

## VIBRATION-BASED DAMAGE DETECTION USING TIME SERIES ANALYSIS

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### ABSTRACT :

In this paper, a statistical pattern recognition method based on time series analysis is implemented to a 5 Story steel frame model. This method uses a combination of Auto-Regressive (AR) and Auto-Regressive with eXogenous inputs (ARX) prediction models. The response of the system was obtained for a linear system subjected to white-noise input. The standard deviation of Mahalanobis squared distance between healthy and damaged state is used to locate structural damaged sites. Three damage scenarios were studied. The occurrences and location of damage were identified for all cases.

**KEYWORDS:** damage detection, structural health monitoring, vibration-based method, time series analysis

### 1. INTRODUCTION

Civil structures have been deteriorated due to ageing or strong ground motions of earthquakes. The attention paid to this deterioration and the urgent need for their rehabilitation is relatively recent. The rehabilitation of structures clearly revealed the limit of knowledge in the field of the evaluation to gather the data necessary to the analyses.

Although damage monitoring of civil structures has generated a lot of research, there is still a debate whether the measured deviations are significant enough to be a good indicator of structural degradation.

Between all methods based on frequency and time domain analysis that have been proposed, an innovative approach to assess the current health state of a structure is the statistical analysis of its measured vibration data. This approach offers several advantages over existing modal-based damage detection methods. Modeling errors and modal identification limitations are avoided in this approach making it more attractive for vibration-based damage detection.

In this paper, a numerical scheme to predict the dynamic response of a 5 Story Steel Frame is implemented using the response of numerical simulation obtained from a linear system excited by a white-noise input excitation.

### 2. AR-ARX PREDICTION MODEL

Sohn et al. presented a comprehensive report providing an overview of existing damage detection methods. The main conclusion that can be drawn from this report is that modal-based damage detection methods usually require large amount of high-quality data and considerable number of sensors strategically located, requirements that are almost impossible to get in the field. Therefore, the research community has been recently exploring the use of pattern recognition approaches to tackle the problem of reliable damage detection when vibration data are measured at limited locations.

Sohn et al developed an AR-ARX prediction model, which is solely based on signal analysis of measured vibration data. This model has been successfully implemented in various damage detection problems as reported by Sohn et al. the mathematical derivation of the model begins by using standardized time signals as shown in Eqn. 2.1

$$x(t) = x_i(t) - \mu_{x_i} / \sigma_{x_i} \quad (2.1)$$

where  $x(t)$  is the standardized signal of the initial signal  $x_i(t)$  at time step  $t$  and  $\mu_{x_i}$  and  $\sigma_{x_i}$  are the mean and the standard deviation of  $x_i(t)$ , respectively. The next step consists on the construction of AR(p) models for each measured location. One of the damage identification features involves the use of the coefficients of the AR(p) models. Therefore, a computationally efficient stepwise least square algorithm for the estimation of AR(p) parameters is used herein in conjunction with the AR-ARX model proposed by Sohn et al.

An AR model using the Yule-Walker method as proposed by Sohn is then replaced by the ARfit algorithm proposed by Neumaier and Schneider. This algorithm computes the model order,  $p_{opt}$ , that optimizes the order selection criteria using QR factorization of a data matrix to evaluate, for a sequence of successive orders, the model order and to compute the parameters of the AR( $p_{opt}$ ) model. Then, the AR( $p_{opt}$ ) model can be represented as shown in Eqn. 2.2.

$$x(t) = \sum_{j=1}^{p_{opt}} \phi_{xj} x(t-j) + e_x(t) \quad (2.2)$$

Once the AR( $p_{opt}$ ) model has been constructed, the residual error of the model,  $e_x(t)$ , is computed by subtracting the data obtained from the AR( $p_{opt}$ ) model from the standardized signal,  $x(t)$ . The AR( $p_{opt}$ ) coefficients,  $\phi_{xj}$ , will later be used to locate damaged sites. Finally the residual error,  $e_x(t)$ , is employed in the construction of the ARX model as shown in Eqn. 2.3 by assuming that this residual error, defined by the difference between the measured and the predicted values obtained from AR model, is mainly caused by unknown external input.

$$x(t) = \sum_{i=1}^a \alpha_i x(t-i) + \sum_{j=1}^b \beta_j e_x(t-j) + \varepsilon_x(t) \quad (2.3)$$

where  $\varepsilon_x(t)$  is the residual error after subtracting the ARX(a,b) model from the standardized signal,  $x(t)$ . Similar results are obtained for different values of  $a$  and  $b$  as long as the sum of  $a$  and  $b$  is kept smaller than  $p_{opt}$  as reported by Sohn et al. The residual errors from healthy state are defined as  $\varepsilon_x(t)$  and the residual errors after the occurrence of structural damaged are defined as  $\varepsilon_y(t)$ . Finally, using the standard deviations of  $\varepsilon_x(t)$  and  $\varepsilon_y(t)$ , the ratio,  $\sigma(\varepsilon_y)/\sigma(\varepsilon_x)$ , is then defined as the first damage sensitivity feature. A threshold value for this ratio must be computed using measured vibration data obtained from different operational conditions. Therefore, a value of this ratio larger than the computed threshold value indicated the occurrence of damage. The standard deviation of the Mahalanobis squared distance between healthy and damaged AR( $p_{opt}$ ) coefficients is then used to locate structural damaged sites as shown in Eqn. 2.4.

$$D = \sigma((\phi_{xj}^d - \bar{\phi}_{xj}^h)^T s^{-1} (\phi_{xj}^d - \bar{\phi}_{xj}^h)) \quad (2.4)$$

where  $\phi_{xj}^d$  are the AR( $p_{opt}$ ) coefficients from the damaged state,  $\bar{\phi}_{xj}^h$  are the mean values from the healthy state and  $s$  is the covariance matrix of  $\bar{\phi}_{xj}^h$ .

