VIBRATION CHARACTERISTICS OF A CABLE-STAYED FOOTBRIDGE VIA MODAL TESTING

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ABSTRACT :

The dynamics of footbridges needs an adequate individuation of modal properties, in the perspective of design methodologies for reducing vibrations. Therefore, modal testing after construction can be an important tool in updating and validation of FE models. In this work ambient vibration testing has been applied to the estimation of modal properties of the Forchheim Cable-Stayed Footbridge. The recorded data have been analysed through Frequency Domain Decomposition (FDD) technique. A FE model has been developed and updated on the basis of experimental data. In addition, a numerical simulation has been performed, with the aim of showing differences in dynamic response of the bridge when the human mass is considered, with respect to the usually adopted methods in which human mass is neglected in the calculation. In fact, for light structures, such as the analyzed footbridge, service live loads can be of the same order of dead loads.

KEYWORDS: Vibration, Footbridge dynamics, Modal testing, Dynamic properties.

1 INTRODUCTION

Dynamics of cable-stayed footbridges have had great attention in recent years, after the unexpected high level vibration under pedestrian load of the Millennium footbridge in London (Strogatz et al., 2005). After the opening of Millennium Bridge several papers were published, which deal with crowd-induced vibration of footbridges (Dallard et al., 2001; Roberts, 2005; Nakamura & Kawasaki, 2006; Venuti et al., 2007; Blekherman, 2005). An interesting review paper on the vibration serviceability of footbridges under human-induced excitation was presented by Živanovic et al. (2005). The dynamics of slender structures, such as footbridges, needs an adequate individuation of modal properties, in the perspective of design methodologies for reducing vibration. Therefore, modal testing after construction can be an important tool in updating and validation of FE models. In this work ambient vibration testing has been applied to the estimation of modal properties of the Forchheim Cable-Stayed Footbridge. The recorded data have been analysed through Frequency Domain Decomposition (FDD) technique (Brincker et al., 2000). A FE model has been developed and updated on the basis of experimental data. In addition, a numerical simulation has been performed, with the aim of showing differences in dynamic response of the bridge when the human mass is considered, with respect to the usually adopted methods in which human mass is neglected in the calculation. In fact, for the analyzed footbridge such as for many others similar structures, live loads can be of the same order of dead loads in the case of crowd.

2 EXPERIMENTAL DEFINITION OF FINITE ELEMENT MODEL

2.1 The Forchheim footbridge

The Footbridge over the Regnitz River in Forchheim, Germany (Fig. 1), is a cable stayed bridge composed of a main span of 88.1 *m* and one side span of 29.4 *m*. Two truss beams compose the cross-section. They also compose the parapets and are connected at their intrados by means of a reticular horizontal structure, which also supports the wooden deck, which is about 4.0 *m* width. In the main span the girder is suspended to three couples of fan-shaped stays, starting at the top of a steel tower and connected to the deck by means of transversal beams. Cables are almost regularly spread along the deck. Two anchor cables are constrained to the abutment, i.e., to an external gravity anchoring. The girder can be supposed to be hinged to the A-shaped pylon. This is composed by two steel circular pillars (diameter ≈ 900 *mm*) and has a height of about 31.0 *m* from the ground. Each pillar is hinged at its base (Clemente et al., 2003).

