APPLICATION OF A ROBUST QFT LINEAR CONTROL IN CIVIL ENGINEERING

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ABSTRACT:

This paper describes in detail the design methodology of a robust Quantitative Feedback Theory (QFT) controller for the control of the structure excited by earthquake motion. QFT is a frequency-domain-based robust control design approach that was developed in the early 1970s and has received considerable attention over the past two decades in mechanical and electronic fields. However, QFT application's in civil engineering is a novel technique; the main advantage of QFT is its robustness and stability that is against the other defects. The basic concepts, principles, design procedure, and features of QFT are reviewed and summarized for single-input single-output systems. The feasibility of the purposed method is shown by a numerical example.

KEYWORDS: Structural Control, Frequency Domain, QFT, Robust Control

1. INTRODUCTION

In the 1960s, Isaac Horowitz introduced an efficient robust control design technique in the frequency domain, known as the "Quantitative Feedback Theory" or QFT. This technique considers a priori the uncertainty that may be present in the process and its environment and establishes a balance between the quantity of feedback required and the design complexity. The controller designed with this method is of minimum cost, does not have a large gain and minimizes the control effort. Moreover, it has a smaller bandwidth than that obtained using any other design technique dealing with special structures and their uncertainties, disturbances and/or specifications.

The QFT method has been applied already in the design of different types of control systems, for example, flight control (Houpis et al., 1994), and vibration control of a smart beam structure (Choi, 2006), robot control systems (Yaniv and Horowitz, 1990; Kelemen and Bagchi, 1993; Piedmonteet al., 1998),controller design for an electro hydraulic actuator (Niksefat and Sepehri , 2000), control of an activated sludge wastewater treatment plant (Ostolaza and Garcia Sanz, 1997), this paper presents the application of this method to the control of civil structure excited by earthquake motion. Application of QFT in control of building is a new idea and this robust method can guarantee stability, disturbance rejection and other objective, whereas the structure models and actuator have a variety of uncertainty.

2. QFT DESIGN TECHNIQUE FOR LINEAR SINGLE-INPUT SINGLE-OUTPUT SYSTEM (SISO):

The QFT design method is specified by its consideration a priori of the uncertainty of the system, caused by the variations in the parameters of the equipment to be control and by external disturbances and takes into account in the controller design process both the gain and its phase. It tries to minimize the control effort in order to avoid saturations in the actuators or in the plant, which can be cause by the amplification of the sensor noise required to reach the desired specifications with a minimum bandwidth. With this method, a robust controller is obtains which is insensitive to the uncertainties of the process. The system model may be given as a transfer function or using experimental data. The state variables representation is not normally use, since it is rather more complex. This technique makes it possible to predict quite simply whether some desired behavior, specification will not be fulfills and to rectify the design accordingly without using complex mathematical tools. With this design method, a controller can be select in graphical form in the frequency domain.

The QFT method demonstrates a general control strategy with two degrees of freedom structure that presented in Figure1. In this block diagram of system, the transfer function P(s) belongs to a set {P} of plants with uncertainties, and the G(s) and F(s) denote the controller and prefilter to be synthesized in order to meet robust stability and closed loop specification and H(s) denote the transfer function of sensor. Also R(s), W(s), V(s), D(s), Y(s), N(s) and U are sequence represented the reference input, controller input disturbance, Plant input disturbance, Plant output disturbance, Plant output, measurement noise and controller output.

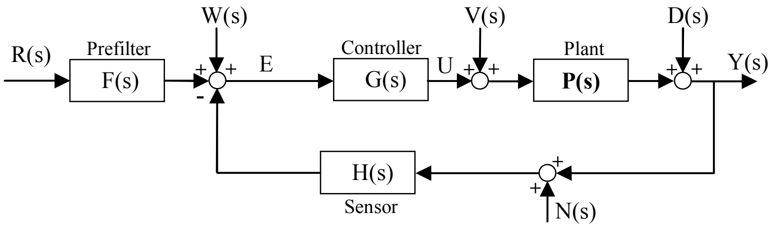


Figure 1 Two degrees of freedom control structure (F and G)

