The Effective Pressure Law for Permeability in Northern Hubei Low Permeability Sandstone Rocks

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ABSTRACT :

Some new cognition has not been acquired from the experimental research on the effective pressure law for permeability for a long time. Therefore, reports about the experimental research for the effective pressure law have not been seen recent years. To obtain new view on the effective pressure law, laboratory experiments with two modified factorial designs were performed to determine the effective pressure law for permeability of two samples from Northern Hubei low permeability sandstone formation. One modified factorial design for one sample included four cycles from which pore pressure was different. Each cycle was run through loading and unloading confining pressure in constant-pore-pressure condition. Another design for the second sample contained three cycles from which confining pressure was different. Each cycle was run through raising and lowering pore pressure in constant-confining-pressure condition. Permeability data were taken with the steady-state method. The response-surface method was used which supposed that nothing was known about the material behavior, and a model was built empirically by matching an approximate $k-p_c-p_p$ surface to the data. The coefficients describing the surface reflected information about material behavior and were transformed into $\alpha-p_c-p_p$ response surface. The $\alpha-p_c-p_p$ surfaces showed that at intervals along the pressure path "local" values of the effective pressure coefficient α in the effective pressure law varied with pore and confining pressure. At the highest confining pressure 44MPa in the test, values of α of two samples were all significantly less than 1.0 and varied below 0.1 at different pore pressure interval in most cases. Small values of α at high confining pressure are contrary to conventional view α=1.0. The data suggest a decrease of α with increasing confining pressure and small values of α at high confining pressure. This is interpreted in terms of change in the geometry of the micro-cracks during closure with increasing confining pressure in the paper.

KEYWORDS: permeability, effective pressure, effective pressure coefficient, modified factorial design, response-surface method

1. Introduction

Permeability is one of the most important properties for porous rock. The property has been chosen for the investigation on the effective pressure law for permeability in the tight sandstone rock, which is called as the permeability effective pressure law. The permeability effective pressure law defines a relationship for the interplay of confining pressure and pore pressure on permeability. The effective pressure law is presented as follows

$$k = f(p_{eff}) = f(p_c - \alpha p_p) \qquad 1.1$$

Where k is permeability, μm^2; p_{eff} is the effective pressure, MPa; p_C is the confining pressure on the samples, MPa; p_p is the pore pressure, MPa; $f()$ is a function that describes the effective pressure on permeability, and α is the coefficient that reflects the effect of pore pressure on the effective confining pressure. The value of α is usually taken to be constant, which gives linear effective pressure laws.

Terzaghi[1923] developed an effective pressure law for use with soil analysis. The effective pressure law is accurately expressed by α=1.0 for soil, so that Terzaghi effective confining pressure, p^T_{eff}, is given by

$$p^T_{eff} = p_c - p_p \qquad 1.2$$

This is usually referred to the classic effective pressure law, and is considered to be valid for most properties of soils. The effective pressure law is also often used to evaluate rock behavior.

Laboratory work on effective pressure behavior for permeability has been seen in some literature(Zoback and Byerlee[1975], Walls and Nur[1979], and Bernable[1986,1987,1988], and Warpinski and Teufel[1992]). Zoback and Byerlee[1975] and Walls and Nur[1979] measured permeability on sandstones by using liquid as pore fluid and observed α values that were fairly constant but generally greater than 1.0, showing that pore pressure had much more effect than confining pressure did. Coyner[1984] obtained similar results for the first cycle. He showed that α value tended to be 1.0 after several cycles. Bernable[1986,1987,1988] conducted mostly on crystalline rocks and found that α→1.0 for permeability, using water as the pore fluid. He pointed out that tremendous loading-path and seasoning effects that needed attention before acquiring accurate, reproducible results. Morrow *et al*[1986] performed permeability measurements on Westerly granite and found that α≈1.0. Warpinski and Teufel[1992] conducted laboratory work for studying the effective pressure law for permeability and deformation. Nitrogen was used as pore fluid. They took that α →1.0 for one chalk sample. That α value for one sample tended to fall off at the high confining pressure was considered resulting from measurement error.

