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Evaluating the quality of hybrid simulation test using an energy-based approach

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ABSTRACT
Hybrid simulation is an experimental method which combines the well known substructure testing techniques with numerical finite element simulation to study the response of a structural system to an excitation. The quality of a hybrid simulation test depends strongly on the magnitude of measurement and control errors associated with the experimental substructures. An on-line error monitoring method is proposed to asses the severity of experimental errors during a hybrid simulation, This method predicts the accuracy of the simulation results during the simulation. Quantitative estimates of error are derived based on energy added to the hybrid model due to measurable errors in the experimental substructures. A recently completed hybrid simulation of seismic response of an innovative lateral load resisting system, a suspended zipper braced frame, is used to demonstrate the effectiveness of the proposed on-line error monitoring indicators. It is shown that a well-calibrated simulation conducted using state-of-the-art equipment can deliver accurate results.

KEYWORDS: Hybrid simulation, Experimental error, NEES.

1. INTRODUCTION

Hybrid simulation is an experimentally based method for investigating the response of structure to static or dynamic excitation using a hybrid model. A hybrid model is an assemblage of one or more physical and one or more numerical, consistently scaled, substructures. The equation of motion of a hybrid model under dynamic excitation is solved during a hybrid simulation test. Experiments have shown that results from hybrid simulation and shaking table tests are comparable, but only when propagation of experimental errors is successfully mitigated in the hybrid simulation (Takanashi and Nakashima 1987). Errors are introduced into hybrid simulations through the structural model idealization, the idealization of the equations of motions, the approximate numerical integration methods, and the experimental errors. The results of a hybrid simulation are most sensitive to experimental errors, because these errors are not known prior to testing and fed back to the hybrid model, which may accumulate and lead to unstable responses.

In an effort to reduce experimental errors, researchers have focused on reducing actuator tracking errors through increasing the accuracy of the instrumentations and improve the servo-hydraulic actuator control loop (Takanashi and Nakashima 1987; Thewalt and Mahin 1987). Actuator tracking errors can be measured during a hybrid simulation and this information has been used to correct the measured restoring force based on the initial stiffness of the structure (Shing et al. 2002) or by adjusting the time increment of the simulation step to accommodate the measured displacement (Yi and Peek 1993). To mitigate the response lag of the actuators, researchers have proposed constant (Horiouchi et al. 1999) and variable (Darby et al. 2002, Ahmadizadeh et al. 2007) compensation schemes. Even though the magnitude of experimental errors has been reduced using methods in these previous studies, the errors cannot be completely eliminated. Relatively small systematic experimental errors can have a cumulative effect on the simulation result that can compromise the stability of executing the simulation, particularly for high frequency modes, and bring into question the validity of the simulation (Mosqueda et al. 2007b).

Hybrid Simulation Error Monitors ($HSEM$) can be used to predict the quality of the hybrid simulation results and detect unacceptable levels of systematic experimental errors in real-time (Mosqueda et al. 2007a). $HSEM$ are based on normalized measures of cumulative energy added to the hybrid simulation as a result of actuator tracking errors. They provide instantaneous information on the control state of each actuator or the cumulative effect of the actuators control errors on the overall accuracy of the computed structural response. $HSEM$ is applied here to a hybrid simulation of a suspended zipper braced frame to demonstrate the accuracy and reliability of the results and to showcase the minimal propagation of errors in modern hybrid simulations using state-of-the-art hardware and control algorithms.

2. Experimental Errors

Errors are introduced into hybrid simulations through the structural model idealization, the approximate numerical integration methods, and the experimental errors. Modeling and numerical errors, resulting primarily from the assumptions in the analytical modeling techniques, are described by Shing and Mahin (1983, 1984). Thewalt and Mahin (1987) provide a thorough discussion of hardware components and sources of experimental errors. Of these various sources of errors, experimental errors can have the most substantial impact on the simulation results, mostly because these errors are not known prior to testing and can be large for improperly tuned experimental setups.

