NUMERICAL MODELING AND EXPERIMENTAL IDENTIFICATION OF THE EUCENTRE TREES LAB SHAKE TABLE

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ABSTRACT :

This paper present two models of the EUCENTRE TREESLab high performance uni-axial shake table based on their numerical implementation using the Matlab system-modeling package Simulink. The first model uses a simplified linearized servovalve actuator system while the second uses a realistic detailed nonlinear system. These numerical models incorporate the inherent dynamic characteristics of the various components of the shake table system and their interaction.
A comprehensive set of random and periodic tests were conducted on the shake table in order to determine all of the parameters involved in the numerical models. Finally, both models were validated with the real shake table by comparing their transfer functions with the experimentally calculated transfer function.

KEYWORDS: shake table , experimental testing, numerical model, characterization, identification

1. INTRODUCTION

Experimental testing is an essential tool for understanding how structures respond to dynamic excitation. Indeed, although numerical methods have experienced significant improvements, the use of experimental testing remains indispensable, particularly for complex problems involving structural non-linear behavior, rate of loading effects, and failure mechanisms. Such problems are common in the field of earthquake engineering and are still difficult to investigate using conventional analytical methods.
Shake table testing, as part of experimental testing, is an effective and practical technique to evaluate the response of the structures. Its primary function is to replicate in the laboratory the true nature of earthquake input as well as artificial ground motion and a wide range of vibration signals, in order to simulate the dynamic excitation of a specimen mounted on the table platform. Because the test is conducted in real time, dynamic effects and rate dependent behavior can be completely modeled. However, one of the key challenges to be overcome in performing an accurate shake table test is the faithful reproduction of the desired table motion. It is therefore essential to develop a joint experimental-analytical approach in order to better understand the dynamics of the shake table system.
This paper describes the experimental procedure for the identification of the TREES Lab at Eucentre shake table and its components (servovalve, actuator, payload….). Two numerical models of the shake table system were implemented using Simulink. The first one was obtained for the case of a simplified model of actuator and servovalve, while the second uses a more sophisticated analytical model including more detailed characteristics of the servovalve and actuator. The total shake table transfer function of the two models were then compared to that obtained experimentally.

2. SHAKE TABLE HARDWARE

The TREES Lab at Eucentre is equipped with an MTS Systems Corporation servo-hydraulic uni-axial shake table shown in Figure 1. It consists of 5.6x7.0 m² moving steel platform that is attached to a servo-hydraulic dynamic actuator of ± 1700 kN force capacity. The system is capable of simulating earthquake events and other

ground vibration with ±500mm maximum stroke and ±2200 mm/sec pick velocity. Accelerations of ±1.8 g are possible with maximum test specimens of 140 tons, and up to ±6.0 g for bare table condition. The maximum over turning moment and yaw moment are 4000 kNm and 400 kNm respectively and the bandwidth of operating frequency is 0-50 Hz. The hydraulic power supply that supplies the shake table consists of 8 high pressure pumps that can deliver a total of 1360 liters per minute at 28 MPa and 900 liters of accumulators for peak demands.
The Table is controlled by advanced Digital Controller MTS system 469D. It provides for the shake table a high-level fixed control techniques such as Three-Variable Control (TVC: displacement, velocity, and acceleration), built-in filtering and adaptive compensation techniques for high fidelity and faithful reproduction of the desired table motions.

Figure 1: TREES Lab at Eucentre shake table

3. SIMULATION MODEL

The shake table system is represented using block diagrams. Figure 2 shows the relations between the different subsystems of the shake table. The major components that comprise the model are the valve and actuator system and the payload.