This paper presents laboratory work which was performed to determine the value α in the tight sandstone formation in E-bei gas reservoir. Tests were run at constant-confining-presssure cycles through decreasing and increasing pore pressure for one sample and at constant-pore-pressure cycles through loading and unloading for another sample. Permeability data were obtained at various confining pressures and pore pressures and were analyzed with Box and Draper statistical response-surface approach. Results show the variability of the α in many aspects is not in conformity with present theories and laboratory work.

2. Experimental Technique

The simplified schematic of laboratory apparatus and more detailes on the apparatus have been discussed in Min' work[2008]. In our tests, the pore fluid was nitrogen gas so that any chemical and capillary effects that might cause from clay/water reactions and imperfect saturation of the pores could be minimized.Usually, using nitrogen gas as pore fluid causes a problem at low pressure due to Klinkenberg effect[*RP*,1956]. However, Klinkenberg corrections are not needed at high pressures because the compressed nitrogen gas behaves more like a liquid and gas slippage hardly exists. In the trials, the minimum pressure was 6.399MPa and hence the error in neglecting Klinkenberg correction was about 8% according to Warpinski and Teufel' viewpoint[1992], within the standard error for the tests.

The experimental designs were two modified factorials, one of which for one sample showing tests run at constant-pore-pressure cycles through loading and unloading(Sample **NO**.4)could be found in previous work[Min Li, 2008] and the other for one sample (Sample **NO**.8)was seen in the **Fig.1**.

Fig.1 shows tests that were run at constant-confining-pressure cycles through decreasing and increasing pore pressure. Constant-confining-confining pressure cycles were changed from high confining pressure to low confining presssure. The low right corner of the factorial design was cut off for pore pressure could never exceed confining pressure. The design could not reduce equilibration time between tests because time was needed to establish pore pressure within the samples and perturbations from pore pressure changes were a little bit great and would take relative long time to be decayed.

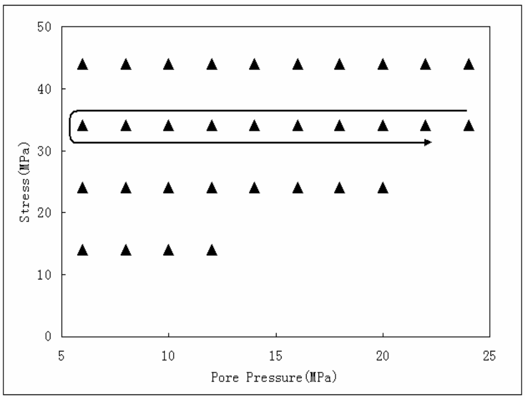

Fig. 1- Modified factorial test design

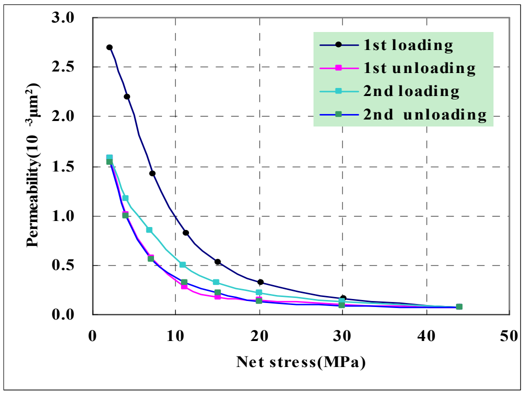

Fig. 2- Permeability seasoning of sample No.4

Permeability measurements were conducted with the steady-state method. Viscosities at different pressure and temperature were obtained from looking up a table in which viscosity at different pressure and temperature is presented for nitrogen gas[*Cryogenic Handbook*,1979]. Time that the same gas volume flowed out from cores were measured five times and averaged for determination of flow rates. Bernable[1986,1987,1988] and Ping[2006] believed that a "seasoning process" before testing is important because results are variable until seasoning is finished. Therefore, all samples were seasoned before testing.