Figure 1 Forchheim footbridge

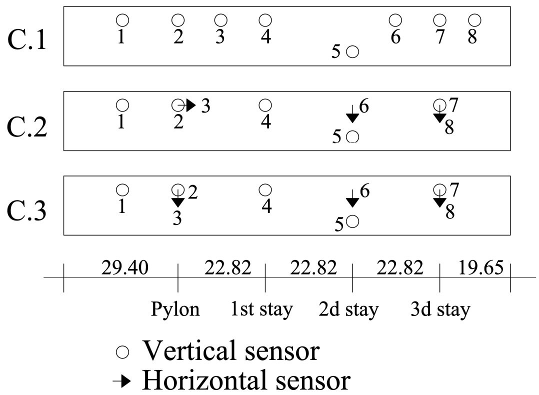


Figure 2 Sensor layout: plan view

2.2 *Experimental modal analysis*

Eight seismometers Kinemetrix SS1, a HP3566A signal conditioner and a laptop composed the experimental set-up. The signals recorded by the eight seismometers, used in synchronized way, were collected by the acquisition system and analysed in real time by HP software in order to have a first glance at the experimental data. Transducers have been temporarily installed in several locations of the structure in three different configurations (Fig. 2). Several time-histories were recorded for each configuration. This was done to show repeatability of the vibrational characteristics and to get average values of the characteristics. The structure was excited by means of ambient vibrations, pedestrian- and bicycle-induced vibrations. Vibration amplitudes were very high, often out of the allowable range for the used velocimeters. Therefore, only low level recordings, which were included into seismometers allowable range, has been used for the identification reported in this paper. Several peaks can be observed in the spectra, but only four of them, which have been related to the structural modes, have been used with the purpose of identification; actually, the few measurement points used did not allow a clear identification of higher modes. In particular, the first lateral mode, the first and second vertical modes have been clearly identified; the first torsional mode can be observed also. The FE model has been updated on the basis of this four modal shapes and frequencies.

Figure 3 Connection girder-pylon

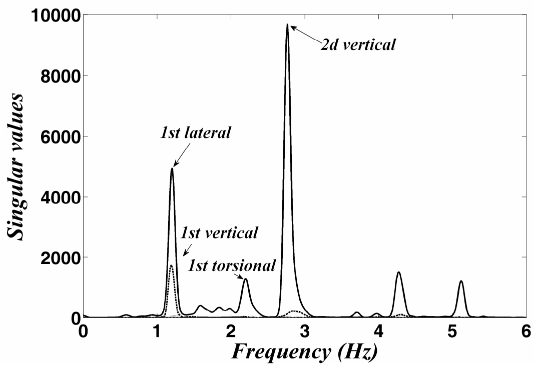


Figure 4 FDD of one recording

2.3 *Finite Element model*

A FE model has been developed through the assemblage of spatial frame elements (six degree of freedom per node), for the bridge truss girder and the A-shaped pylon. Cables have been modelled by means of axially resistant elements, with a reduced elastic modulus of the material, in order to take into account the sag effect. The first model was not suitable to fit the measured values of certain frequencies correctly. In particular, the numerical frequency relative to the first lateral mode was much higher than the experimental one. So the model has been manually updated, by introducing elastic connections between the pylon and the girder (Fig. 3). The stiffness of these connection spring elements has been changed in order to obtain a good match between experimental and numerical results. In table 1 the measured frequencies are compared with the numerical ones, finally deduced by standard FE modal analysis performed by means of the computer code Matlab. Fig. 5 shows the first four modal shapes.