3. MATHEMATICAL MODEL FOR CONTROL OF CIVIL STRUCTUR

Taking into account that in structural control the main control objective is to reduce vibrations, the reference signal R(s) can be set to zero all time and a prefilter does not have sense in this kind of problems. Therefore, the structural control in block diagram format is shown in figure 2. In which V(s) represents the external disturbance of the structure such as earthquake motion. The output Y(s) represent the state variable to be controlled and U define the force control that provided by the control devices at the position where the output Y(s) is measured. The plant P(s) involve the structural parameters that directly related with the variable to be controlled, while the other immeasurable states are not included or included as a finite value.

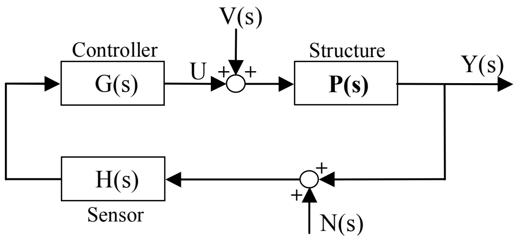


Figure 2 LTI structure for structural control

The motion equation ofs ingle-input single-output system describe by:

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = 0 \qquad (3.1)$$

The Laplace transformation of Eqn.3.1 whit initial condition $\dot{y}(0) = 0, y(0) = 0$ is:

$$P(s) = \frac{Y(s)}{R(s)} = \frac{1}{ms^2 + cs + k} \tag{3.2}$$

Then transfer function of plant with uncertainty is:

$$\{\boldsymbol{P(s)}\} = \left\{ \frac{1}{ms^2 + cs + k} \;\middle|\; k \in [k_{min}, k_{max}] \;\;\&\;\; c \in [c_{min}, c_{max}] \right\} \tag{3.3}$$

Finding G(s) is the main target of this method. The following steps are required to calculate G(s).

4. QFT DESIGN STEPS

4.1. Definition of Design Objectives

The objectives to be performed by a control design in structural domain are: a) The displacement at the certain position is less than certain value for the uncertain disturbance (disturbance rejection) and b) The displacement is less than certain value for all working frequencies (robust stability).

4.2. Selection of Design Parameters (Define Nominal Plant)

The second step in the design process is to select a nominal plant $P_0(s)$ from among the family of plants **P(s)**. An adequate and finite set of frequencies Ω must also be selected. This set is determined by the bandwidth of the system and by the frequencies of interest, for which the different desired behavior specifications are defined. In this case, the nominal plant selected is:

$$P_0(s) = \frac{1}{ms^2 + c_0 s + k_0} \tag{4.1}$$

$$k_0 = (k_{min} + k_{max})/2 \;,\;\; c_0 = (c_{min} + c_{max})/2 \tag{4.2}$$

4.3. Representation of Plant Uncertainty and Template Computation

The third step in the design process is to represent as accurately as possible the uncertainty of the system. When the system is not determine by a single model, the frequency responses of the system for a given frequency is represented by a set of points, as there are different models. All of these points define a region of uncertainty known as template. There will be as many templates as frequencies in the set Ω. The most common way to calculate a template is to perform a sweep of the values that the model parameters can take. In this study, a sweep is made of the values that can be taken by the parameters k and c.

4.4. Obtaining the Bounds

The fourth step in QFT design terminology is to define the appropriate behavior limitations. The specifications given, combined with the uncertainty of the system, form what are termed bounds. They are represented on the magnitude (dB)-phase (deg) plane, and there is one for each frequency and specification; they are denoted as $B\;(\omega)$. These curves are the objects that define the bounds of the regions prohibited for the adjustment of the controller. If the transfer function of the controller is denoted as $G(j\omega)$ and the transfer function of the nominal plant as $P_0(j\omega)$, the bounds are those regions that the open loop function frequency response $L_0(j\omega)$ that $L_0(j\omega) = G(j\omega)P_0(j\omega)$ must avoid in order to guarantee the fulfillment of the design specifications for the whole set of plants $P(j\omega)$. In order to use the QFT method, the bounds need to be defined in the frequency range.