3. Analysis Approach

The paper applies the response-surface method[Box and Draper,1987]. The advantage about this method has been discussed by Warpinski and Teufel[1992]. Briefly, this method include three steps, as follows:

1)Transform the permeability data to a simple form and appropriately weight the variance.

2)Match the resulting transformed data to a quadratic surface in both p_c and p_p.

3)Adopt standard statistical and graphic techniques to obtain the coefficients in the effective pressure law at various p_c and p_p.

The transformed permeability is least-squares matching by the full quadratic surface

$$k^{(\beta)} = a_1 + a_2 p_c + a_3 p_p + a_4 p_c^p + a_5 p_c p_p + a_6 p_p^2 \qquad 3.1$$

Usually, transformations of the form $k^{(\beta)} = k^{\beta}$ will be applied to the data. Where k is experimental data and $k^{(\beta)}$ is transformed data. β is a power generally between -3 and 3, with β=0 being a log transformation. the coefficients a_i are determined from the match. This produces a 3D response surface in $k^{(\beta)} - p_c - p_p$ space. This is a linear regression, which is an important point because all standard statistical analyses can now be applied to the results.

Having been transformed and matched the data, the suitability of the match is determined by performing an analysis of variance. Box and Draper[1987] suggest using *F* tests to evaluate the adequacy of the match. Where *F* is the ratio of the regression mean square to the error mean square. Normal *F* tests simply reveal whether there is any statistical significance to the regression. Box and Draper came up with an analysis that considered that *F* value equals to 10 times the *F*-distribution percentage point is needed to ensure that the surface is adequately matching the data. Generally, the 95% significant *F*-distribution percentage-point will be accepted for any data set.

Assured of a suitable match, a surface of value as a function of confining pressure and pressure can be described as follows[Warpinski and Teufel,1992]

$$\alpha = -\frac{\frac{\partial k}{\partial p_p}}{\frac{\partial k}{\partial p_c}} = -\frac{a_3 + a_5 p_c + 2a_6 p_p}{a_2 + 2a_4 p_c + a_5 p_p} \qquad 3.2$$

Above expression can determine the coefficient α in the effective pressure law at different pore pressure and confining pressure after the coefficient a_i is obtained.

4. Description of the samples

Two samples tested were sandstones from Northern Hubei tight formation of well Da-47 at depth 2760m with estimated overburden confining pressure 62.857MPa. Northern Hubei gas reservoir is low permeability and low pressure sandstone reservoir. Reservoir pressure at 2760m is about 24MPa. The equivalent confining pressure loaded in laboratory is about 44MPa empirically. Consequently, the maximum confining pressure 44MPa is adopted in the laboratory work to represent the in situ stress condition.

Table 1 Propertities of the sampls

Sample No	Length (cm)	Diameter (cm)	Permeability (μm^2)	Porosity (%)
4	3.962	2.528	1.90×10^{-4}	5.48
8	4.022	2.529	2.06×10^{-4}	2.42

Basic properties of the sample No.4 and No.8 were listed in **Table.1.** The samples had had longer length than the length listed in **Table.1**. The longest segments about 4cm long were used for the tests, the other segments were uesd for micro-analysis(SEM and capillary pressure tests), the results of micro-analysis show that the two samples have microcracks that might not exist in situ..

5. Experimental Results

Fig.2 presents permeability results of the seasoning cycles on sample No.4. It was found that the unloading cycle stabilize swiftly, requiring only two cycles. Both two samples were put through a "seasoning process"for two cycles. Usually the loading cycles will take three or four times before the samples are adequately seasoned. To provide stable samples, we loaded confining pressure to maximum confining pressure 44MPa for one night after the seasoning cycles were finished. The seasoning shown in **Fig.2** was performed at an average pressure less than 1.15MPa with no Klinkenberg corrections[*RP*,1956].The results are meant to be rather a diagnostic of sample seasoning than accurate permeability measurements.