Experimental errors result from the displacement control of hydraulic actuators, force relaxation or strain rate effects due to the slow rates of testing, calibration errors in the instrumentation, and noise generated in the instrumentation and analog to digital converters. Experimental errors can be classified as either random or systematic in nature. A major source of random errors is low amplitude, high frequency noise in experimental measurements, which can excite spurious response in lightly damped higher modes in the hybrid model. To mitigate this effect, higher modes can be damped numerically by using specific integration algorithms (Hilbert et al. 1977, Shing and Mahin 1984) or by solving the governing equation of motion in its integral form (Chang et al. 1999).

Systematic errors can have a greater influence on the simulation results compared to random errors (Thewalt and Roman 1994; Mosqueda et al. 2005). Sources of systematic errors include load-history effects on the experimental substructures and displacement control errors in the servo-hydraulic actuators. For example, in a conventional ramp-and-hold pseudo-dynamic test, the specimen may exhibit force-relaxation during the hold portions of the test. Continuous (Magonette 2001; Stojadinovic et al. 2006) and real-time (Nakashima 2001) testing techniques successfully mitigate this source of errors in pseudo-dynamic tests. Further, real-time testing methods can be used to load the experimental substructures at realistic seismic rates for strain-rate sensitive materials. Alternatively, strain-rate effects have been compensated mathematically during slow continuous tests (Molina et al. 2002b).

3. Energy Balance for Hybrid Simulation

The hybrid simulation test method comprises numerical simulations and simultaneous experimental testing of substructure components by integrating the dynamic equation of motion for the hybrid model. A time-stepping integration procedure is used to solve the discretized equation of motion for displacements, $\mathbf{u}$, at the degrees of freedom of the structure at time intervals $t_i = i\,\Delta t$ for $i = 1 \cdots N$.

$$\mathbf{M}\ddot{\mathbf{u}}_i + \mathbf{C}\dot{\mathbf{u}}_i + \mathbf{r}_i + \mathbf{r}_i^E = \mathbf{f}_i \tag{3.1}$$

The subscript i denotes the time-dependant variables at time t_i, Δt = integration time step and N is the number of integration steps. The mass matrix, $\mathbf{M}$, damping matrix, $\mathbf{C}$, and applied loading, $\mathbf{f}$, are typically modeled numerically in the computer. For the hybrid model, the restoring force vector is assembled using forces measured at the degrees of freedom of experimental substructures, $\mathbf{r}_i^E$, or computed for numerical substructures, $\mathbf{r}_i$.

The energy balance equation is obtained by determining the work done by each of the force components in Eq. (3.1). Integrating with respect to the displacement $\mathbf{u}$, and using relative displacements formulation results in (Uang and Bertero 1990):

$$\int (\mathbf{M}\ddot{\mathbf{u}})^T d\mathbf{u} + \int (\mathbf{C}\dot{\mathbf{u}})^T d\mathbf{u} + \int (\mathbf{r})^T d\mathbf{u} + \int \left(\mathbf{r}^E\right)^T d\mathbf{u} = \int (\mathbf{f})^T d\mathbf{u} \tag{3.2}$$

In addition to kinetic energy and viscous damping dissipation, energy can be stored and dissipated by both numerical $(\mathbf{r})$ and experimental $\left(\mathbf{r}^E\right)$ substructures. The energy associated with experimental substructures is of particular interest: this energy corresponds to the work done by the measured resisting forces on the displacements of the experimental substructure.

$$E^E = \int \left(\mathbf{r}^E\right)^T d\mathbf{u} = \int \left(\mathbf{r}^m + \mathbf{r}^{error}\right)^T d\mathbf{u} \tag{3.3}$$