4.4.1 Performance Specifications

4.4.1.1. Robust Stability Specification

Relative stability is defined normally in terms of certain desired gain margins and phases. These related with a value in decibels δ, known as the M-circle because it takes this shape if represented in a magnitude-phase diagram. This circle identifies a forbidden zone around the point [-180º, 0dB], which the loop function must not cross ($\forall \omega \in \Omega, \forall P \in \boldsymbol{P}$) in order to ensure the margin of minimum stability. The specification of robust stability is written as:

$$\left| \frac{P(j\omega)G(j\omega)H(j\omega)}{1 + P(j\omega)G(j\omega)H(j\omega)} \right| \le W_{S_1} = \delta \tag{4.3}$$

If transfer function of sensor, H, $H(j\omega) = 1$ then:

$$\left| \frac{PG}{1 + PG} \right| \le \delta \tag{4.4}$$

Relating it with the gain margin (GM) and phase (PM) as follows:

$$GM = 1 + \frac{1}{\delta}, PM = 180 - \boldsymbol{cos}^{-1}(0.5/\delta^2 - 1) \tag{4.5}$$

4.4.1.2. Disturbance Rejection Specification

Plant input disturbance rejection means for any $P \in \boldsymbol{P}$ the transfer function from the disturbance at the Plant input to the plant output is bounded by:

$$\left| \frac{Y(j\omega)}{V(j\omega)} \right| = \left| \frac{P(j\omega)}{1 + P(j\omega)G(j\omega)H(j\omega)} \right| \le W_{S_3} = \delta_p(\omega) \tag{4.6}$$

If transfer function of sensor, H, $H(j\omega) = 1$ then:

$$\left| \frac{P}{1 + PG} \right| \le \delta_p(\omega) \tag{4.7}$$

4.5. Tuning of the Controller

The fifth step in the design of the control system consists in finding a controller with which all of the desired specifications are fulfilled. It is also known as the synthesis or "loop-shaping" phase. The method consists in assuming an initial value of the controller function $G_0(j\omega)$, and adjusting the loop function $L_0(j\omega)$ that verifies the imposed restrictions and minimizes the control effort. The adjustment is made by shifting the loop curve vertically and horizontally on the magnitude-phase plane, until it is situated in such a way as to not violate the bounds and as to have the lowest gain possible. For the example and assume an initial controller of a constant value, $G_0(s)=1$ the representation of the loop function is a curve with several points marked in colors. These points correspond to the response of the loop for the various frequencies defined in Ω, following the same color code as in the bounds. The loop adjustment must be done in such a way that each colored point is close to the bound of the same color, and same frequency. The good design is greatly depend on the skill of designer. There is no single or perfect solution. The controller is related with the loop function as follows:

$$L_0(j\omega)=G(j\omega)P_0(j\omega) \tag{4.8}$$

In this way, once the loop is adjusted, it is simple to obtain the transfer function of the compensator. The controller obtained is robust, that is, it provides good results for all of the family of plants defined by the uncertainty, not only for the nominal plant used in the loop-shaping stage. It is recommended, in this loop-shaping stage, to always begin by adjusting the point corresponding to the lowest frequency, continuing upwards and modifying the function progressively.

4.6. Design Validation

The last step in the design process is validation of the results, which is done by checking the specifications in the frequency and time domain graphically. Moreover, this validation is essential, since the design has made only for a finite set of frequencies and hence it cannot ensured, a priori, that it will fulfilled for any other frequency, inside or outside this range.

