FORMULATIONS OF MASS FRACTION OF THE MULTI-STORY FLEXIBLE STRUCTURES ISOLATED BY FRICTION PENDULUM SLIDING BEARINGS

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ABSTRACT:

In this paper, dynamic response analysis of the isolated structure performed by the mathematical models to investigate functional relationship between natural periods of isolated flexible structure and its mass fraction. Fundamental Frequency of the isolated flexible structure depends on the mass fraction of the structure that some fraction of mass vibrates at the frequency of isolator and rest vibrates at its own frequency. The maximum displacement response of the isolated structure at isolation level is the key design parameter for the application of base isolation. Base isolator can reduce inertia forces generated by ground shaking. However, since natural frequency of a flexible structure directly related to its mass fraction; it is the case that the natural frequency of isolator would match to the one of the mode of the natural frequency of the isolated flexible structure. As a result inertia forces of the isolated structure would be amplified instead of reduced. Therefore, this paper underlines the importance of the mass fraction, and to obtain reliable parameters when flexible structure is capable to translate at base. The outcomes of the mathematical models are consistent with the experimental results of two and three story model of structure.

KEYWORDS:

Mass Fraction, Isolated Structure, Fundamental Frequency, Maximum Displacement

INTRODUCTION

The Primary objective of isolating structure is to reduce inertia forces induce on structure during ground shaking. To achieve this objective, the isolated structure translates at base relative to ground when excited by ground motion. The magnitude of the lateral force that causes the lateral displacement of the supported structure depends on primarily natural frequency of isolated structure. And natural frequency of the isolated flexible structure depends on mass fraction. The following mathematical models are used to demonstrate the functional relationship between natural frequency of isolated flexible structure and its mass fraction.

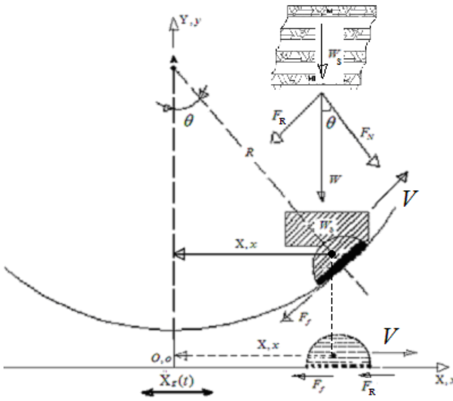

Fig: 1 Pendulum motion model

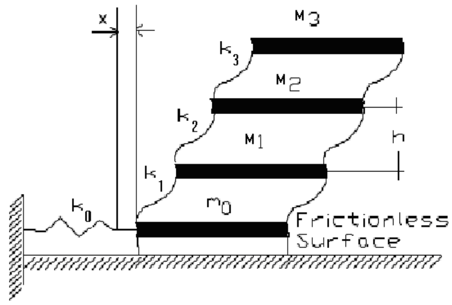

Fig: 2 sliding model at horizontal surface

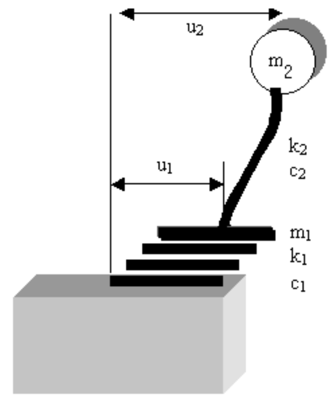

Fig: 3 Lump-mass model

Pendulum motion of structure supported by Friction Pendulum bearing:

As an isolation device, Friction Pendulum Sliding (FPS) bearings are most efficient and cost effective seismic protection system once properly designed. This system simply alters the force-response characteristics of the structures at the expense of a large displacement at isolation level. The isolated structure supported by (FBS) bearings undergoes friction pendulum motion when excited by ground motion. Maximum value of the displacement response of the isolated structure is a crucial parameter for designing proper and reliable seismic protection system.

The mathematical model shown in Fig: 1 represents the motion of the structure when such structure excited by harmonic ground accelerations. With this model, M_b denotes of the total mass of the structure if structure is totally rigid; if not it denotes the mass portion of the structure that vibrates at the frequency of friction pendulum bearings. In this case the structure behaves as a flexible structure, and the rest portion of the mass of the structure, M_s, which vibrates at its own frequency. The equations of the motion are derived as follows.