In Eq. (3.3), $\mathbf{r}^m$ = measured force and $\mathbf{r}^{error}$ = presumed error in the force measurements, which are then introduced into the energy balance equation. Substituting Eq. (3.3) and moving the error term to the right-hand side in Eq. (3.2) results in

$$\int \left(\mathbf{M}\ddot{\mathbf{u}}\right)^T d\mathbf{u} + \int \left(\mathbf{C}\dot{\mathbf{u}}\right)^T d\mathbf{u} + \int \left(\mathbf{r}\right)^T d\mathbf{u} + \int \left(\mathbf{r}^m\right)^T d\mathbf{u} = \int \left(\mathbf{f}\right)^T d\mathbf{u} - E^{error} \tag{3.4}$$

The negative sign on the experimental substructure energy error, E^{error}, indicates that a negative energy error adds energy to the structural model, similar to the effect of negative damping.

4. Sources of Energy Errors

In a typical experimental setup for hybrid simulation, the displacements are imposed at the actuator degrees of freedom and the forces are measured by load cells on the actuators. Therefore, it is useful to express the energy in the experimental substructures in terms of the actuator degrees of freedom coordinates. This approach allows for the energy errors to be computed separately for each actuator and related to the energy in the global structural model using similitude scaling factors and geometric transformations from conventional finite element models.

The geometric transformation matrix, $\mathbf{T}$, is obtained by transforming the global structural degrees of freedom, $\mathbf{u}$, to the actuator degrees of freedom, $\mathbf{u}^{ac} = \mathbf{T}\mathbf{u}$, where $\mathbf{u}^{ac}$ is a vector containing the displacement command signals for the actuators. The transformation $\mathbf{r}^{\mathrm{m}} = \mathbf{T}^{\mathrm{T}}\mathbf{r}^{\mathrm{am}}$, can be used to transform the measured forces at the actuator degrees of freedom, $\mathbf{r}^{\mathrm{am}}$, to the global degrees of freedom in the restoring force vector $\mathbf{r}^{\mathrm{m}}$. Substituting the transformations and their derivatives into Eq.3.3, an expression of the experimental substructures energy in terms of the actuator degrees of freedom is obtained.

$$\mathbf{E}^{\mathrm{E}} = \int \left(\mathbf{r}^{\mathrm{am}}\right)^{\mathrm{T}} \mathbf{T}\, d\mathbf{u} \tag{4.1}$$

In the above expression, the vector multiplication suggests that energy in the experimental substructures is simply the sum of energy contributions produced by each individual actuator. Further, Eq. (4.1) states that the numerical integration algorithm considers the response of the experimental elements as the measured restoring forces resulting from the command displacements. Since the command displacement is not necessarily the same as the applied or measured displacement, the behavior of the specimen may not be correctly modeled.

A more accurate representation of the energy in experimental substructures can be obtained from the measured displacement and the measured force data. This data pair is the best representation of the experimental substructure available and is typically used to evaluate a structure after a quasi-static test. The best estimate of energy stored and dissipated by the experimental substructure is

$$\mathbf{E}^{\mathrm{BE}} = \int \left(\mathbf{r}^{\mathrm{am}}\right)^{\mathrm{T}} d\mathbf{u}^{\mathrm{am}} \tag{4.2}$$

where $\mathbf{u}^{\mathrm{am}}$ = measured displacement of the specimen at the actuator degrees of freedom. It is important to consider that in a hybrid simulation, the load path of the experimental element should coincide with the computed response history of the structural model. Consequently, $\mathbf{E}^{\mathrm{BE}}$ is not necessarily the best energy estimate with respect to numerical simulation since the load history on the experimental substructure may differ from that in the numerical model. In spite of this, the energy error introduced into a hybrid simulation can be estimated as

$$\mathbf{E}^{\mathrm{error}} = \mathbf{E}^{\mathrm{BE}} - \mathbf{E}^{\mathrm{E}} \tag{4.3}$$

Eq. (4.3) shows the difference between the energy that is dissipated by the experimental substructures and the energy dissipated by the numerical integration algorithm. The energy error, $\mathbf{E}^{\text{error}}$, will be close to zero if there is no displacement control errors.