5. NUMRERICAL EXAMPLE

In this section the QFT design steps are shown by an example. For this example $W=17.5ton$, $k\in[360,1200]$ ton/m and $c\in[4.60,17.2]$. The nominal plant is obtained from Eqn.4.1 and Eqn.4.2:

$$P_0(s)=\frac{0.56}{s^2+6.121s+437.20} \tag{5.1}$$

Natural frequency for the example is $\omega_n=2.6rad/s$ then the finite set of frequencies Ω is $\Omega=[0.1\omega_n,10\omega_n]=[0.26,26]$
The templates obtained for the family of plants P(s) and for the set of frequencies Ω are as shown in Figure 3.
Each point represents the frequency response of one plant of the family and each color distinguishes the response for each value of the frequency range. The shape of the templates varies with the frequency and its size decreases when the frequency increases.
Robust stability specification is obtained from Eqn.4.4 and Eqn.4.5. For the example, a phase margin of at least 45º is assumed. Thus, the following should fulfill:

$$\left|\frac{P_0G}{1+P_0G}\right|\leq\delta=1.2 \tag{5.2}$$

For disturbance rejection specification from Eqn.4.7:

$$\left|\frac{Y}{V}\right| = \left|\frac{P}{1 + PG}\right| \le \frac{2\left(0.1s^2 + 0.3s + 0.03\right)}{s^2 + 420\,s + 50} \tag{5.3}$$

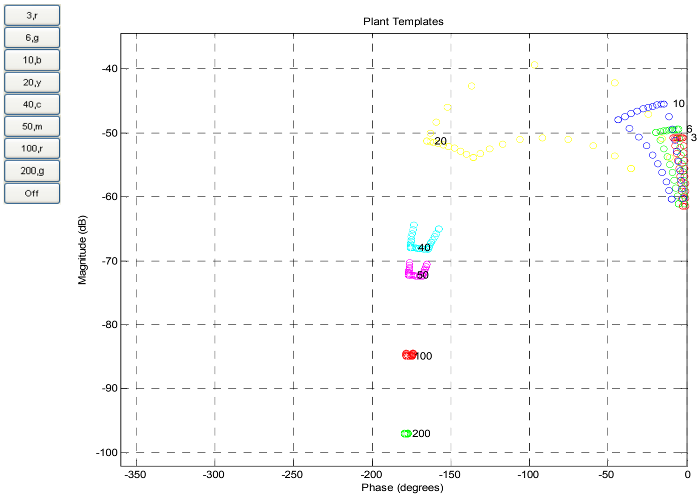
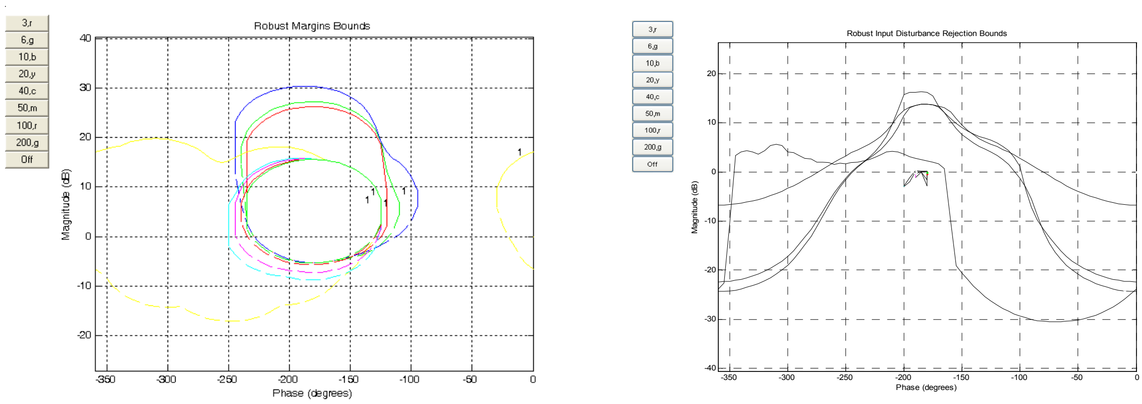


Figure 3 Templates

Taking into account the specifications imposed and the uncertainty of the model, the bounds for robust stability are shown in Figure 4-A.When the bounds are represented by a continuous line and closed, the specification is verified if the frequency response of the loop function for each frequency is outside the curve corresponding to the same frequency.



A) Robust stability bounds B) Robust disturbance rejection bounds

Figure 4 Templates

Taking into account the specifications imposed and the uncertainty of the model, the bounds for robust disturbance rejection bounds are as shown in Figure 4-B.