$$\sin\theta = x/R \qquad (1)$$

$$\sin\theta = F_R/W \ , \ \cos\theta = F_N/W \ , \ F_f = \mu F_N$$

$$F_R = W(x/R) \qquad (2)$$

$$F_f = \mu W \cos\theta \qquad (3)$$

$$\ddot{X}_g(t) = \ddot{X}_{g0}\sin(wt)\,, \quad \ddot{X} = \ddot{x} + \ddot{X}_g(t)\,,\ W = W_b + W_s\,, \quad V = F_R + F_f$$

Dynamic equilibrium of the structure supported by (FPS) bearings is as follows:

$$M_b\ddot{X} + V = 0 \ \text{ Or } \ M_b(\ddot{x} + \ddot{X}_g) + F_R + V = 0 \qquad (4)$$

By replacing equation (2) and equation (3) into equation (4), the governing equation of structure, which is supported by FPB and subjected to harmonic ground excitation, would be derived as follows:

$$M_b\ddot{x} + \mu W\cos\theta + (W/R)x = -M_b\ddot{X}_g \qquad (5)$$

It is extremely difficult to solve the above second-order non-linear differential equation (5). Therefore, nonlinear trigonometric term, $\cos\theta$ is eliminated by redefining it with the terms of $R, \dot{x}$ and w_D. In order to obtain an ordinary second-order linear differential equation from this non-linear differential equation (5), the procedure starts by taking derivative of equation (1) as follows:

$$d(\sin\theta) = d(x/R), \qquad \cos\theta = \dot{x}\Big/(\dot{\theta}R) \tag{6}$$

$\dot{\theta}$, the angular velocity, which is the natural vibration frequency, w_D, of damp structure supported by friction pendulum bearings (FPB) subjected to harmonic ground excitation.

The equation (5) is reorganized by replacing $W = W_b + W_s$, $M = M_b + M_s$, $\cos\theta = \dot{x}\Big/(\dot{\theta}R)$,

$\dot{\theta} = w_D$ And $w_n = \sqrt{g/R}$ as follows:

$$\ddot{x} + \mu\frac{(M_b + M_s)}{M_b}\left(\frac{w_n^2}{w_D}\right)\dot{x} + \frac{(M_b + M_s)}{M_b}w_n^2 x = -\ddot{X}_g$$

Since $w_D = w_n\sqrt{1-\xi^2}$, and $w_D \cong w_n$ when $\xi << 1$, the motion of the supported structure with (FPS) bearings is derived as a second order ODE as follows

$$\ddot{x} + \mu\left(1 + \frac{M_s}{M_b}\right)w_n\dot{x} + \left(1 + \frac{M_s}{M_b}\right)w_n^2 x = -\ddot{X}_g \tag{7}$$

Where,

W, M The total weight, mass of the supported structure

W_b, M_b The weight, mass of the structure that vibrates at the frequency of FPB

W_s, M_s The weight, mass of the structure that vibrates at its own frequency

X, Y Imaginary, fix coordinate system

x, y Translating coordinate system, which is attached to (FPS) bearings?

V The lateral force-shear force- of bearing at the isolation level,

F_R Restoring force

F_f Friction force

$\dot{X}, \dot{x}$ Absolute, relative velocity of bearing

$\ddot{X}, \ddot{x}$ Absolute, relative acceleration of bearing

$\ddot{X}_g(t)$ Harmonic ground acceleration,

$\ddot{X}_{g0}$ Peak ground acceleration,

R The radius of curvature of bearing

μ The coefficient of friction mobilized during sliding (assumed constant)

w_n Natural frequency of FPB

w_D Damp frequency of isolated structure

w Excitation frequency

ξ Damping ($\xi = \mu / 2$, for rigid structure), ($\xi > \mu / 2$, for flexible structure)

ϖ_n Dominant natural frequency of flexible structure supported with FPB

The equation of the motion for flexible structure in the form of linear (ODE)

$M_s > 0$, and ϖ_n , which is dominant natural frequency of system is $\varpi_n^2 = (1 + \frac{M_s}{M_b}) w_n^2$. The governing equation of the motion of a supported flexible structure by (FPB) would be derived from equation (7) as follows:

$$\ddot{x} + 2\xi\varpi_n \dot{x} + \varpi_n^2 x = -\ddot{X}_g \quad (8)$$

Where, $\xi = \frac{\mu}{2}\sqrt{(1 + \frac{M_s}{M_b})}$

Since, $(1 + \frac{M_s}{M_b}) > 0$ then $\varpi > w_n$, $\xi_{rigid} < \zeta_{flexible}$

Indeed, the dynamic response of flexible structure is a combination of two vibrations. Those vibrations are the vibration of some mass of structure that vibrates at the frequency of FPB, and the vibration of the rest of the mass of the structure that vibrates its own dominant frequency.