It is important to note that Eq. (4.3) does not account for scaling factors between the numerical model (global degrees of freedom) and the experimental substructure (actuator degrees of freedom). Such scaling factors, if not included in the transformation matrix, $\mathbf{T}$, should be included in the energy calculations to provide the correct comparison between the computed energy errors and the relevant energy measure in Eq. (3.2).

5. Hybrid Simulation Error Monitors

The experimental energy error estimate presented above can be computed in each step of the numerical integration algorithm. Further, the error estimate only requires information from the current and previous steps. Thus, energy errors can be computed throughout the simulation and used to monitor their severity in real-time. The use of $HSEM$ to asses the quality of a hybrid simulation requires that a threshold value be specified as the level of unacceptable error, corresponding to the point when the accuracy of simulation results is exceeding an allowable tolerance limits. To this end, a normalization of the energy error by the input energy from the earthquake excitation is considered here. A global view of the experimental substructure energy errors is given by the energy balance Eq. (3.4), where the input energy may be defined as

$$\mathbf{E}^{\text{input}} = \int (\mathbf{f})^{\mathrm{T}}\, d\mathbf{u} \tag{5.1}$$

A measure of energy error relative to the input energy is defined as follows:

$$HSEM = \frac{E^{error}}{E^{input} + E^{strain}} \tag{5.2}$$

The strain energy, E^{strain} is included in the denominator because the input energy is zero at the beginning of a simulation, which can result in an infinite value for $HSEM$. E^{strain} is defined here as the maximum recoverable strain energy assuming the structural behavior is elastoplastic. Note that for an elastic system, the energy input from the earthquake is dissipated only by viscous damping. Therefore, in the linear range, a comparison of the energy error to the energy dissipated by viscous damping will provide similar results.

6. Large-Scale Test of a Suspended Zipper Braced Frame

Suspended zipper braced frame is an innovative steel concentrically braced frame proposed by Leon and Yang (2003). The structural system has very similar configuration to that of the conventional inverted-V braced frame, except for a zipper column, a structural element added between the beam mid-span points from the second to the top stories of the frame. In the event of severe earthquake shaking, the inverted-V braces are designed to buckle to dissipate earthquake energy. This creates unbalanced vertical forces at the mid span of the beams. The zipper columns are designed to transfer the unbalanced vertical forces to the higher stories by engage the remaining unbuckled braces and beams to resist the unbalanced vertical force. If the earthquake shaking increases, brace buckling will propagate to the higher stories until all compressed braces buckle. To prevent the system from entering a force-deformation response softening range, the top-story braces are capacity designed to remain linearly-elastic under the total unbalanced vertical forces in the system.

The response of the suspended zipper braced frame is complex, with interacting nonlinear response of the braces and nonlinear response of the system. Hybrid simulation was selected to examine the seismic response of this structural system because it provides insight into the minutia of the local buckling response of the braces simultaneously with the global view of the system response at every instant of the earthquake loading. The hybrid simulation was conducted at the *nees@berkeley* laboratory, an equipment site of the George E. Brown Jr. Network for Earthquake Engineering Simulation (http://nees.berkeley.edu). Relatively new state-of-the-art high performance actuators, controllers and data acquisition system were used in this simulation, providing a unique opportunity to evaluate the effectiveness of the proposed $HSEM$ and evaluate the accuracy of a hybrid simulation using modern equipment.