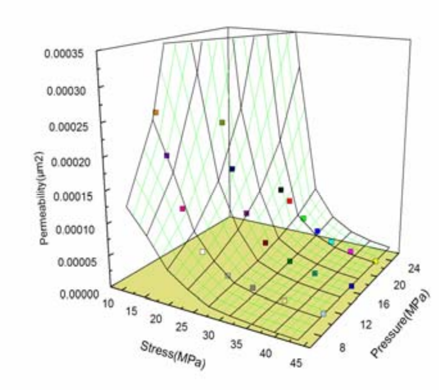

Fig. 3 $k-p_c-p_p$ surface of sample No.4 during loading

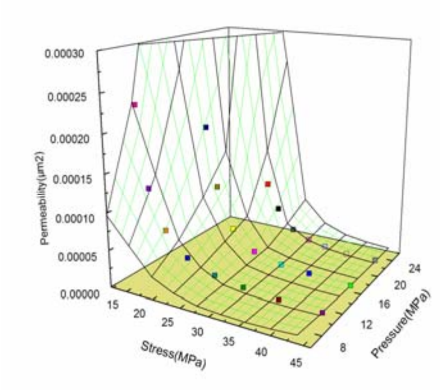

Fig. 4 $k-p_c-p_p$ surface of sample No.4 during unloading

Fig.3 and **Fig.4** shows 3D plots of the permeability results of sample No.4 for the loading and the unloading cycles respectively. The maximum confining pressure was 44MPa and the minimum was 12MPa. These plots show the surfaces in $k-p_c-p_p$ space and the square signs (the experimental data) superposed. These surfaces are called inverse response surfaces because they were transformed from the $k^{(\beta)}-p_c-p_p$ response surface. The data points that pore pressure surpasses confining pressures are drawed on all $k-p_c-p_p$ surfaces in the paper although these points do not exist. The transformation forms and the coefficients a_i describing the response surface are listed in **Table 2** for the cases of the constant-pore-pressure cycles. The maximum 95% significant *F*-distribution percentage-point value multiplied by 10 would not exceed 41 for all these tests. The large *F*-ratios shown in **Table.2** ensure an adequate match of the data as shown on the $k-p_c-p_p$ response surface.A power for the transformation was determined as log transformation by the likelihood Function[Box and Draper, 1987].

Table 2 Transformation forms and coefficients for response surfaces

Sample	Cycle	Transform	Coefficients for response surfaces						
			a_1	a_2	a_3	a_4	a_5	a_6	*F*-ratio
No.4	Load	ln(*k*)	-7.8543	-0.1881	0.2022	2.13E-03	-4.80E-03	4.69E-04	152.04
No.4	Unload	ln(*k*)	-7.5089	-0.2857	0.2558	4.75E-03	-8.27E-03	1.94E-03	103.44
No.8	Decrease	ln(*k*)	-8.7726	-0.2414	0.331	3.23E-03	-7.75E-03	5.07E-04	372.81
No.8	Increase	ln(*k*)	-7.7106	-0.285	0.2208	3.91E-03	-6.73E-03	2.47E-03	100.33

Fig.5 shows 3D plots of the permeability results of sample No.8 for the decreasing pore pressure cycles. **Fig.6** presents 3D plots of the permeability results for the increasing pore pressure cycles. The maximum pore pressure was about 25MPa and the minimum was close to 6MPa. These plots show the $k-p_c-p_p$ surface and the square signs (the experimental data) superposed. The transformation forms and the coefficients a_i describing the response surface are listed in **Table 2**. The large *F*-ratios ensure an appropriate match of the data as seen on the plotted surfaces from **Fig.5** to **Fig.6**. All these plots show that the results match the experimental data quite well.