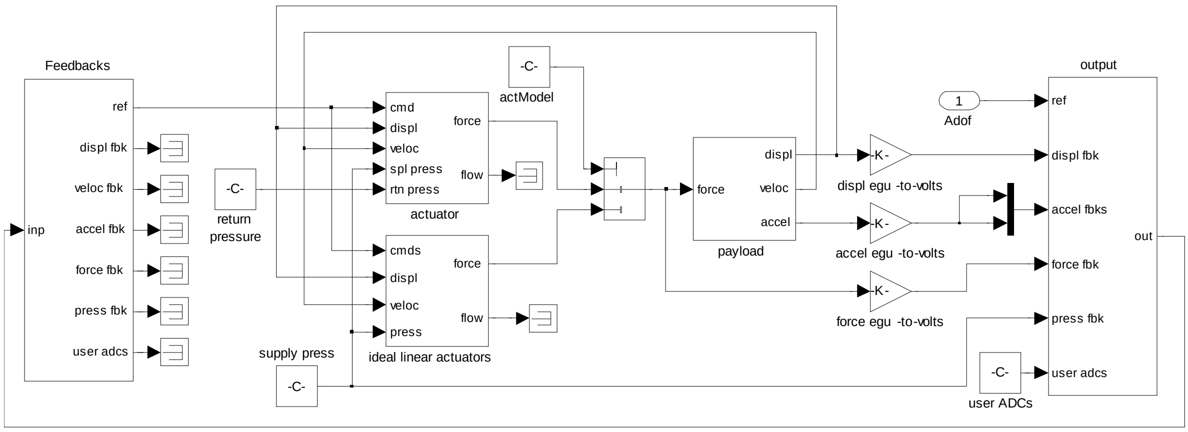

Figure 2: Seismic Table Model detail view

3.1 Valve & Actuator Model

Two types of valve and actuator models were implemented, the former, named “ideal actuator”, in which the servovalve is represented as a time delay and the actuator with an approximate linearized flow continuity equation, while the latter, named “real actuator”, uses a more sophisticated model that includes realistic representations of a large number of parameters.

3.2.Payload Model

The payload block models the relationship between the table motion and the actuator force (minus friction force). Both the effect due to the flexibility of the foundation reaction mass and the presence on the table of flexible specimens have been modeled. The kinematics of the table (acceleration, velocity and displacement) are also calculated in this block model. Refer to Thoen and Laplace (2004) for a detailed description of the major components of the payload model.

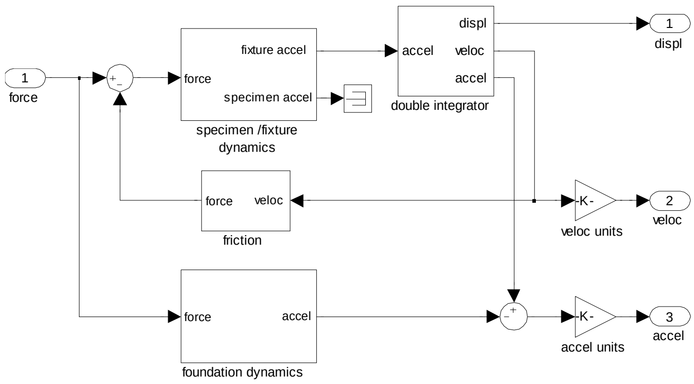

Figure 3: Payload model

4. PARAMETER IDENTIFICATION

Modeling of the shake table requires the determination of many parameters. Some of them, mainly concerning the geometric and physical proprieties of the various system components, can be obtained from the manufacturer or through direct measurement. Some other parameters need to be determined experimentally, like nominal flow, effective bulk modulus, servovalve spool dynamics, table rigid mass, friction coefficient, and foundation dynamics. These parameters can be obtained by conducting bare table tests. The final set of parameters relates to the specimen dynamic characteristics, which can be determined using one of the experimental dynamic identification techniques such as hammer test.

In this study, a series of random and periodic (sine and triangular waves) tests were conducted on the EUCENTRE TREES Lab shake table in order to determine the bare table system parameters. Data acquisition was done using the MTS 469D digital controller. The selected channels to be recorded during the tests were stored in user selected file with sampling rate set to 512 Hz.

The parameter estimation process has been fully described in Thoen and Laplace (2004), the same procedure was used here and only the results are presented in the following sections, except for the servovalve spool dynamics, for which more detailed descriptions are given in this paper.