The obvious question would be asked about what fraction of the total mass of structure vibrates at the frequency of friction pendulum bearings (FPB), or what fraction of the total mass of structure vibrates at its own dominant frequency? The stiffness level of the structure would be the answer of this question.

For example, if stiffness of structure is too large, then whole structure would be assumed rigid, and $M_s = 0$.

The complete solution of the above ordinary differential equation (8) would be as follows:

$$x = e^{-\xi w_n t}(A\cos\varpi_D t + B\sin\varpi_D t) + C\sin wt + D\cos wt$$

This general solution contains two distinct vibration components: Transient vibration and steady state vibration, where the constants A, B, C and D are defined as follows:

$$A = (-\ddot{X}_{g0} / w_n^2)\frac{2\xi(w/w_n)}{\left[1-(w/w_n)^2\right]^2 + \left[2\xi(w/w_n)^2\right]^2}(1+\frac{M_b}{M_s})$$

$$B = (-\ddot{X}_{g0} / w_n^2)\frac{w\left[1-(w/w_n)^2\right] - 2\xi^2 w_n (w/w_n)}{\left[1-(w/w_n)^2\right]^2 + \left[2\xi(w/w_n)^2\right]^2}(1+\frac{M_b}{M_s})(\frac{1}{w_D})$$

$$C = (-\ddot{X}_{g0} / w_n^2)\frac{1-(w/w_n)^2}{\left[1-(w/w_n)^2\right]^2 + \left[2\xi(w/w_n)^2\right]^2}(1+\frac{M_b}{M_s})$$

$$D = (\ddot{X}_{g0} / w_n^2)\frac{2\xi(w/w_n)}{\left[1-(w/w_n)^2\right]^2 + \left[2\xi(w/w_n)^2\right]^2}(1+\frac{M_b}{M_s})$$

For flexible structure, the natural frequency in the above equations will be replaced by

$$\varpi_n^2 = (1+\frac{M_s}{M_b})w_n^2 \quad , \quad \varpi_n = w_n\sqrt{1+M_s / M_b}$$

Parameter study

Parameter studies performed for the rigid and flexible structures when the structures are excited by harmonic ground acceleration. Excitation frequency is used as $w = 3\pi$. Maximum peak acceleration is applied as 1.0g. For flexible structure mass fraction is assumed $(M_s / M_b) = 4$. Mat lab figures below illustrate dynamic and free responses of the relative displacements, absolute velocities and absolute accelerations of rigid and flexible structures supported by (FPS) bearings of radius of curvature of R=90 in.

The excitation frequency is chosen as w=3*π that is dominant frequency of most major earthquakes $\ddot{X}_{g0}$ =1.0g (peak ground acceleration)

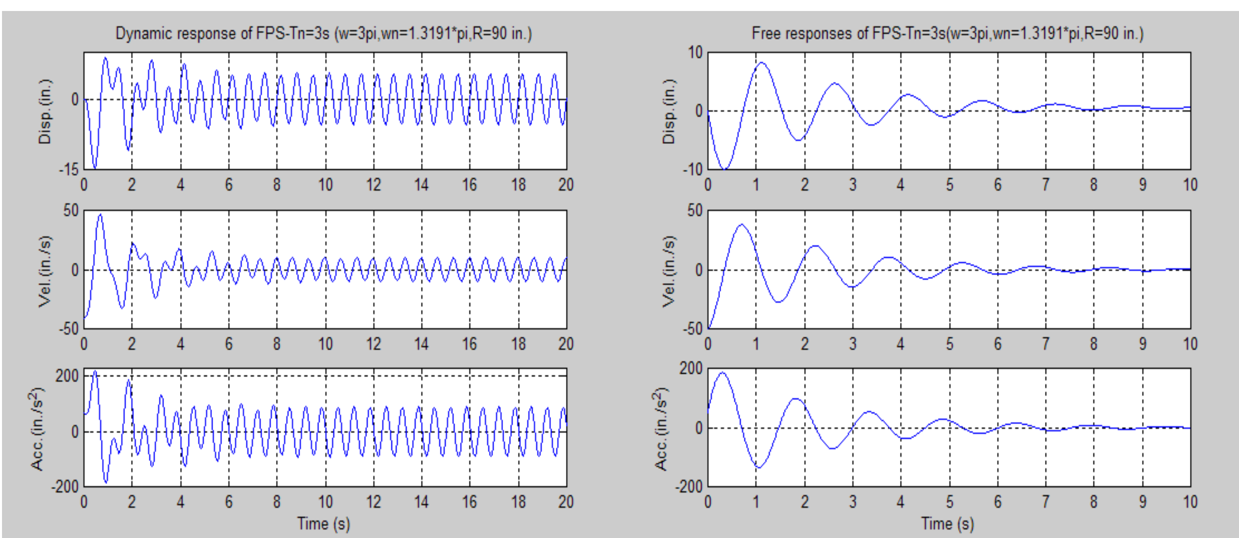

Fig. 4 Total Dynamic and free responses of the a flexible structure supported by (FPS) bearings

The motion of the structure when sliding at frictionless surface:

The mathematical model in Fig: 2 represent the structure capable to translate at the frictionless surface.

