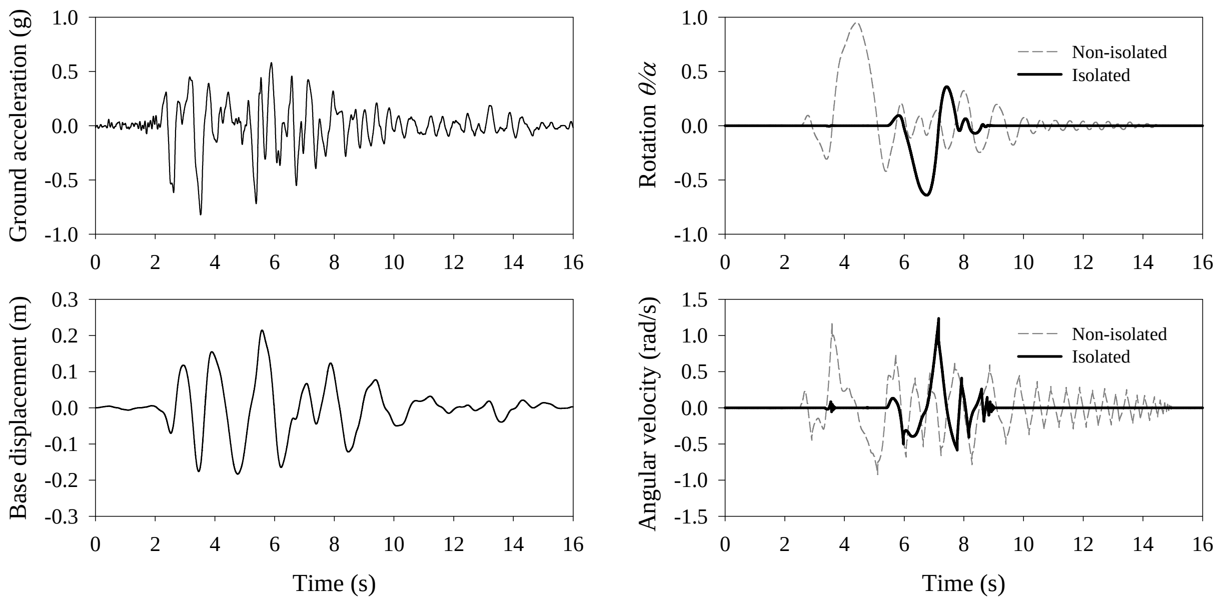


Figure 5: Response of the system to the N-S component of 1995 Kobe, Japan earthquake

5. CONCLUDING REMARKS

The dynamic response of base-isolated block-like slender objects, such as statues, subjected to horizontal ground excitation is investigated. The model considered consists of a rigid block supported on a rigid base, beneath which the isolation system is placed. Under the assumption of sufficient friction to prevent sliding of the block relative to the supporting base, when subjected to base excitation the system may exhibit two possible patterns of motion, each being governed by highly nonlinear differential equation(s). The system can be set in pure translation, in which the system as a whole oscillates horizontally (1-DOF response), or rocking, in which the rigid block pivots on its edges with respect to the horizontally-moving base (2-DOF response). The dynamic response of the system is strongly affected by the occurrence of impact between the block and the moving base, as it can modify not only the energy but also the degrees of freedom of the system by virtue of the discontinuity introduced in the response. Therefore, the critical role of impact in the dynamics of the system necessitates a rigorous formulation of the impact problem. In this paper, a model governing impact from the rocking mode is derived from first principles using classical impact theory. Numerical results are obtained via an ad hoc computational scheme developed to determine the response of the system under horizontal ground excitation.

ACKNOWLEDGEMENTS

The project has received research funding from the European Community´s Sixth Framework Programme under the Marie Curie Action (IRG-2007-046577).

REFERENCES

Agbabian, M.S., Masri, S.F., Nigbor, R.L. and Ginell, W.S. (1988). Seismic damage mitigation concepts for art objects in museums. *Ninth World Conference on Earthquake Engineering*, Tokyo-Kyoto, Japan.

Augusti, G., Ciampoli, M. and Airoldi, L. (1992). Mitigation of seismic risk for museum contents: An introductory investigation. *Proceedings of the Tenth World Conference on Earthquake Engineering*, Balkema, Rotterdam.

Caliò, I. and Marletta, M. (2003). Passive control of the seismic rocking response of art objects. *Engineering Structures* **25:** 1009-1018.

Ishiyama, Y. (1982). Motions of rigid bodies and criteria for overturning by earthquake excitations. *Earthquake Engineering and Structural Dynamics* **10:** 635-650.

Makris, N. and Roussos, Y.S. (2000). Rocking response of rigid blocks under near-source ground motions. *Géotechnique* **50: 3,** 243-262.

MATLAB 7.3 (2006). The Language of Technical Computing The Mathworks, Inc.: Natick, MA.

Myslimaj, B., Gamble, S., Chin-Quee, D., Davies, A. and Breukelman, B. (2003). Base isolation technologies for seismic protection of museum artifacts. *The 2003 IAMFA Annual Conference*, San Francisco, California.

Psycharis, I.N. (1990). Dynamic behaviour of rocking two-block assemblies. *Earthquake Engineering and Structural Dynamics* **19:** 555-575.

Shenton III, H.W. and Jones, N.P. (1991). Base excitation of rigid bodies. I: Formulation. *Journal of Engineering Mechanics (ASCE)* **117: 10,** 2286-2306.

Spanos, P.D. and Koh, A.-S. (1984). Rocking of rigid blocks due to harmonic shaking. *Journal of Engineering Mechanics (ASCE)* **110: 11,** 1627-1642.

Spanos, P.D., Roussis, P.C. and Politis, N.P.A. (2001). Dynamic analysis of stacked rigid blocks. *Soil Dynamics and Earthquake Engineering* **21,** 559-578.

Vestroni, F. and Di Cintio, S. (2000). Base isolation for seismic protection of statues. *12th World Conference on Earthquake Engineering*, Auckland, New Zealand.

Yim, C.-S., Chopra, A.K. and Penzien, J. (1980). Rocking response of rigid blocks to earthquakes. *Earthquake Engineering and Structural Dynamics* **8: 6,** 565-587.