4.1. Parameter estimation using random tests

Table 4.1 Random test program

Test	R1	R2	R3
RMS	0.005 m	0.1 g	0.05 g
Freq range (Hz)	[0 -125]	[0.5 -50]	[0.5 -125]

The test program used in this study consisted of three wideband white noise tests, one in displacement control and the other two in acceleration control. Detail of these are given in table 4.1.

4.1.1.Servovalve spool dynamics

Typically the servovalve dynamics can be represented either by first order or second order models. Laplace transfer functions of these models are given below, for the first order and the second order models, respectively:

$$H(s) = \frac{k_s}{\tau s + 1} \tag{4.1}$$

$$H(s) = k_s \frac{\omega_v^2}{s^2 + 2\xi\omega_v + \omega_v^2} \tag{4.2}$$

where s is the Laplace transform variable, k_s is the valve gain and τ is the response delay, while ξ and ω_v represent the damping ration and natural frequency of the servovalve, respectively.

To identify the dynamic parameters mentioned above a test was conducted to generate a frequency response plot of the servovalve. The table was excited with a wideband random program in displacement control mode (R1 in Table 4.1). The conditioned servovalve command and the third stage spool position of the servovalve were recorded. The experimental transfer function of the servovalve was calculated and shown in fig. 4.

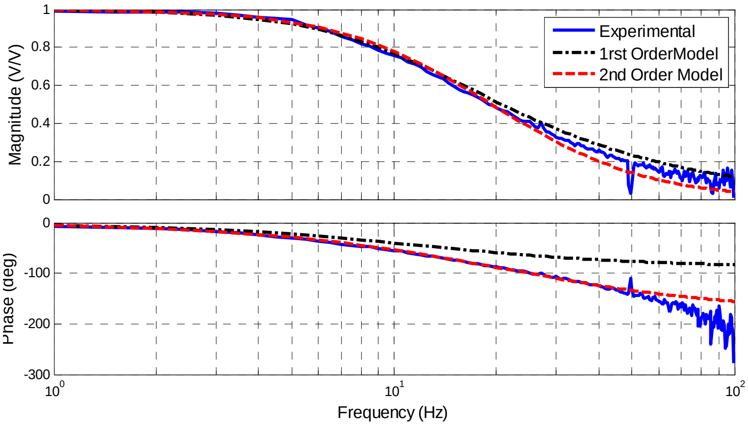

Figure 4: Measured servovalve dynamics

The following information was extracted from the experimental frequency response function:

- The valve gain, which corresponds to the asymptotical line of the magnitude response, was found equal to one (K_s = 1).
- The time delay for the first order model was found to be 0.0137 sec.
- An equivalent natural frequency of 20.5 Hz and a damping of 105% were found.

The frequency responses of the first and second order models of the servovalve are compared in Fig. 4. It can be seen from the figure that the magnitude responses of both the models match very closely the magnitude of the actual table. The phase response of the second order model fits perfectly the experimental phase response over wideband frequency range (0-50 Hz), while the first order model matches the experimental phase response up to 10 Hz with reasonable accuracy.

4.1.2.Rigid mass of the table

The table was excited with a random acceleration command (R2 in Table 4.1) and acceleration and force feedbacks were recorded. The second-order filter was fit between acceleration as input and force as output to yield rigid mass as a function of frequency. The typical transfer function is shown in Figure 5. The rigid mass parameter is the magnitude of this filter at lower frequencies and is found to be 41.258 tons.

4.1.3.Oil column frequency and damping and oil bulk modulus

A random command (R3 in Table 4.1) in acceleration control mode was carried out on the table and servovalve spool position and force feedbacks were recorded. The oil column frequency and damping were then obtained by fitting the second-order filter to the recorded data. Once these parameters were estimated and knowing the total effective rigid mass already calculated in the previous section, the bulk modulus was then calculated.

These parameters were estimated to be: oil column frequency f_{oil} = 14.169 Hz, damping ξ = 3.36% and bulk modulus β = 1.1920e+003 MPa.