The Mahalanobis squared distance is independent of the scale of the AR( $p_{opt}$ ) coefficients. Therefore, vibration data measurement point closest to the location of the structural damage would have the largest values of  $D$ . The proposed AR-ARX prediction model uses two damage sensitivity features,  $\sigma(\varepsilon_y)/\sigma(\varepsilon_x)$  and  $D$ , to identify and locate structural damage respectively.

### 3. FIVE-STORY STEEL FRAME MODEL

The numerical model of the frame is based on a full scale five-story steel structure shown in Figure 1 and elevation views, with cross section of members are shown in Figure 2.

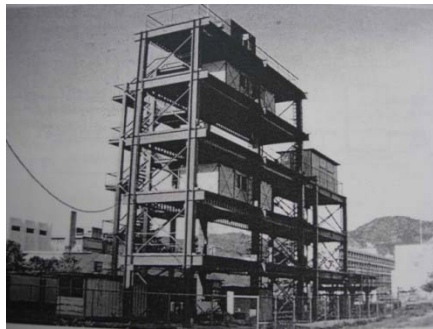


Figure 1 View of five story steel frame

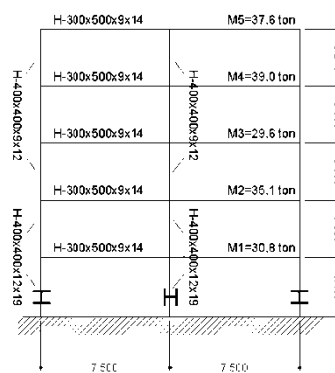


Figure 2 Elevation view of the five story steel frame

This structure was modeled with two-dimensional beam elements, in view of obtaining the responses. All members with rigid connections. As for the external columns, 1<sup>st</sup> and 2<sup>nd</sup> floors, the total geometric moment of inertia was considered  $4.054 \times 10^{-4} \text{ m}^4$  and from 3<sup>rd</sup> to 5<sup>th</sup> floors,  $2.560 \times 10^{-4} \text{ m}^4$ . As for the internal columns, 1<sup>st</sup> and 2<sup>nd</sup> floors,  $1.199 \times 10^{-3} \text{ m}^4$  and from 3<sup>rd</sup> to 5<sup>th</sup> floors,  $8.026 \times 10^{-4} \text{ m}^4$ .

The floors are consisted of two steel beams and a reinforced concrete slab. According to previous monitoring test presented by Kuroiwa & Iemura (2007), the interaction between the slab concrete and the beams have to be ignored, when defining the inertia, that results in  $1.15 \times 10^{-3} \text{ m}^4$ . The Young's modulus of concrete and steel was considered respectively  $1.8 \times 10^{10} \text{ N/m}^2$  and  $2.1 \times 10^{11} \text{ N/m}^2$ .

The total mass per floor, including equipments, was considered according to Bae (1999) as following: 1<sup>st</sup> floor,  $m_1=37600 \text{ kg}$ ; 2<sup>nd</sup> floor,  $m_2=39000 \text{ kg}$ ; 3<sup>rd</sup> floor,  $m_3=29600 \text{ kg}$ ; 4<sup>th</sup> floor,  $m_4=35100 \text{ kg}$ ; 5<sup>th</sup> floor,  $m_5=30800 \text{ kg}$ .

In view of changing the configuration of the system, three sceneries are considered as described in Table 1.

The excitation on the structure is imposed at the base by a white-noise, with standard deviation of 1. The acceleration responses at masses  $m_1$  to  $m_5$  are recorded from numerical simulations.

Table 1- Damage patterns

	Description
(1)	Columns between first and second floor reduced by 20%
(2)	Columns between third and forth floor reduced by 30%
(3)	(1) and (2) damaged cases simultaneous

#### 4. SIMULATION RESULTS

The acceleration records are standardized according to Eqn. 1. The threshold value selected for this study is 1.0.

The results of the ratio  $\sigma(\varepsilon_y)/\sigma(\varepsilon_x)$  are plotted in Figure 3 for each damage scenarios

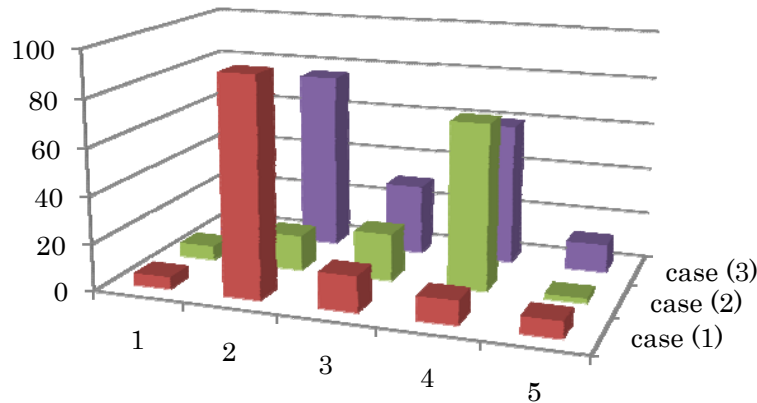


Figure 3  $\sigma(\varepsilon_y)/\sigma(\varepsilon_x)$  ratio for 4 damage scenarios

#### 4. CONCLUSIONS

The method implemented for the analytical model exemplified in this paper showed to be able to determine the damage states. The method has the advantage of being based on only output measurements. It is important to investigate what would happen if environmental excitation were a colored noise.

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