SHEAR MECHANISM AND CAPACITY CALCULATION OF STEEL REINFORCED CONCRETE SPECIAL-SHAPED COLUMNS

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ABSTRACT :

Through experiments on 17 steel reinforced concrete (SRC) special-shaped column specimens under low-cyclic reversed loading, loading process and failure patterns of SRC special-shaped columns with different steel reinforcement are observed. From the experiments it is shown that the failure patterns of these columns include shear-diagonal compression failure, shear-bond failure, shear-flexure failure and flexural failure. The failure mechanism and characteristics of SRC special-shaped columns are also analyzed. For different SRC special-shaped columns, based on the failure characteristics and mechanism observed from the test, formulas for calculating ultimate shear capacity in shear-diagonal compression failure and shear-bond failure under horizontal axis and oblique loading are derived. The calculated results show very good agreement with the actual experimental measurement. Both the theoretical analysis and the experimental results show that, shear capacity of T、L shaped columns under oblique loading are larger than that under horizontal axis loading, whereas the shear capacity of ┼-shaped columns under oblique loading are less than that under horizontal axis loading.

KEYWORDS: steel reinforced concrete (SRC), special-shaped columns, shear mechanism, shear capacity

1. EXPERIMENT OUTLINE

1.1 Specimen design and production

Totally 17 specimens, which include 9 T-shaped, 4 L-shaped and 4 ┼-shaped columns are designed(Chen, 2007). 4 parameters, which are shape steel, loading direction, axial compressive ratio and shear span ratio, are used. Shape steel includes T-shape steel truss, channel steel truss, and solid shape steel. Loading directions include horizontal axis and oblique axis. Shape steel and cross-sectional geometry of the specimens are shown in Figure 1. Test parameters are listed in Table 1.

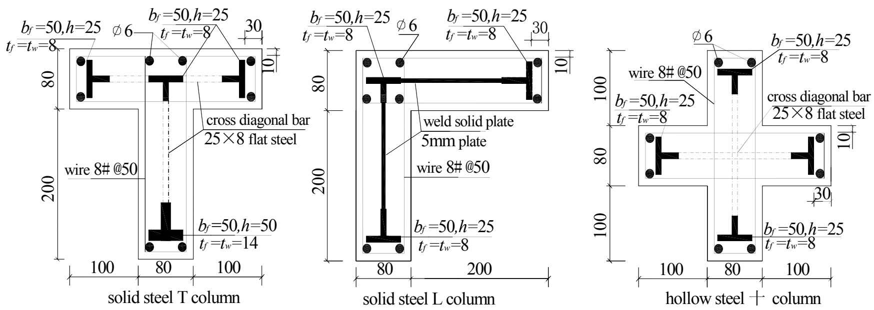


Figure 1 Geometry and steel details of specimens

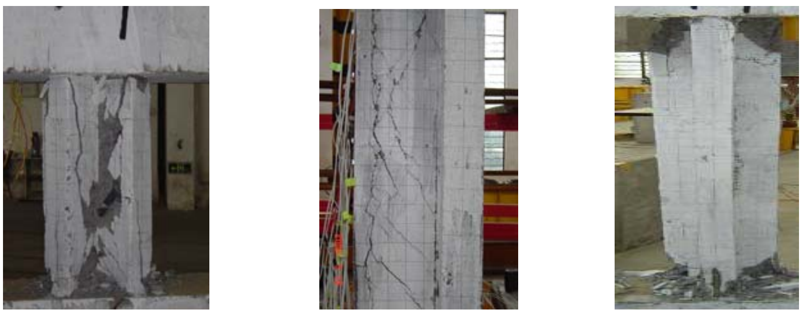
Table 1 Test parameters

Specimen ID	Shape steel	Loading angle	Axial compressive ratio n	Shear span ratio λ	Cubic compressive strength f_{cu}/N·mm^{-2}	ρ_{ss} /%	ρ_{sw} /%
T1	T-shape	along flange	0.3	1	25.48	8.33	3.868
T2	T-shape	along web	0.5	2	27.03	8.33	3.765
T3	T-shape	45°	0.7	2.5	28.09	8.33	3.744
T4	channel	along flange	0.5	2.5	28.09	8.49	3.665
T5	channel	45^0	0.7	1	25.48	8.49	3.990
T6	channel	along web	0.3	2	27.03	8.49	3.720
T7	solid	along flange	0.7	2	24.88	11.21	0
T8	solid	45^0	0.3	2.5	22.81	11.21	0
T9	solid	along web	0.5	1	27.24	11.21	0
L1	channel	along horizontal axis	0.3	1	27.24	5.39	3.990
L2	channel	45^0	0.7	2	27.03	5.39	3.720
L3	solid	along horizontal axis	0.7	2	24.88	7.55	0
L4	solid	45^0	0.3	1	25.48	7.55	0
┼1	T-shape	along horizontal axis	0.5	1	25.48	6.25	1.620
┼2	T-shape	45^0	0.7	2.5	28.09	6.25	3.068
┼3	channel	along horizontal axis	0.7	2.5	28.09	7.18	3.750
┼4	channel	45^0	0.5	1	27.24	7.18	4.090