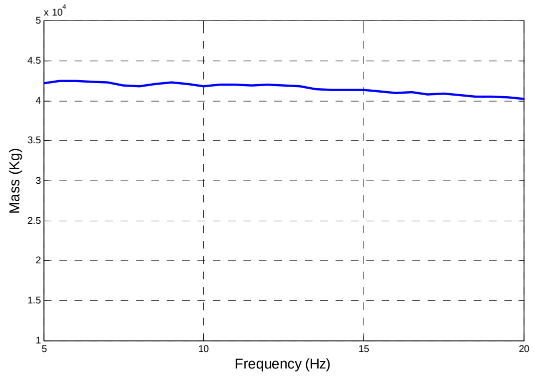

Figure 5: Table mass versus frequency

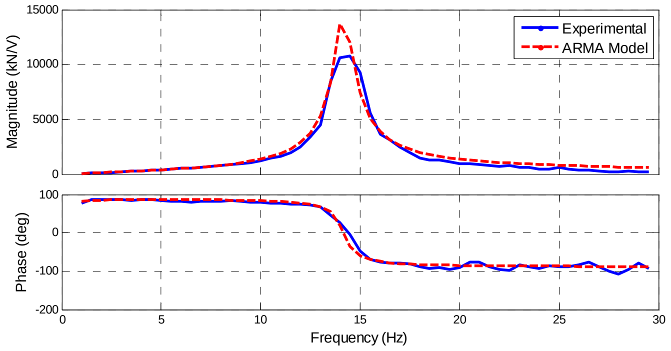

Figure 6: Oil column frequency response.

4.2.Parameter estimation using periodic tests

The testing program using periodic (sinusoidal and triangular) excitations is given in tables 4.2 and 4.3.

Table 4.2 sinusoidal program tests

Test	S1	S2	S3	S4	S5	S6	S7	S8	S9
Frequency (Hz)	0.05	0.10	0.5	0.05	0.10	0.5	0.05	0.10	0.5
$u_{\max}$ (m)	0.05	0.05	0.05	0.10	0.10	0.10	0.20	0.20	0.20

Table 4.3 triangular program tests

Test	T1	T2	T3	T4	T5	T6
Frequency (Hz)	0.05	0.10	0.05	0.10	0.05	0.10
$u_{\max}$ (m)	0.05	0.05	0.10	0.10	0.20	0.20

The dynamic equilibrium equation is used in this section in order to represent the equation of motion for the mechanical sub-system of the shake table. The purpose is to take advantage from the periodic nature of the displacement, velocity and acceleration of the triangular or sinusoidal tests to calculate the horizontal stiffness, the friction and the total rigid mass of the shake table.
The basic simplified conceptual model of the system, can be expressed by:

$$F_I(t) + F_D(t) + F_E(t) = F_A(t) \tag{4.3}$$

Where $F_A(t)$ is actuator force, and $F_I(t), F_D(t)$ and $F_E(t)$ are the table inertia, elastic, and damping forces, respectively. Equation (4.3) can be written as:

$$M_T(u).\ddot{u}(t) + F_D(\dot{u}) + F_E(u) = F_A(t) \tag{4.4}$$

Where M_T is the total effective rigid mass of the table and $u(t)$, $\dot{u}(t)$ and $\ddot{u}(t)$ represent the table kinematics (displacement, velocity and acceleration respectively).
It should be noted that the mechanical sub-system considered here does not include the compressible oil columns in the actuator chambers. The recorded actuator forces obtained from the pressures on both sides of the pistons already account for the oil column effect (Ozcelik *et al.*, 2007).
For the identification process one cycle of test data is selected in which the displacement $u(t)$ is positive over the first half cycle $(0 < t < T/2)$. Then four time instants are considered so that, $0 < t_1 < T/2$, $t_2 = T/2 - t_1$,

$t_3 = T/2 + t_1$ and $t_4 = T - t_1$.