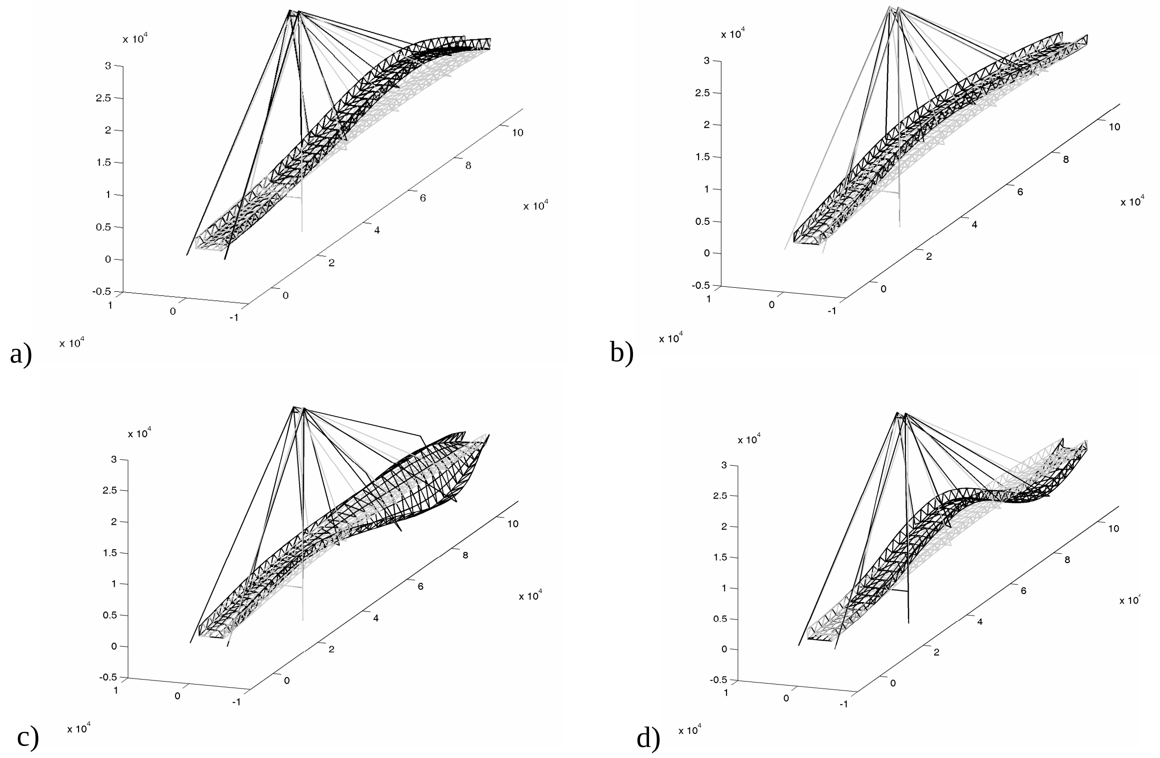


Figure 5 FE model: a) Mode 1; b) Mode 2; c) Mode 3; d) Mode 4

Table 1 Numerical and experimental frequencies

Mode	Prevalent Displ.	FE Freq. (Hz)	Exp. Freq. (Hz)	Error (%)
1	Vertical	1.22	1.19	2.52
2	Lateral	1.24	1.20	3.33
3	Torsional	2.06	2.19	5.93
4	Vertical	2.77	2.76	0.36

3 NUMERICAL SIMULATION OF RESPONSE UNDER HUMAN LOADS

3.1 Modeling of walking people

On the basis of the updated FE model, a numerical simulation has been performed in order to evaluate the bridge response to human loads. Even though several published papers were dealing with modeling of walking people, some aspects are still to clarify. For example it is important to account for the synchronization phenomenon, due to the fact that when the bridge is crowed, every pedestrian adjusts its walking speed on the

speed of each other, so determining a synchronization of loads. In addition, the mass of people, which is usually neglected, becomes very important when bridge dead loads are similar to the travelling loads. Therefore, in this paper human mass is considered by assuming a time-varying mass-matrix in the equation of motion.
The simulation of walking people is based on the following consideration. For simplicity, it is supposed that people move in one direction only. The arrival times are probabilistically defined through a Poisson distribution with parameter λ (Ricciardi, 1994):

$$p(N,t) = \frac{e^{-\lambda t}\left(\lambda t\right)^N}{N!} \tag{1}$$

The human weight has been defined as a random variable with uniform distribution between 500 and 1000 *N*. According to Živanovic et al. (2007), the step frequency and length have been supposed to be random parameters with Gaussian distribution; mean values and standard deviation are, respectively:

$$\mu_f = 1.87\,Hz; \qquad \sigma_f = 0.186\,Hz; \qquad \mu_s = 0.71\,m; \qquad \sigma_s = 0.071\,m \tag{2}$$

The step frequency determines the velocity. The position of the *i-th* person at time *t* on the bridge is defined by two coordinates, collected into vector $\boldsymbol{X}_i(t)$. Each person moves along a straight line and, if a person moves more slowly than the person behind him, the latter changes the velocity, in accordance with the person in front; this assumption simulates the synchronization phenomenon.