After several attempts, for this example, the compensator G(s) obtained finally is (Figure 5):

$$G(s) = 701.12\frac{s + 10.209}{s + 61.011} \tag{5.4}$$

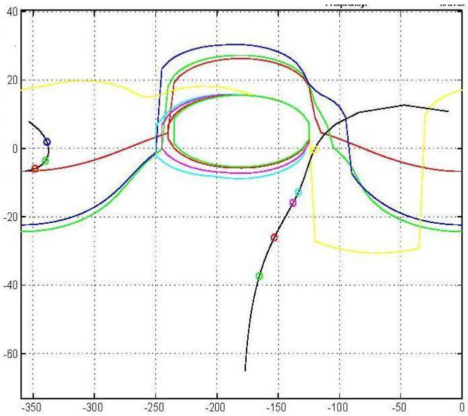


Figure 5 loop shaping

Design validation in the frequency and time domain is necessary. Figure 6 (left) shows with a dotted line the desired stability value (δ = 1.2dB) and with a continuous line the system response in the frequency domain. As this latter value is below the specification line, the required robust stability condition is fulfilled and Figure 6 (right) shows ratification of robust disturbance rejection in the frequency domain.

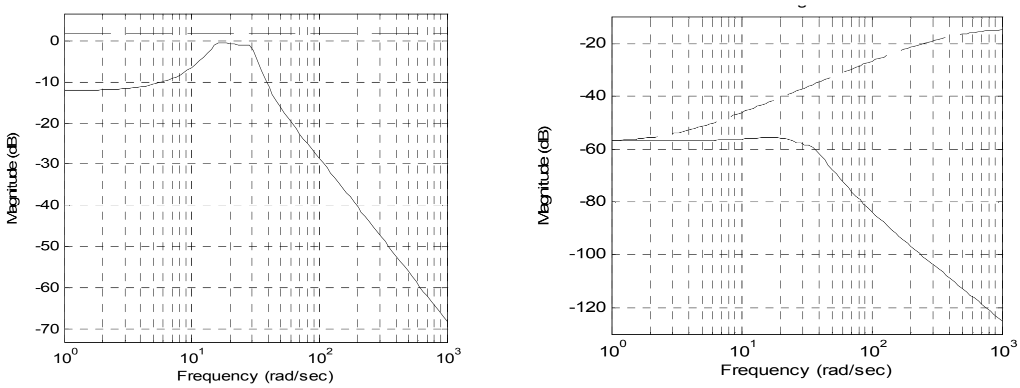


Figure 6 design validation

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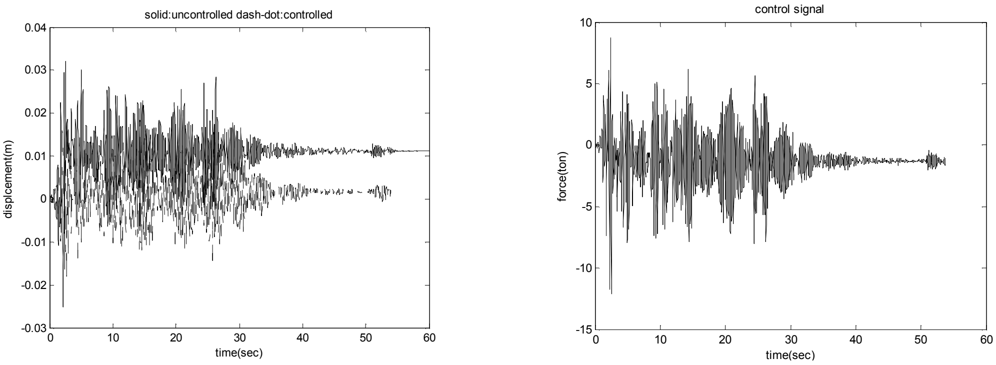

Figure 7 comparison of displacement (left) - control signal (right)