Notes: axial compressive ratio $n=N/f_cA$, N is axial compressive force, A is area of section, shear span ratio $\lambda=L/2H$, L is length of specimens, H is height of cross section, ρ_{ss} is ratio of shape steel to concrete area, ρ_{sw} is ratio of diagonal and level steel bar to concrete volume.

1.2 Failure pattern and mechanism

Low cyclic reversed loading test is carried out on the parallel crosshead equipment. Failure patterns of these specimens can be grouped into four categories: shear-diagonal compression failure, shear-bond failure, shear-flexure failure and flexural failure, which are shown in Figure 2.



(a) shear-diagonal compression failure (b) shear-bond failure (c) flexural failure

Figure 2 Failure patterns of specimens

（1）Shear-diagonal compression failure mechanism

Shear-diagonal compression failure occurs mainly on specimens with low shear span ratio（λ=1）. Its failure procedure can be divided into three stages as elastic, elastic-plastic, and failure. In elastic stage before cracking, deformations of shape steel and concrete are the same. It gets into the elastic-plastic stage when cracks appear. Firstly cross diagonal cracks appear in the middle segment. The numbers of cracks increase with load increasing, and divide surface concrete into small rhombic pieces. While load continues to increase, specimens get into the failure stage. Surface concrete is divided into several small short diagonal columns through the formation of several main cross diagonal cracks among the multiple diagonal cracks. Concrete gradually crushes and falls off. The periphery part outside stirrups is crushed first and the crushing expands inside until web or diagonal bars are exposed. In the end web steel yields, lateral load drops rapidly and specimens are destroyed. According to the test, strain of shape steel and stirrups are both small in the elastic stage, while shape steel and concrete work together as a whole element to carry load, and have the same deformation. Concrete quit the work after it cracks. It releases elastic energy and transfers it to the web steel it crossed with. Shear stress redistribution occurs, and shear strain of shape steel and stirrups increases obviously and shows nonlinear behavior. When load reaches its peak value, shape steel and stirrups yield.

（2）Shear-bond failure mechanism

For SRC special-shaped columns with moderate shear span ratio（λ=2）, shear-bond failure occurs easily. The failure process includes the initial appearing of horizontal cracks at both ends of columns, and then cross diagonal cracks appear in middle segments. The cross diagonal cracks extend when load increases. They become steep abruptly when extend to the vicinity of steel flanges, and then form vertical bond cracks. Since then, the bond cracks develop faster than the diagonal cracks, and they interconnect from above to below to form a single vertical bond crack through the height of a column. The crack splits the protective concrete cover, load drops rapidly and specimens are destroyed. When load reaches its peak value, web members of shape steel and stirrup almost approach yield but not yet so.

（3）Shear-flexure failure mechanism

For L4 specimens with low shear span ratio (λ=2), reinforced with solid web steel and loaded in 45º, shear-flexure failure occurs. Cross diagonal cracks firstly appear in middle segment. Having the characteristics of large in numbers and small in sizes, these cracks develop and increase in numbers continuously with the increase of loads. In the meantime, horizontal cracks appear due to moment at both ends. Then with the continuous increase of loads, these horizontal cracks develop more rapid than the diagonal cracks. At last, vertical cracks appear at both ends of specimens and concrete is crushed. From failure pattern, it can be seen that the ultimate load capacity depends on the flexural capacity of normal section. Neither the web steel nor stirrups yield when failure occurs.

（4）Flexural failure mechanism

Flexural failure often occur in specimens with high shear span ratio (λ=2.5). Horizontal flexure cracks or vertical flexure cracks firstly appear at both ends of specimens. With the increase of loads, minor diagonal cracks appear in some specimens. Failure pattern is shown as the crush of concrete at both ends. Longitudinal steel and bars yield, but not web steel and stirrups.