Applying equation (4.4) at times t_1 and t_2, t_3 and t_4, t_1 and t_4, t_2 and t_3 and by considering the periodic nature of the table kinematics, we have:

$$M_T(u(t_1))\ddot{u}(t_1) + F_E(u(t_1)) = \left[F_A(t_1) + F_A(t_2)\right]/2 \tag{4.5}$$

$$M_T(u(t_3))\ddot{u}(t_3) + F_E(u(t_3)) = \left[F_A(t_3) + F_A(t_4)\right]/2 \tag{4.6}$$

$$F_D(\dot{u}(t_1)) = \left[F_A(t_1) + F_A(t_4)\right]/2 \tag{4.7}$$

$$F_D(\dot{u}(t_2)) = \left[F_A(t_2) + F_A(t_3)\right]/2 \tag{4.8}$$

These equations are used to determine the most important characteristic of the shake table, which are the effective horizontal stiffness, the rigid mass and the dissipative force.

4.2.1.Estimation of elastic force and effective horizontal stiffness

For the particular case of triangular wave, the table acceleration is zero. Therefore equations (4.5) and (4.6) are reduced to:

$$\begin{aligned} F_E(\bar{u}(t)) &= \left[F_A(t) + F_A(T/2 - t)\right]/2 \\ \bar{u}(t) &= \left[u(t) + u(T/2 - t)\right]/2 \end{aligned} \qquad (0 < t < T/4) \tag{4.9}$$

and

$$\begin{aligned} F_E(u(t_3)) &= \left[F_A(t_3) + F_A(t_4)\right]/2 \\ \bar{u}(t) &= \left[u(t) + u(3T/2 - t)\right]/2 \end{aligned} \qquad (T/2 < t < 3T/4) \tag{4.10}$$

The elastic force and the stiffness of the system can be easily calculated using equations (4.9) and (4.10). Figure 7 shows the results obtained for tests T1 to T6. It can be seen that the elastic force is nearly zero for all six tests. All of the above considerations stand for displacements not close to the point of motion inversion, say for displacements within $0.8d_{max}$. At the inversion of the motion in fact, a number of factors influence the stability of the purely triangular response: (i) first, sudden inversion of motion cause the oil column to be excited, then (ii) when the table invert the motion the effect of the static coefficient of friction cause the typical 'stick-slip' effect, which at the motion inversion increases the force required to move the table (which is immediately decreased to the steady value as soon as motion begins), and (iii) finally the actual inversion of motion is not characterized by the theoretical step in the velocity function, but by a very steep velocity change. For all of these reasons, mass, friction and table stiffness are estimated out of the motion inversion region.

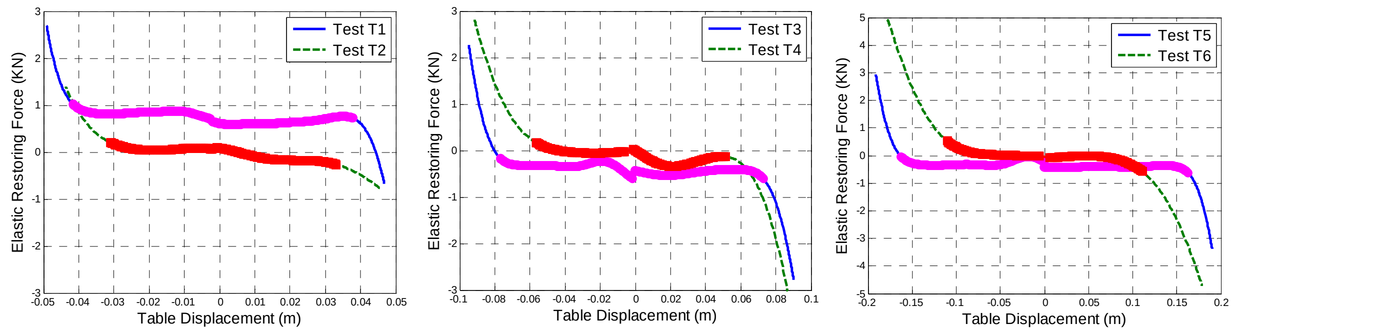

Figure 7: Estimates of the horizontal stiffness from triangular tests (T1 to T6)