The hybrid model of the suspended zipper braced frame consists of a physical first-story inverted-V braced sub-assembly, set up in the laboratory, and a finite element model of the remainder of the frame (Figure 1). The substructures were selected to model the complex brace buckling behavior experimentally, while the remainder of the structure is modeled using a state-of-the-art finite element software, OpenSees (UCB 1997). The computer model consists of flexibility-based nonlinear beam-column elements (de Souza 2000; Filippou and Fenves 2004) with fiber sections and zero-length elements to model the beams, braces, zipper columns and columns of the suspended zipper braced frame. The foundations of the columns and the beam-to-column connections were modeled using the semi-rigid connection model suggested by Astaneh-Asl (2005). Large nonlinear geometric transformation were included using the corotational element formulation in OpenSees. Table 1 shows the element sizes of the tested 1/3-scale suspended zipper braced frame model. The reduced scale size of the experimental model was governed by collaborative earthquake simulator testing of an identical model at University at Buffalo (Schachter and Reinhorn 2006). The floor heights of the 1/3-scale suspended zipper braced frame are 1.34 m, 1.29 m and 1.29 m for the first, second and third story, respectively. The bay width is 2.03 m. A floor mass of 9063 kg was assigned as two lumped masses at the exterior nodes at each floor. Rayleigh mass and stiffness proportional damping of 5 percent was assigned to the first and second vibration modes of the hybrid model.

Table 1: Element sizes for the 1/3-scale suspended zipper braced frame model

Story	Braces	Column	Beam	Zipper column
3	HSS 3x3x3/16	S4x9.5	S3x5.7	HSS2x2x3/16
2	HSS 2x2x1/8	S4x9.5	S5x10	HSS1.25x1.25x3/16
1	HSS 2x2x1/8	S4x9.5	S3x7.5	

The ground motion used in this study, LA22, was selected from the suite of ground motions used in the SAC Joint Venture project (Somerville et. al. 1997) for buildings located in Los Angeles. The ground motion was selected and scaled according to the procedure presented in Schachter and Reinhorn (2006) for the comparative earthquake simulator studies of a similar frame. Figure 2 shows the test setup for the suspended zipper braced frame hybrid simulation at the *nees@berkeley* laboratory. The setup consists of two vertical dynamic actuators, one horizontal dynamic actuator, and a guiding system to allow the intersection of the inverted-V braces to only move in-plane. The nonlinear geometry transformation of the actuator movement is accounted for in this hybrid simulation test setup (Yang 2006).

This hybrid simulation used a displacement controlled algorithm to solve the dynamic equations of motion for the hybrid model. The integrator (Newmark average-acceleration time-step integration) implemented in OpenSees calculated the structural deformation due to external excitation at the beginning of each time step. The test setup then executes the displacement and samples the force feedback from the experimental subassembly. Simultaneously, the analytical elements in OpenSees calculate the force feedback at the displacement state. The integrator then combines the force feedback from the experimental and the analytical elements and the external excitation to calculate the structural deformations at the next time step. Detailed procedures for the hybrid simulation algorithm and architecture are presented in Yang (2006).

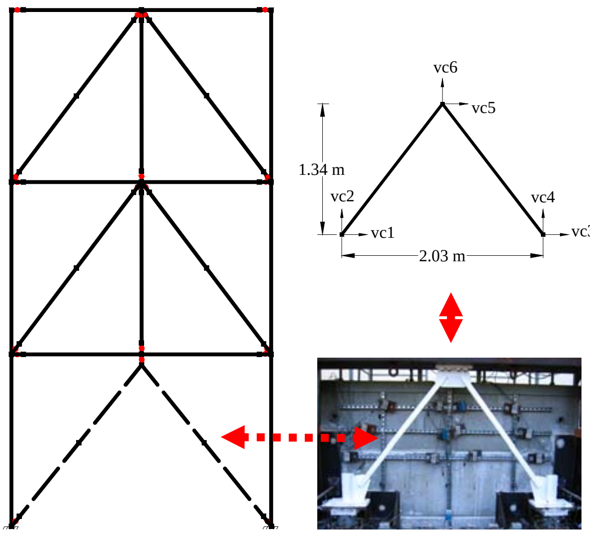

Figure 1: Hybrid simulation model.