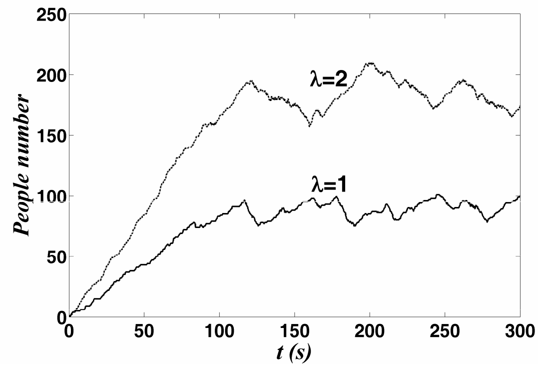


Figure 6 People number on the bridge

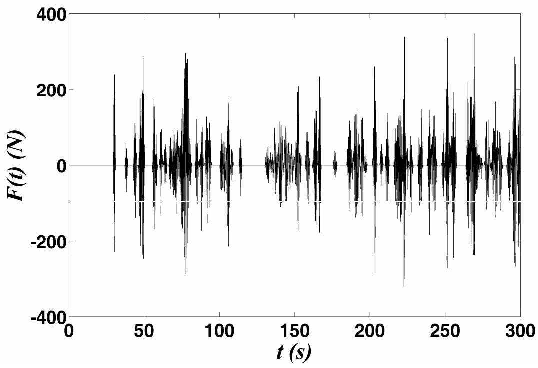


Figure 7 Dynamic component of force upon a node at first stay anchor point

Mathematically speaking, the distance between human beings must respect the following condition:

$$\left\|\boldsymbol{X}_i - \boldsymbol{X}_j\right\| > 1 \;\; m \tag{3}$$

where $\|\cdot\|$ is the Euclidean norm. Fig. 6 shows the people number on the bridge versus time, for two parameter λ. The human load have been modeled in the time domain by a superposition of harmonic and sub-harmonic components (Živanovic et al.. 2007). The dynamic part of the load is:

$$F(t) = \sum_{i=1}^{5} F_i(t) + \sum_{i=1}^{5} F_i^s(t) \tag{4}$$

where:

$$F_i(t) = W\,DLF_i \sum_{\bar{f}_j = i-0.25}^{i+0.25} \overline{DLF}_i(\bar{f}_j)\cos(2\pi\,\bar{f}_j\,f_s\,t - \theta(\bar{f}_j)) \tag{5.a}$$

$$F_i^s(t) = W\,DLF_i^s \sum_{\bar{f}_j = i-0.75}^{i-0.25} \overline{DLF}_i^s(\bar{f}_j^s)\cos(2\pi\,\bar{f}_j^s\,f_s\,t - \theta(\bar{f}_j^s)) \tag{5.b}$$

The dynamic loading factors DLF for harmonics and sub-harmonics and the other quantities have been defined according to the suggestions of Živanovic et al. (2007). However, according to person movement, the force have been applied in a correspondent point on the bridge, and have been lumped in the FE sense. In the same way, also human mass has been applied, leading to a modification in the mass matrix of the system, as will be shown later. Only the vertical component of human load has been taken into account. The simulated force on a node located at first stay-cables anchor point, is plotted in Fig. 7.

3.2 Evaluation of Bridge response

The equation of motion is based on the following expression:

$$\boldsymbol{M}(t)\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}(t)\boldsymbol{u}(t) = \boldsymbol{F}(t) \tag{6}$$

in which $\boldsymbol{M}$(t) is the time-variable mass matrix, $\boldsymbol{C}$ is the damping matrix, $\boldsymbol{K}$(t) is the stiffness matrix, $\boldsymbol{u}$(t) the displacements vector, $\boldsymbol{F}$(t) the load vector dependent from human walking. The dependence of $\boldsymbol{K}$(t) on the time is due to cables stiffness. The damping matrix is determined assuming a modal damping ratio equal to 1% for all modes; this modal damping ratio has been estimated as an averaged value by means of half-power bandwidth method for the identified modes. If $\boldsymbol{\Xi}$ indicates the modal damping matrix, the matrix $\boldsymbol{C}$ is determined by means of the following relation:

$$\boldsymbol{C} = \boldsymbol{M}_0\boldsymbol{\Phi}\boldsymbol{\Xi}\boldsymbol{\Phi}^T\boldsymbol{M}_0 \tag{7}$$

where $\boldsymbol{M}_0$ is the initial mass matrix and $\boldsymbol{\Phi}$ is the modal matrix. The model is composed by means of 912 frame elements, and the total number of degrees of freedom is equal to 1524, after condensation of the rotational degrees of freedom. Considering forces only in vertical direction, an additional simplification has been done by condensation of all non-vertical degrees of freedom. The reduced dynamic model possesses 508 degrees of freedom. Even if the cables are modeled as frame elements, the elastic modulus, variable with cable tension, is defined by the well known Ernst modulus. Instead of solving the equation of motion by the modal analysis approach, with the aim of considering large variability in the mass matrix a direct time integration approach has been used. On this purpose, the equation of motion is written in first order form:

$$\dot{\boldsymbol{Y}}(t) = \boldsymbol{A}(t)\boldsymbol{Y}(t) + \boldsymbol{H}(t) \tag{8}$$

$$\boldsymbol{Y}(t) = \begin{Bmatrix} \boldsymbol{u}(t) \\ \dot{\boldsymbol{u}}(t) \end{Bmatrix} \qquad \boldsymbol{A}(t) = \begin{Bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}(t)\boldsymbol{K}(t) & -\boldsymbol{M}^{-1}(t)\boldsymbol{C} \end{Bmatrix} \qquad \boldsymbol{H}(t) = \begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{F}(t) \end{Bmatrix} \tag{9a-c}$$