Feasibility of selected compensator in time domain is evaluated by structure response that the external disturbance is El Centro (NS) and Northridge ground motion and the structure equipped with QFT controller. Figure 7 (right) shows Comparison of controlled and uncontrolled displacement under El Centro NS excitation and Figure 7 (left) shows the control signal. Figure 8 (right) shows Comparison of controlled and uncontrolled displacement under Northridge excitation and Figure 8 (left) shows the control signal

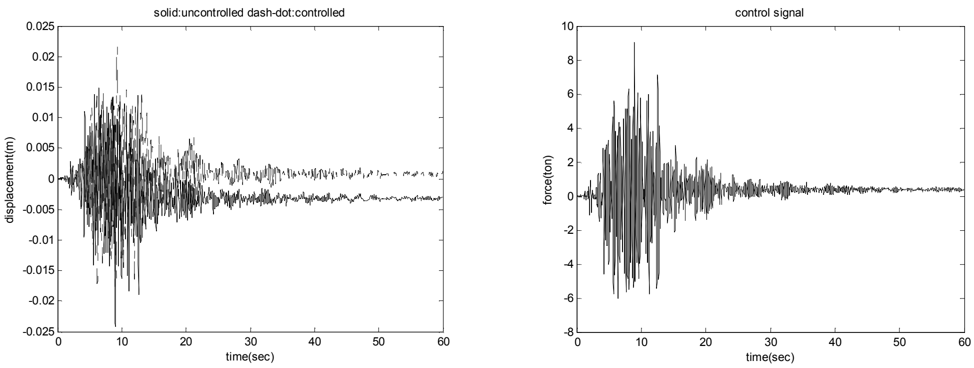

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Figure 8 comparison of displacement (left) - control signal (right)

6. CONCLUSION

An adaptive linear QFT robust control methodology has been applied to civil structure control. It has been demonstrated that this technique is applicable for civil structures, which have uncertainties in the parameters. It has been verified, by means of simulation, that the required specifications of robust stability and disturbance rejection are fulfilled. Unchanged design of QFT for different earthquakes is another important characteristic of QFT controller in civil engineering.

REFERENCES

AZvine, B., Wynne R. J. (1992). A review of Quantitative feedback theory as a robust control system design technique. *Transactions Inst. Measurement and Control* **14**, 265-279.

Choi, S.B., Cho, S S., Park, Y.P. (1999). Vibration and Position Tracking Control of Piezoceramic based Smart Structures via QFT. *ASME J. Dynamic System Measurement Control* **121**, 27-33.

Choi, S.B. (2006). Vibration control of a smart beam structure subjected to actuator uncertainty: Experimental Verification. *Acta Mechanica* **181**, 19–30.

Horowitz, I. (1959). Fundamental Theory of Linear Feedback Control System. *Transactions IRE on Automatic Control* **Ac-4**.

Horowitz, I. (1991). Survey of Quantitative Feedback Theory. *Int. Journal of Control* **53**, 255-294.

Houpis, C.H., Sating, R., Rasmussen, S., Sheldon, S. (1994). Quantitative Feedback Theory Technique and Applications. *Int. Journal of Control* **59**, 3970-3984.

Keating, M.S., Pachter, M., Houpis, C.H. (1995). QFT Applied to Fault Flight Control System Design, *Proceedings of ACC* 184–188.

Kelemen, M., Bagchi, A. (1993). Modeling and Feedback Control of a Flexible Arm of a Robot for Prescribed Frequency Domain Tolerances. *Automatic* **29**, 899-909.

Niksefat N., Sepehri N. (2000). Design and experimental evaluation of a robust force controller for an electro-hydraulic actuator via quantitative feedback theory. *Control Engineering Practice* **8**, 1335-1345.

Piedmonte, M.D., Mechl, P.H., Nwokah, O.D., Francheck, M.A. (1998). Multivariable Vibration Control of a Coupled Flexible Structure using QFT. *Int. Journal of Control* **69**, 475-498.

Yaniv, O., Horowitz, I. (1990). Quantitative Feedback Theory for Active Vibration Control Synthesis. *Int. Journal of Control* **51**, 1251-1258.