4.2.2.Estimation of effective rigid mass

Since the elastic force previously calculated is essentially zero, equations (4.5) and (4.6) become as follows:

$$M_T(\bar{u}(t))\ddot{\bar{u}}(t_1) = [F_A(t) + F_A(T/2-t)]/2 \quad (0 < t < T/4) \qquad (4.11)$$
$$\ddot{\bar{u}}(t) = [\ddot{u}(t) + \ddot{u}\ (T/2-t)]/2$$

for $\ddot{\bar{u}}_x(t) > 0$ and

$$M_T(\bar{u}(t))\ddot{\bar{u}}(t) = [F_A(t) + F_A(3T/2-t)]/2 \quad (T/2 < t < 3T/4) \qquad (14.12)$$
$$\ddot{\bar{u}}(t) = [\ddot{u}(t) + \ddot{u}(3T/2-t)]/2$$

for $\ddot{\bar{u}}(t) < 0$.

Sinusoidal input waveform tests were used to calculate the effective rigid mass of the shake table.

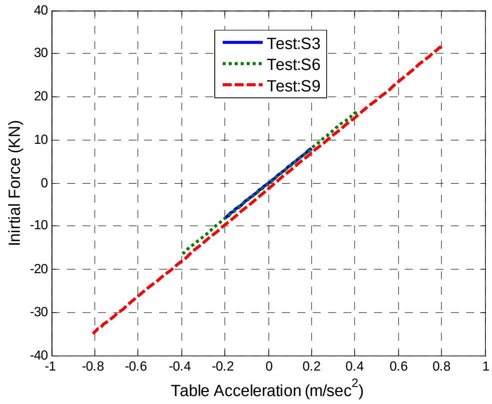

Figure 8: Estimate of effective mass obtained by loop approach from sinusoidal tests S3, S6 and S9

Figure 8 shows the relation between the table inertial force with the table acceleration for the sinusoidal tests S3, S6 and S9. As shown in the curves the inertial forces vary linearly with the total acceleration of the table for the three tests. The effective rigid mass of the table is the slope of the curves, and is found to be M_T = 41 tons: such value is in good agreement with the effective rigid mass of the table estimated with the random excitation in section *4.1.2*.

4.2.3.Estimation of the effective total dissipative force

For the estimation of the total dissipative force we can use either equations (4.7) and (4.8), or simply replace the values of the rigid mass and stiffness, found in the previous sections, in equation (4.4). The latter methods is used here, and equation (4.4) becomes for the case (M_T = 41 tons and F_E = 0 kN):

$$F_D(\dot{u}) = F_A(t) - M_T(u).\ddot{u}(t) \qquad (4.13)$$

In theory the triangular waves are preferred to sinusoidal waves for the dissipative force calculation, mainly for our case in which the elastic forces are essentially zero, so the dissipative force are simply equal to the total actuator force. However, this is not the case in practice, because of the occurrence of acceleration spikes at the time of change in velocity, producing additional forces difficult to quantify. For this reason sinusoidal tests are chosen for the dissipative force calculation.

Figure 9 shows the relationship between the total dissipative force and the table displacement as well as the relationship between the total dissipative force and the table velocity for sinusoidal tests S2, S4 and S5.

The experimental results are compared to the simulation of the friction model which uses the continuous viscoplastic friction law described by Bondonet and Filiatrault (1997) and defined as:

$$F_D = \mu Z \qquad (4.14)$$

where μ is the coefficient of friction, Z is hysteretic dimensionless parameter that is the solution to:

$$Y\frac{dZ}{dt} = \left(1 - Z^2\left(\beta + (1-\beta)sign\left(Z\dot{X}\right)\right)\right)\frac{dX}{dt} \tag{4.15}$$

where Y is the equivalent yield displacement, β is a dimensionless constant, and X is the displacement. Figure 9 indicates a good agreement between the measured and the simulated friction force namely for small velocities.