If a unit force is placed at each floor, the influence coefficients are as follows:

$$a_{00} = 1/k_0$$

$$a_{11} = 1/k_0 + 1/k_1 = 1/k_0 + h^3/24EI$$

$$a_{22} = 1/k_0 + 1/k_1 + 1/k_2 = 1/k_0 + 2h^3/24EI$$

$$a_{33} = 1/k_0 + 1/k_1 + 1/k_2 + 1/k_3 = 1/k_0 + 3h^3/24EI$$

Where,

$k_1 = k_2 = k_3 = k$ (assumed)

$M_1 = M_2 = M_3 = m$ (assumed)

$k = 24EI/h^3$

Fundamental frequency of the flexible structure derived is as follows:

$$\varpi_n = \sqrt{\frac{4k_0 EI}{(3m + m_0)4EI + k_0 mh^3}}, \qquad \varpi_n = \sqrt{\frac{4k_0 EI / 4EI}{(3m + m_0)4EI / 4EI + (1/4)k_0 mh^3 / EI}}$$

$h^3/4EI \approx 0$ then, fundamental frequency of isolated flexible structure becomes as follows:

$$\varpi_n = \sqrt{\frac{k_0}{(3m + m_0)}}, \qquad \varpi_n = \sqrt{\frac{k_0 / m_o}{(3m / m_o + 1)}}, \qquad \varpi_n = w / \sqrt{3m / m_o + 1}$$

Where,

$w_n = {}^{k_o}\!/\!_{m_o}$ (Fundamental frequency of the structure fixed at base)

Parameter study:

Let assume:

$m \le mo \le 3*m$, and 5 kip/in $\le k_0 \le 20$ kip/in

$M_1 = M_2 = M_3 = m = 1000/g$ (kips s^2/in)

$h = 12$ ft. $g = 386$ in./s^2 $EI = (29000$ ksi$)*(140$ in$^4)$

For above example, following mat lab Fig: 5 shows that the fundamental period of structure increases significantly when the foundation of structure is capable to translate. It also illustrates that the mass addition to basement increases the fundamental period of the structure as well.

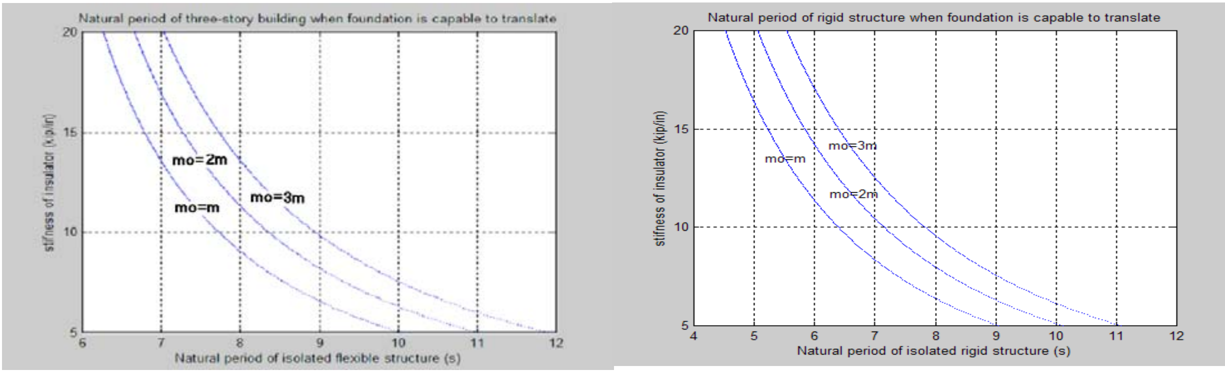

Fig: 5 Natural Period of the isolated flexible structure (left) and rigid structure (right)

Motion of the structure when supported by lead rubber type of isolator:

A base isolated structure can also be modeled as a two DOF system as shown in Fig.3. In this model m_2 represents the whole mass of structure as a lump mass. In order to compare the outcomes of models for the isolated frequency of structure; $m_2= 3*m$, $m_1=m_0$ and $k_1=k_0$ are assumed with the same parameter used in previous model. The equation of the motion is derived as follows