2. FAILURE MECHANISM OF SRC SPECIAL-SHAPED COLUMNS UNDER OBLIQUE LOADING

Experiments are carried out on T-shape specimens under 0º (along the flange), 45º and 90º (along the web) load, L-shape and ┼-shape specimens under 0º (along the flange) and 45º load. In polar coordinate system, α denotes the load angle and polar radius denotes characteristic value of shear capacity $V_{u\alpha} / f_t bh_0$. Due to the low-cyclic reversed loading, in the directions of 45º（45º and 225º）, 0º（0º and 180º）and 90º（90º and 270º）, two shear capacities are obtained in each direction. For L-shape and ┼-shape specimens, the shear capacity at 0º is equal to that at 90º. Based on the test results, all the data points describing shear-diagonal compression failure pattern are depicted in the same polar coordinate system and then the polar coordinate is transferred to rectangular coordinate following the relations of $x = \rho \sin \alpha$, $y = \rho \cos \alpha$. The result is shown in Figure 3. The correlation curves of L-shape and T-shape specimens are close to ellipse, and the correlation curves of ┼-shape specimens is close to rhombic.

Figure 4 shows shear stress distributions of different SRC special-shaped columns. It can be seen that, for T-shape and L-shape specimens, the maximum shear stress under oblique load is less than that under horizontal axis load. This is mainly due to the enlargement of shear area under oblique load. But for ┼-shape specimens, the maximum shear stress under horizontal axis load is less than that under oblique load. This is mainly because that, when specimens are loaded at horizontal axis, the vertical branch locates in the vicinity of centroid of cross section, so the shear stress here decreases. When specimens are under oblique loads, the intersection of two vertical branches is in the vicinity of centroid of the cross section, in this place the width of shear-resistance cross section changes abruptly (from double sections to single section to resist shear force), which results in the abrupt increase in the maximum shear stress. Although the widths of cross sections with the maximum shear stress in the two loading directions are the same, the distance between the critical section and centroid of cross section under horizontal axis load is longer than that under oblique load, therefore its maximum shear stress is less than that under oblique load.

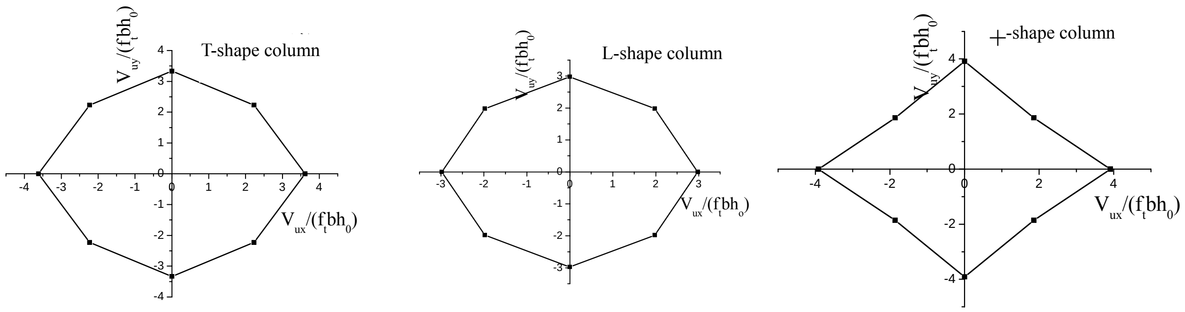


Figure 3 Relationship between shear-bearing capacity and loading direction for SRC special-shaped columns

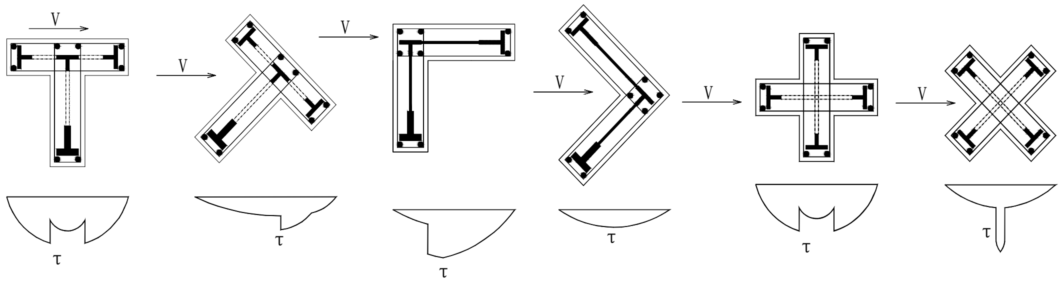


Figure 4 Shear stress distributions in deferent loading direction of SRC special-shaped columns