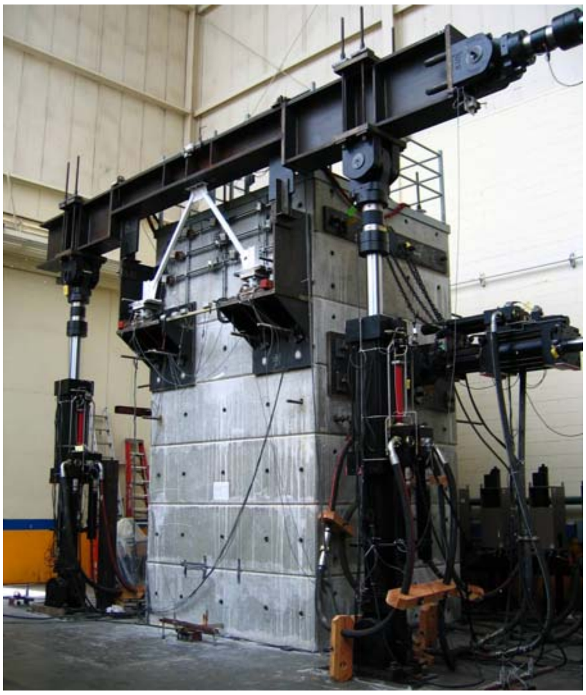

Figure 2: Test setup for the suspended zipper braced frame hybrid simulation at the *nees@berkeley* laboratory.

7. Experimental Results and Assessment of Errors

The first-story drift response of the structure, applied on the experimental substructures by the horizontal actuator is shown in Figure 3. Both, the displacement command and feedback displacement measurements corresponding to the horizontal actuator are shown to demonstrate the performance of the actuator controller. Due to careful tuning of the system prior to testing, actuator tracking errors are negligible, and thus the experimental errors are expected to be acceptably small.

The proposed $HSEM$ are used to better assess the effects of experimental errors on the response of the hybrid model in this simulation. Figures 4 shows the energy stored as strain energy or dissipated by hysteric damping in the experimental substructure. These plots include the energy input in the substructure from all three actuators and compare the energy estimates using Eqs 4.1 and 4.2. The difference between these two curves is the estimated energy error according to Eq. 4.3, which is shown in Figure 5. The error is negative, which indicates that energy is being added to the experiment as a result of the actuator tracking errors. Note that the energy error is approximately 5% of the energy input by the actuators into the experimental substructure.

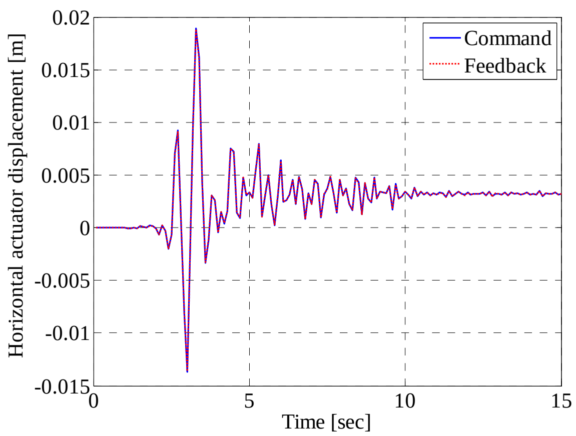

Figure 3: Displacement command and feedback histories of the horizontal actuator.

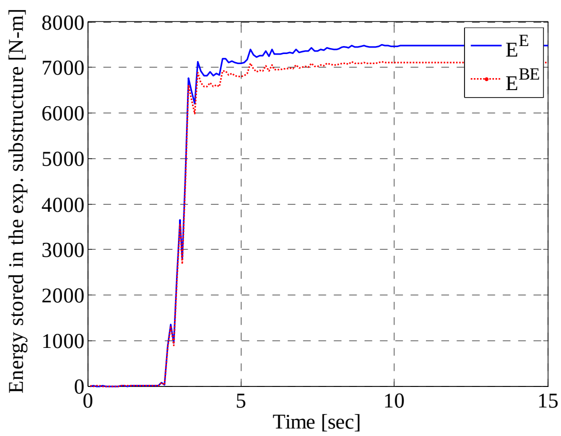

Figure 4: Energy stored and dissipated by experimental substructure.