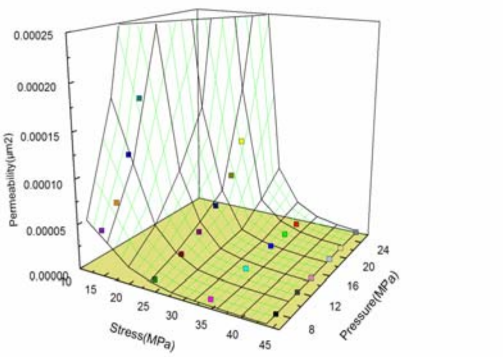

Fig. 7 $k-p_c-p_p$ surface of sample No.8 during decreasing pore pressure

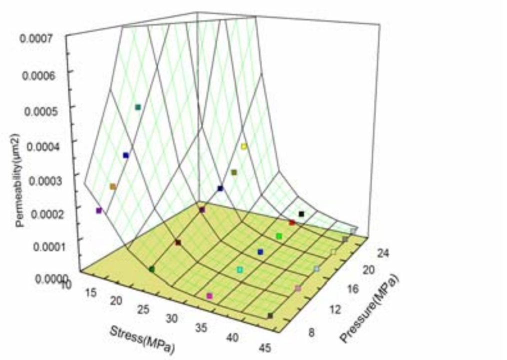

Fig. 8 $k-p_c-p_p$ surface of sample No.8 during increasing pore pressure

6. Result analysis

Sample No.4. The response surfaces resulted from the statistical analysis are applied to determine the values of α by using **Eq. 3.2** over the domain of interest and the result are drawn as a surface in $\alpha-p_c-p_p$ space. **Fig**.7 and **Fig.8** show these results in loading and unloading respectively. From the figures, we can read the α values at various confining pressures and pore pressures, and analyz the changing trends of the coefficient α.

Fig.7 and **Fig.8** indicate that the coefficient α depends on confining pressure and pore pressure. For the loading cycle at pore pressure 22MPa shown in **Fig.7**, the value α decreases from 0.561 to 0.108 as the confining pressure increases from 24MPa to 44MPa. At pore pressure 12MPa, the value α decreases from 0.785 to 0.0365 as the confining pressure increases from 14 MPa to 44MPa. The small α values at the high confining pressure show that permeability hardly varies with pore fluid pressure because of the small α value in situ. **Fig.7** also shows that the value α increases slightly as the pore pressure decreases at relatively low confining pressure, but the value α decreases as the pore pressure decreases at the high confining pressure (say 44MPa).