In Fig: 3 model for base isolated structures as a two DOF system:
$c_2 \approx 0$ (Assumed)

$m_2 \ddot{u}_2 + k_2(u_2 - u_1) = -m_2 \ddot{u}_g$; (1) $[(k_1 + k_2) - m_1 w^2]u_1 - k_2 u_2 = 0$ (3)

$m_1 \ddot{u}_1 + c_1 \dot{u}_1 + (k_1 + k_2)u_1 - k_2 u_2 = -m_1 \ddot{u}_g \dot{u}$ (2) $-k_2 u_1 + (k_2 - m_2 w^2)u_2 = 0$ (4)

$(m_2 w^2 - k_2)[m_2 w^2 - (k_1 + k_2)] - k_2{}^2 = 0$ (Second order characteristic polynomial)

$T_F = 2\pi\sqrt{\dfrac{m_2}{k_2}}$ (Fix base period of the structure)

$T_I = 2\pi\sqrt{\dfrac{(m_1 + m_2)}{k_1}}$ (The isolator period)

The natural modes and periods of vibration T_1 and T_2 of the isolated structure can be derived from the

characteristic simultaneous equation (3) and (4):

$T_{1,2} = [(2T_F^2 T_I^2/(1+n))/[(T_F^2+T_I^2) \pm (T_F^4+T_I^4+(T_I^2 T_F^2)(2-4/(1+\eta))]^{0.5}$

Where;

$\eta = m_2 / m_1$ (Mass fraction)

$\alpha = T_F / T_I$

$T_1 = T_I \sqrt{(1+\alpha^2)}$ (First mode)

When $0.01 \le \alpha \le 0.5$ then $T_1 \cong T_I$ which the fundamental period of a rigid structure supported by friction pendulum bearing is equal to fundamental period of friction pendulum system (FPS).

$T_2 = T_F / \sqrt{(1+\eta+\eta\alpha)}$ (Second mode)

***Parameter study*:**

By using same parameters, in previous parameter study (m_2= 3*m, m_1=m_0 and k_1=k_0), the following Mat Lab figure illustrates natural periods (see Fig. 6) of this three-story building as a 2-DOF base isolated structures with mass lump model illustrated in Fig 3 . By replacing k_2=∞, this model also represents isolated rigid structure. Because of the presence of the isolation system, the participation of the first mode is dominant and flexible structure will primarily oscillate along its firs mode. For rigid structure second mode would not be produced.

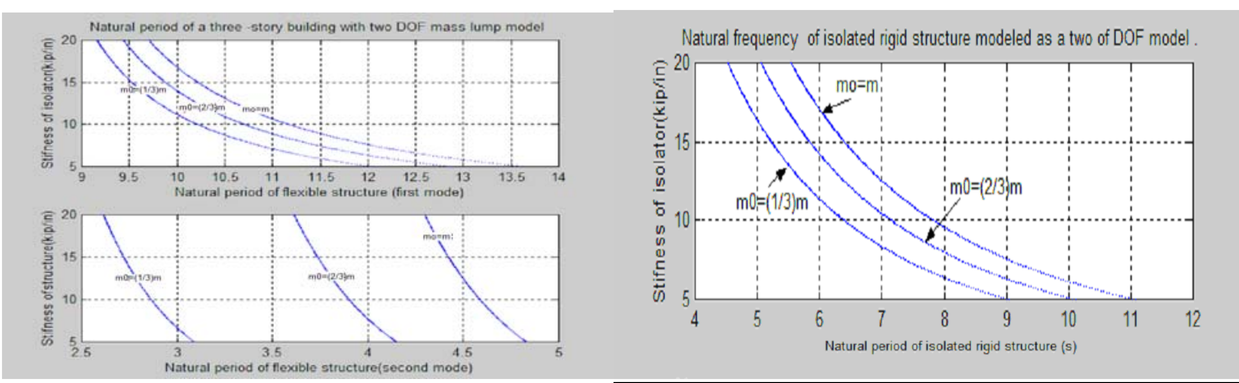

Fig.6 Natural periods of base isolated flexible and rigid structure with 2-DOF mass lump model

Conclusion

It is demonstrated by this paper that isolators used for flexible structures requires careful consideration about its displacement response at base. Inertia reduction achieved by base isolator is always at the cost of the large displacement at base. The possible large displacement must be managed by the isolator. Isolating rigid structure is pretty straight design application since its altered natural frequency is not related to mass of the structure. However, the altered natural frequency of the isolated flexible structure is certainly related to its mass distribution.

REFERENCES

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