Thus, the dimension of the state-space vector $\boldsymbol{Y}$(t) is 1016. The above written is a linear differential equation with variable coefficients, and the solution can be obtained numerically by means of a time-step integration technique:

$$\boldsymbol{Y}(t_{k+1}) = e^{\int_{t_k}^{t_{k+1}} \boldsymbol{A}(t)dt}\boldsymbol{Y}(t_k) + \int_{t_k}^{t_{k+1}} e^{\int_{\tau}^{t_{k+1}} \boldsymbol{A}(t)dt}\boldsymbol{H}(\tau)d\tau \tag{10}$$

Assuming $\boldsymbol{A}(t)$ constant in each time step Δt:

$$\boldsymbol{Y}(t_{k+1}) = \boldsymbol{\Theta}_k(\Delta t)\boldsymbol{Y}(t_k) + \int_{t_k}^{t_{k+1}} \boldsymbol{\Theta}_k(t_{k+1}-\tau)\boldsymbol{H}(\tau)d\tau \tag{11}$$

where:

$$\boldsymbol{\Theta}_k(\Delta t) = e^{\boldsymbol{A}(t_k)\Delta t} \tag{12}$$

This expression differs from the one which can be used for constant $\boldsymbol{A}$(t) only for the fact that at every time step the matrix $\boldsymbol{\Theta}$ has to be redefined. An explicit expression of Eqn. (11) can be derived assuming $\boldsymbol{H}$(t) linear in each time step:

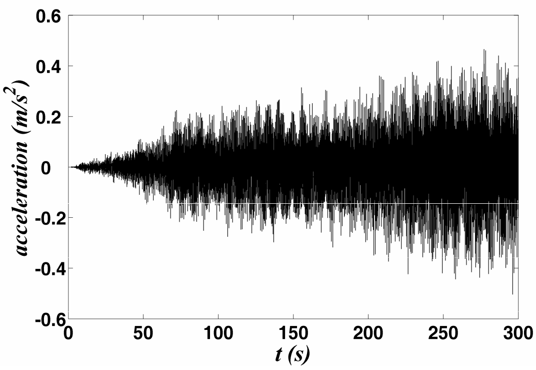
$$\boldsymbol{Y}(t_{k+1}) = \boldsymbol{\Theta}_k(\Delta t)\boldsymbol{Y}(t_k) + \gamma_{0,k}(\Delta t)\boldsymbol{H}(t_k) + \gamma_{1,k}(\Delta t)\boldsymbol{H}(t_{k+1}) \tag{13}$$

in which:

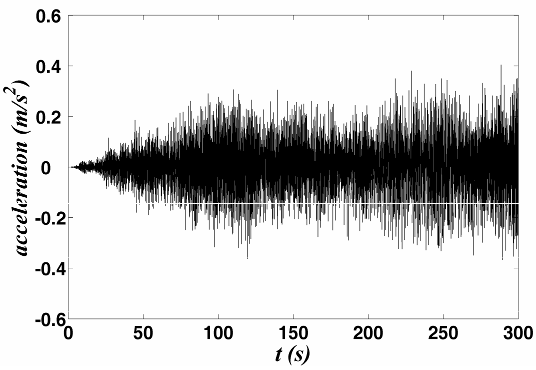
$$\gamma_{0,k}(\Delta t) = \left[\boldsymbol{\Theta}_k(\Delta t) - \frac{1}{\Delta t}\boldsymbol{L}_k(\Delta t)\right]\boldsymbol{A}^{-1}(t_k);\quad \gamma_{1,k}(\Delta t) = \left[\frac{1}{\Delta t}\boldsymbol{L}_k(\Delta t) - \boldsymbol{I}\right]\boldsymbol{A}^{-1}(t_k) \tag{14a,b}$$

$$\boldsymbol{L}_k(\Delta t) = \left[\boldsymbol{\Theta}_k(\Delta t) - \boldsymbol{I}\right]\boldsymbol{A}^{-1}(t_k) \tag{15}$$