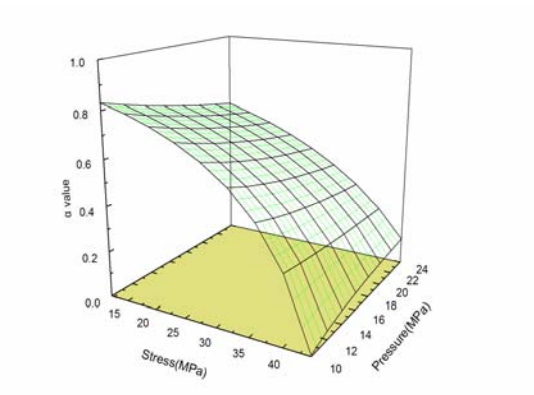

Fig. 7 $\alpha - p_c - p_p$ surface of sample No.4 during loading

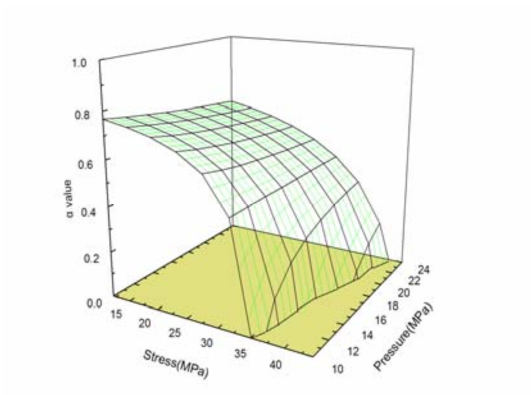

Fig. 8 $\alpha - p_c - p_p$ surface of sample No.4 during unloading

The similar conclusion can be arrived at from **Fig.8** that the value α decreases as the confining pressure increases for the unloading cycle under the constant-pore-pressure condition. However, the α values become so small that they are less than zero at high confining pressure (say 44MPa and 40MPa) and low pore pressure; this may be caused by the measured permeability without Klinkenberg corrections[*RP*,1956] at high confining pressure and low pore pressure. The throats of porous media were compressed and became so small at the high confining pressure that gas slippage effect manifested at low pore pressure, and caused the measured permeability a little bit larger than the permeability with Klinkenberg corrections[*RP*,1956], thus gas slippage will offset the effect of the decrease of pore pressure on permeability to a certain extent and makes the α value smaller than it should be. The value α at high confining pressure being very small, negative values can be worked out by small measurement errors and measured permeability data without Klinkenberg corrections. The negative values are not shown in **Fig.8** and in following figures because the value α can never be below zero in the effective pressure law.

It can be seen by comparing **Fig.8** with **Fig.7** that the α values in **Fig.8** are almost equal to those in **Fig.7** under the same conditions as a whole, even if some data differ a little. It can also be found that the two figures have the same trend by comparing the two surface shapes. These mean that either the loading or the unloading cycles will not make differences in the α values under the constant-pore-pressure conditions.

Sample No.8. Fig.9 and **Fig.10** show $\alpha - p_c - p_p$ surfaces of sample No.8 in decreasing and increasing pore pressure cycles. **Fig.9** also displays that the α depends on confining pressure and pore pressure. For the decreasing pore pressure cycles at the constant-confining-confining pressure 44MPa, the value α increases from 0.0074 to 0.101 as the pore pressure increases from 10MPa to 24MPa, indicating that the α value increases a little as the pore pressure increases at the highest confining pressure. At the constant-confining-confining pressure 20MPa, the value α increases from 0.772 to 0.982 as the pore pressure decreases from 18 MPa to 10MPa. It is found the similar recognition shown in **Fig.7** that the α value increases as the pore pressure decreases at relatively low confining pressure.

Fig.10 shows that α decreases with the increases of the confining pressure under the constant-pore-pressure conditions, yet the value α does not decreases as the pore pressure increases at relatively high confining pressure as displayed in **Fig.7**, **Fig. 8**, **Fig. 9**. It is difficulty to tell what causes the difference. Sample **No.8** has the lowest porosity 2.42% with relatively higher permeability $2.06\times10^{-4}\mu m^2$ than that of sample **No.4**, with permeability $1.9\times10^{-4}\mu m^2$. The extreme low porosity with relatively high permeability shows that pore space is very small and percolation flow is mainly through microcracks. The difference may be one reason that the increasing pore pressure causes the increase in the α value. By the comparison, we are assured that different materials do cause different variability of α.

Compared **Fig.9** with **Fig.10**, we find that the value α differs from each other. The α value in the decreasing pore pressure cycles is greater than that in the increasing pore pressure cycles at relative low confining pressure, yet the value α in the decreasing pore pressure cycles is less than that in the increasing pore pressure cycles at relative high confining pressure. e.g., the α value changes from 0.627 to 0.845 with the average value 0.724 as the pore pressure decreases from 18MPa to 6MPa at constant confining pressure 20MPa. For the same sample at high confining pressure 44MPa, the value α changes from 0.110 to 0.175 with the average value 0.122 as the pore pressure changes from 24MPa to 6MPa, while the α value changes from 0.311 to 0.245 with the average value 0.320 as the pore pressure changes from 24 to 6MPa. The differences hint that pore pressure has different effect on permeability during different cycles on constant confining pressure conditions. Moreover, the $\alpha-p_c-p_p$ surfaces show that the α values in the loading cycles make no great differences with the α values in the unloading cycles under the constant-pore-pressure conditions, but the α values in the decreasing pore pressure cycles do make great differences with the α values in the increasing pore pressure cycles under constant confining pressure conditions. This proves that the α values have differences between changing confining pressure at constant pore pressure and changing pore pressure (mainly in increasing pore pressure) at constant confining pressure.