3. CALCULATION OF SHEAR CAPACITY

3.1 Shear capacity of SRC special shaped columns under horizontal axis load

The experiment results show that main failure patterns of SRC special-shaped columns are shear-diagonal compression failure and shear-bond failure. The former one is due to the crush of diagonal concrete short columns divided by diagonal cracks, the latter one is due to the split of concrete outside the flanges. According to the aforementioned failure mechanism, web steel yield during the shear-diagonal compression failure, while during the shear-bond failure the bond between shape steel and concrete loses, hence superimposition method can be used to calculate the shear capacity(Zhao,2001):

$$V = V_{rc} + V_s \qquad (3.1)$$

Where: V_{rc} is the shear force carried by reinforced concrete, V_s is the shear force carried by shape steel.

3.1.1 Calculation of shear capacity in shear-diagonal compression failure

For different shape steel, the shear force taken by shape steel V_s can be calculated using the formula below:

(1) V_s of solid shape SRC special-shaped columns

$$V_s = \frac{2HM_{fy}V^2_{sy} + 2M_{wy}V_{sy}\sqrt{H^2 \cdot V^2_{sy} - 4(M^2_{fy} - M^2_{wy})}}{H^2V^2_{sy} + 4M^2_{wy}} \qquad (3.2)$$

Where: $N_{wy} = t_w h_w f_s$, $V_{sy} = \frac{1}{\sqrt{3}} t_w h_w f_s$, $M_{wy} = \frac{1}{4} t_w {h_w}^2 f_s$, $M_{fy} = b_f t_f f_s (h_w + t_f)$, are respectively the ultimate axial force, shear force and moment carried by web and ultimate moment of flanges in the plastic stage of solid shape steel in the column branches paralleling to the loading direction. H is the height of specimens.

(2) V_s of hollow shape SRC special-shaped columns

For hollow shape SRC special-shaped columns, treating diagonal members as bent-up steel bars and horizontal members as stirrups, their shear capacity can be calculated using the calculation method for RC members.

$$V_s = f_{sw} A_w \cos\theta + \frac{A_{wh}}{s} f_{sw} h_0 \qquad (3.3)$$

Where: f_{sw} is yield strength of web steel, A_w is area of diagonal web members in the same cross section, θ is the angle between diagonal web member and horizontal web members which parallel to shear force, A_{wh} is cross section area of horizontal web members, s is the spacing of horizontal web members along the height of columns, h_0 is calculated height of the section, let $h_0=h-a_s$, and a_s is depth of concrete cover of longitudinal steel.

To determine shear capacity V_{rc} of reinforced concrete in SRC special-shaped columns, the current Specification for Design of Concrete Special-Shaped Column Structures (JGJ149-2006) in China can be referred, and the contribution of flanges to the increase of shear capacity should also be considered. Shear capacity of RC segment in SRC special-shaped columns under earthquake loading can be determined using the equation below.

$$V_{rc} \le \frac{1}{\gamma_{RE}}\left(\frac{1.05}{\lambda + 1.0} k \cdot f_t b_c h_0 + f_{yv}\frac{A_{sv}}{s} h_0 + 0.056N\right) \qquad (3.4)$$

Where γ_{RE} is the adjustment coefficient of seismic capacity. λ is the shear span ratio. k is increment factor of shear stress considering the contribution of flanges. Their values are shown in Table 2. According to the distribution of shear stress, shear stress of cross section is calculated with or without the consideration of flanges respectively, and the maximum value is picked, consequently the values of k are determined. f_t is the design axial tensile strength of concrete. b_c and h_0 represent the width and effective height of the calculation section respectively. f_{yv}, A_{sv} and s represent the design yield strength, total section area and spacing of the stirrups respectively. N is the design value of axial compressive force. When $N > 0.3(f_c A_c + f_s A_{ss})$, use $N = 0.3(f_c A_c + f_s A_{ss})$. Here A_c is the net area of concrete cross section.