Neglecting human-structure interaction, time histories of 300 *s* have been derived. Two values for λ have been considered, λ=1 and λ=2 respectively, in order to show dynamics responses for different crowd intensity. The first value corresponds, as soon as stationary conditions have been reached, to a people density of about 0.19 *people/m²*, whereas the second value means a density of about 0.42 *people/m²*. It is worth noting that during the opening day of the Millennium Bridge in London, the maximum density was 1.3-1.5 *people/m²*.



a)



b)

Figure 8 Vertical acceleration in a node located at first stay location: a) $\lambda = 1$ and constant mass matrix; b) $\lambda = 1$ and variable mass matrix

In the integration, the time step Δt =0.05 s has been used. Figs. 8 and 9 show the time response in terms of acceleration in a node of the FE model, located at first stay-cable anchorage; in particular, for each examined people densities, two analyses have been done: the first is based on constant mass matrix, whereas the second is based on variable mass matrix. Figs. 8a,b show that, in the case λ=1, the difference in terms of maximum acceleration is low, even if there is some difference in the time history. In the case λ=2 (Figs. 9a,b) differences become apparent: the maximum acceleration reaches values of about 0.8 m/s^2 in the analysis with variable mass matrix, whereas for constant mass matrix the maximum acceleration value is about 0.4 m/s^2.

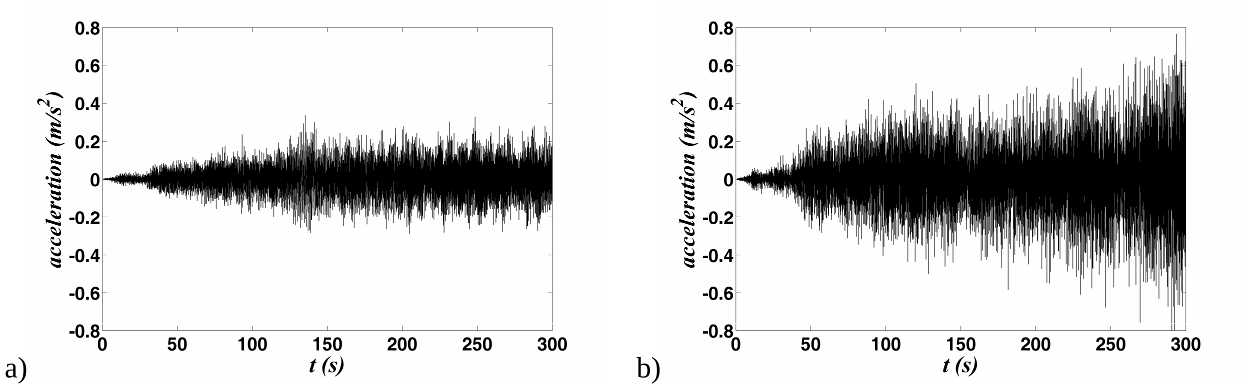


Figure 9 Vertical acceleration in a node located at first stay location: a) $\lambda = 2$ and constant mass matrix; b) $\lambda = 2$ and variable mass matrix

It is interesting to observe that Eurocode 5 indicates the value 0.7 m/s^2 as limit vertical acceleration value for timber bridges with natural frequencies lower than 5 *Hz*. Figs. 10a,b show the amplitude Fourier spectra for the case λ=2, with constant or variable mass matrix. It can be observed some variation in frequency values and significant differences in amplitude. In particular, the second vertical mode (about 2.76 *Hz*) possesses a higher amplitude in the case of variable mass.