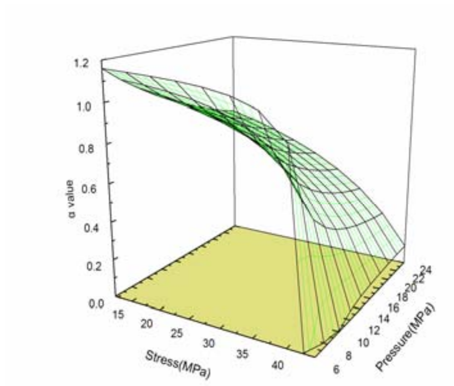

Fig.9 $\alpha-p_c-p_p$ surface of sample No.8 during decreasing pore pressure

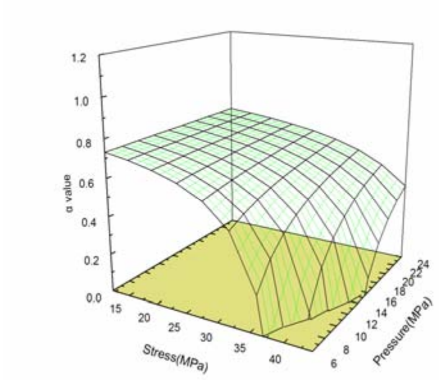

Fig. 10 $\alpha-p_c-p_p$ surface of sample No.8 during increasing pore pressure

It is found from above analysis that (1) the coefficient α in the effective pressure law depends on confining pressure and pore pressure;(2) the value α decreases as the confining pressure increases under constant pore pressure cycles, yet the variability of the α depends on the given confining pressure and the material as well as different cycles(i.e., the increasing or the decreasing pore pressure cycles) under constant confining pressure conditions;(3) the presented α values for permeability are bigger than 0.625 at low confining pressures and high pore pressures, and are below 0.625 at high confining pressures and low pore pressure; According to Bernabe' viewpoint[1986], the α values over 0.625 means the rock deformation is mainly the deformation of microcracks and the limit lower of α value is 0.625 when microcracks close entirely or microcracks become more resistant to pressure; Our samples contain microcracks so that the deformation at low confining pressures and high pore pressures is mainly the deformation of microcracks because the α values are over 0.625; However, the α values are also below 0.625 in the tests and far less than 1.0 at high confining pressures and low pore pressures which could be interpretated in terms of Chuan-liang's view[2], but the effective pressure coefficients vary with confining pressure and pore pressure which does not agree with Chuan-liang's view[1999], so the variability of α values may be difficult to be explained by the present theories; (4) we'd better evaluate α values through changing pore pressure under constant-confining-confining pressure conditions because the α values have differences between changing confining pressure at constant-pore-pressure conditions and changing pore pressure at constant-confining-confining pressure conditions.

7. Conclusion

1) The coefficient α in the effective pressure law depends on confining pressure and pore pressure.
2) The value α decreases as the confining pressure increases under constant pore pressure conditions, yet the variability of the α values depends on the given confining pressure and the material as well as different cycles under constant confining pressure conditions.
3) The α values have differences between changing confining pressure at constant-pore-pressure conditions and changing pore pressure at constant-confining-pressure conditions.
4) At the highest confining presssure 44MPa, the α values are significantly less than 1.0 and vary below 0.1 at fifferent pore pressure in most case, which does not agree with the present theories and exprimental work.

Acknowledgments

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