Table 2 k values in different loading angle directions

Length to width ratio	T-shape		L -shape	┼-shape
	Loaded along web	Loaded along flange	Loaded along horizontal axis	Loaded along horizontal axis
2.5	1.0007	1.305	1.0007	1.305
3	1.0019	1.208	1.0019	1.208
3.5	1.0073	1.152	1.0073	1.152
4	1.0137	1.117	1.0137	1.117

3.1.2 Calculation of shear capacity in shear-bond failure

The test result shows that, for SRC special-shaped columns, when shear-bond failure occurs, web steel and stirrup almost but not yield. Let σ_s=0.8f_s and take it into formulae（3.2） and （3.3）to calculate V_s. Here the calculation of shear capacity of RC segment V_{rc} is different from that in shear-diagonal compression failure. Although they are all consisted of RC segment and stirrup segment, they are expressed differently. V_{rc} here is expressed as

$$V_{rc} = V_c + V_{sv} \tag{3.5}$$

（1）Concrete shear capacity V_c

According to the analysis of shear-bond failure mechanism and model, V_c can be expressed as below：

$$V_c = [\tau_1 (b - b_f) + \tau_2 b_f] \cdot j \tag{3.6}$$

Where: τ_1 is shear stress of concrete on the two sides of shape steel flanges, τ_2 is bond stress between concrete and shape steel flanges, b_f is width of shape steel flanges，j is arm of internal force, which is the distance between midpoint of concrete cover outside the compressive flange and centroid of tensile shape steel in the branch column section paralleling to the direction of shear force.

$$\tau_1 = \frac{1}{2}\sqrt{\left(2f_t + \frac{A_{sv}}{bs}\sigma_s + \frac{N}{A}\right)^2 - \left(\frac{A_{sv}}{bs}\sigma_s - \frac{N}{A}\right)^2} \tag{3.7}$$

$$\tau_2 = \lambda_{cy}(0.2378 + 0.4480 C_{ss}/d) f_t \tag{3.8}$$

Where: C_s is the thickness of concrete cover for steel flange, d is the height of shape steel section, and λ_{cy} is the degeneration coefficient of the bond stress under reversed loading. According to a previous test result(Xue, 2007), λ_{cy}=0.83.

（2）Shear capacity of stirrups V_{sv}

For SRC special-shaped columns, stirrups can not only directly resist shear force but also restrict core concrete to improve bond action between concrete and shape steel. Hence it is necessary to set certain amount of stirrup in such columns. The shear capacity of stirrups is:

$$V_{sv} = \sigma_s \frac{A_{sv}}{s} h_0 \tag{3.9}$$

Where: σ_s is stress of stirrup, from this experimental measurement, σ_s=0.8f_{yv}. A_{sv}, s are respectively total area and spacing of stirrup at longitudinal direction in branch columns parallel to shear force. h_0 is calculated height of the section, h_0=h-a_s， a_s is depth of concrete cover for longitudinal bars.

3.1.3 Comparison between calculated result and test result

From Table 3 we can see that the calculated result agrees well with the test result.

3.2 Shear bearing capacity of SRC special shaped columns under oblique load

3.2.1 Shear capacity calculation of the T-shape and L-shape specimens under oblique load

The test results indicate that the correlative curve of the T-shape and L-shape specimens under oblique load is in the shape of ellipse(Maruyama, 1884; Wang, 2006), so its equation can be written as:

$$\left(\frac{V_x}{V_{ux}}\right)^2 + \left(\frac{V_y}{V_{uy}}\right)^2 = 1 \tag{3.10}$$

Where: V_x is projection of oblique shear force V on x axis, $V_x = V \cdot \cos\theta$（θ is the angle between V and x axis）, V_y is projection of oblique shear force V on y axis，$V_y = V \cdot \sin\theta$. V_{ux} and V_{uy} are the shear capacity in x axis and y axis respectively when they are under shear force alone.

Table 3 Comparison of shear capacity between calculated result and test result under horizontal axis load

Specimen ID	Shape steel	Axial compressive ratio n	Shear span ratio λ	Concrete tensile strength f_t/MPa	Failure pattern	Measured value V_t/kN	Calculated value V_c/kN	V_t/V_c
T1	T-shape (hollow)	0.3	1	2.25	shear-diagonal compression	175.8	176.5	0.99
T2	T-shape (hollow)	0.5	2	2.34	shear-diagonal compression	168.4	169.8	0.99
T9	solid	0.5	1	2.35	shear-diagonal compression	290.4	239.5	1.21
┼1	T-shape (hollow)	0.5	1	2.25	shear-diagonal compression	190.2	182.3	1.04
T6	Channel (hollow)	0.3	2	2.34	shear-bond	151.0	156.4	0.965
L1	channel (hollow)	0.3	1	2.35	shear-bond	151.5	156.5	0.968

In the process of diagonal shear design, assume V is shear design value. If the design is conducted for its projection values V_x and V_y on two main axes and treated like positive direction shear design, the diagonal shear capacity V_u is less than the shear design value V. Therefore shear design under oblique load needs to be conducted excessively in both horizontal axes directions, i.e., increase the design shear resistance values in both horizontal axes directions to $\xi_x V_x$ and $\xi_y V_y$ respectively and treat them as shear design in horizontal axes directions.