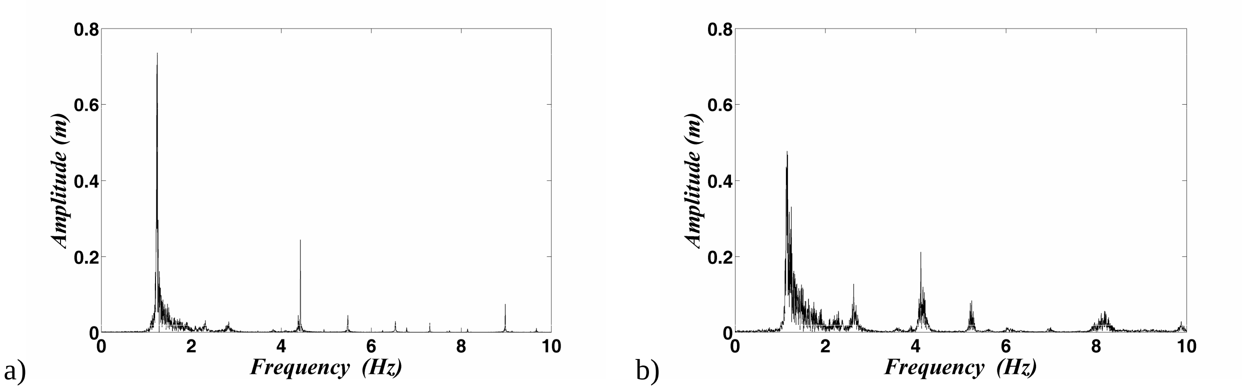


Figure 10 Fourier amplitude of displacement in a node at first stay location: a) $\lambda = 2$ and constant mass matrix; b) $\lambda = 2$ and variable mass matrix

Finally, Figs. 11a,b show the amplitude spectrum for the acceleration, for the case λ=2 and with constant or variable mass matrix. The differences becomes very large. In fact, even though the peaks are lower, the area under the spectrum becomes larger in the case of variable mass matrix.

4 CONCLUSIONS

The experimental set-up of a FE model for the Forchheim Cable-Stayed Footbridge is presented in this paper. The modal identification has been performed by means of the well known FDD method and an analytical model has been developed using spatial frame elements. Based on walking-load models proposed in literature, a numerical simulation has been carried out, in order to point out same aspects of the bridge response to walking people. In particular, with some difference from other literature works, the dynamic problem has been based on a time-variable mass matrix and the step adjustment in presence of crowd have been taken into account. The motivation for varying mass matrix is due to the very low value of dead loads, which is a characteristic common to many footbridges. Calculations, performed via a direct time-step integration method, has shown differences in vertical dynamic response, in the two cases with or without considering human mass; the differences in vertical acceleration can be very large, when people density increases. The research proposed in this paper could be extended to lateral loads, which has not considered here; besides, the probabilistic structure of walking load has still to be investigated in detail, with the purpose of deriving more simplified methods of analysis, less onerous from a computational point of view, with respect to direct simulations.

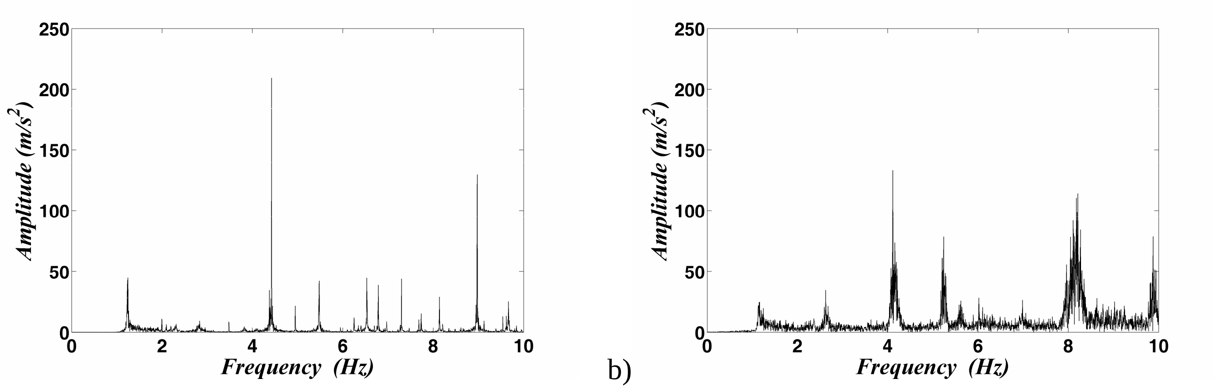


Figure 11 Fourier amplitude of acceleration in a node at first stay location: a) $\lambda = 2$ and constant mass matrix; b) $\lambda = 2$ and variable mass matrix.

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