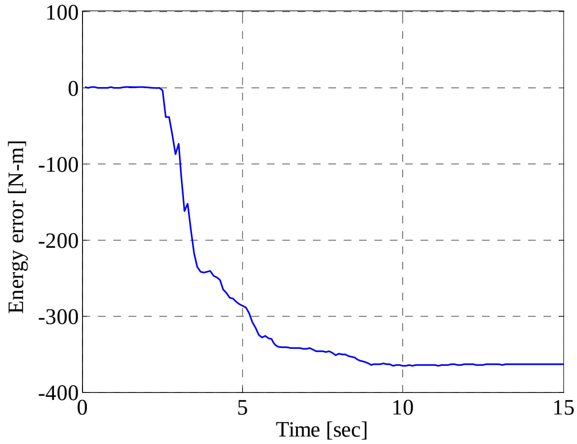

Figure 5: Energy error in the experimental substructure.

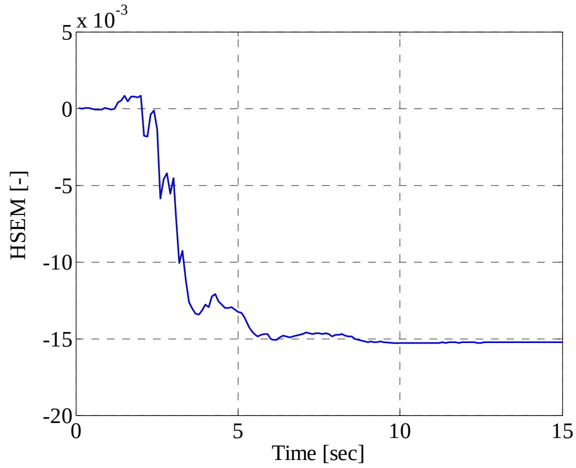

Figure 6: $HSEM$ (Equation 5.2).

The effects of energy error on the overall results of the hybrid simulation are best demonstrated through the proposed $HSEM$. Figure 6 presents the $HSEM$ described in Eq. 5.2, which is the energy error normalized by the total input earthquake energy plus the strain energy. For this purpose, the strain energy is calculated at the yield displacement, estimated to be 0.2% of the inter-story drift ratio. The results indicate that the energy error resulting from the experimental setup is less than 2% of the total energy input to the structural model from the external excitation. The majority of the energy error accumulated during the 3-second long acceleration spike in the ground motion record, corresponding to the large displacements in Figure 3. The remainder of the simulation, imposing less demands on the actuators did not contribute significantly to the $HSEM$. This error is reasonably small, indicating that the performance of the experimental setup is very good and the final results are accurate within an acceptable margin. As previously discussed, sources of errors in a hybrid simulation, other than actuator tracking and control should also be assessed to ensure simulation results are correct.

8. Conclusions

Hybrid simulation outcomes are sensitive to measurement and control errors associated with the experimental substructures. Hybrid simulation error monitors ($HSEM$) are proposed to asses the severity of experimental errors during a hybrid simulation. They are based on quantifying energy added to the hybrid model due to measurable command errors in the experimental substructures. A recently completed hybrid simulation of seismic response of a suspended zipper braced frame is used to demonstrate the effectiveness of the proposed on-line error monitoring indicators. This demonstration was successful: $HSEM$ can be easily implemented and used to measure the experimentally induced simulation error (primarily due to imperfect actuator control). It was shown the suspended zipper braced frame seismic response simulation experienced an addition of approximately 5% of the total excitation energy due to actuation errors. This error level was considered acceptable given the uncertainties associated with ground motion excitation. More important, use of state-of-the-art controllers and actuators, together with accurate calibration and preparation of the test setup is key to conducting highly accurate hybrid simulations, even when the response of the hybrid model is highly nonlinear and quite complex.

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