Based on the present Code for Design of Concrete Structures (GB50010) in China, shear capacity of SRC special-shaped columns under bi-directional shear loading should satisfy the requirements below:

$$V_x \le \frac{V_{ux}}{\xi_x} = \frac{V_{ux}}{\sqrt{1+\left(\frac{V_{ux}}{V_{uy}}\tan\theta\right)^2}}, \quad V_y \le \frac{V_{uy}}{\xi_y} = \frac{V_{uy}}{\sqrt{1+\left(\frac{V_{uy}}{V_{ux}}\cdot\frac{1}{\tan\theta}\right)^2}} \tag{3.11}$$

V_{ux} 、 V_{uy} can be calculated using the aforementioned method under single directional shear load.

3.2.2 Shear capacity calculation of the ┼-shape specimens under oblique load

For ┼-shape specimens, the test results indicate that shear capacity under load in oblique direction is less than that in horizontal axis direction, and the correlative curve is in the shape of rhombus. There is shear stress concentration in the center of the intersection of two column branches, thus it is the weak part of a specimen, as shown in Figure 5. It is mainly here that the cracking from shear-diagonal compression failure and the final failure pattern take place, which has been verified by experimental research. From calculation it is found that the maximum shear stress in the weak part of sections under load in oblique direction is identical to that under load in horizontal axis direction without taking the vertical branch column into effect. However, for a ┼-shape specimen, because its two branches intersect at center, their shear stress distribution in the cross section is affected each other. This is taken into account with the introduction of the aforementioned increment factor k. Therefore, shear capacity under oblique loading should be $1/k$ of the shear capacity in any horizontal axis direction, that is:

$$V = V_{ux} / k \tag{3.12}$$

Where: k is the increment factor considering the influence of flanges. Values can be found in Table 2. V_{ux} and V_{uy} are shear capacity in the directions of x and y axes respectively when they are under shear load individually. They are equal because of the symmetry of cross sections.

Using the above method, shear capacity of the T-shape, ┼-shape specimens under oblique load are calculated and compared with actual measured values from experiments (see Table 4). It can be seen that the two results fit very well.

Table 4 Comparison of shear capacity between calculated results and test results under oblique load

Specimen ID	Shape steel	Axial compressive ratio n	Shear span ratio λ	Concrete tensile strength f_t/MPa	V_{ux}/kN	V_{uy}/kN	Calculated value V_c/kN	Test value V_t/kN	V_t/V_c
T5	channel truss	0.7	1	2.25	224.4	202.3	237.0	241.6	1.02
┼4	channel truss	0.5	1	2.35	197.1	197.1	171.1	176.2	1.03

4. Conclusion

（1）Under low-cyclic reversed loading, failure patterns of SRC special-shaped columns mainly include shear-diagonal compression failure, shear-bond failure, shear-flexure failure and flexural failure. Shear span ratio is the main influencing factor of failure patterns. The failure pattern which is mainly in the form of shear deformation occurs easily in specimens with low shear span ratio.

（2）Flanges in SRC special-shaped columns can enhance the shear capacity. The degree of enhancement is related with loading direction and length to width ratio of column branches, but has nothing to do with width of branches when the length to width ratio is fixed.

（3）Under oblique load, the shear capacity versus loading direction curves are ellipse for T-shape and L-shape columns and rhombic for ┼-shape columns. For L-shape and T-shape columns, as long as the shear capacities in the two horizontal axes directions are satisfied respectively, the shear capacity in the oblique direction can also be satisfied. But for ┼-shape columns, because the diagonal shear capacity is slightly less than the shear capacities in two horizontal axes directions, fulfilling the required shear capacities in two horizontal axes directions does not mean the shear capacity in oblique direction is satisfied.

（4）For SRC special shaped columns with different steel reinforcement, based on the shear mechanism in shear-diagonal compression failure and shear-bond failure patterns, formulae for calculating shear capacity under horizontal axis load and oblique load are derived. The calculated results show very good agreement with the test results.

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