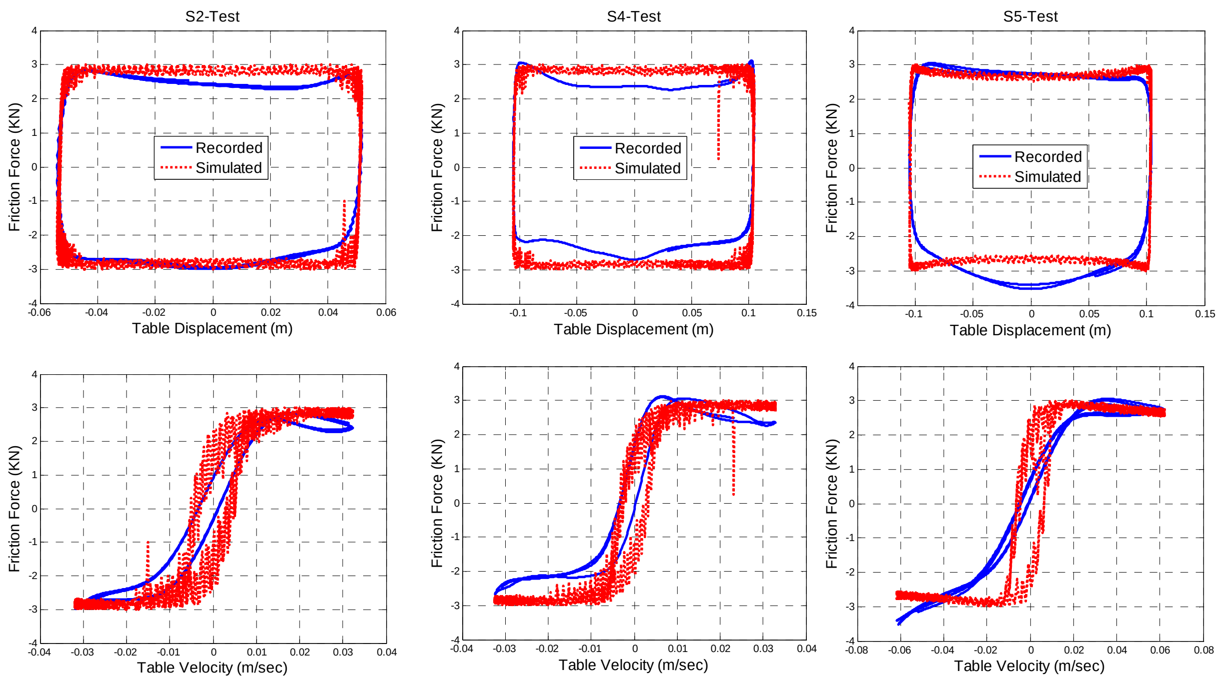

Figure 9: Comparison of recorded and simulated total dissipative forces *vs* table displacement and velocity for tests S2, S4 and S5

5. TABLE TRANSFER FUNCTION ESTIMATION

At this stage, all the parameters involved in the numerical model of the EUCENTRE TREES Lab shake table are clearly defined. The transfer functions estimated by the Simulink model for both the cases of ideal linear actuator and real actuator are compared to the experimental one.

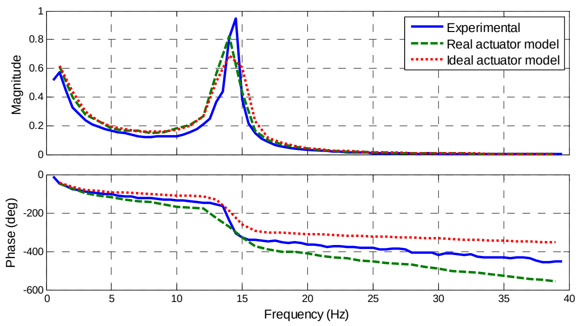

Figure 10: shake table transfer functions

Figure 10 shows that the analytical transfer functions obtained for both the models of the shake table are in very good agreement with the experimental one. Moreover, the magnitude of the transfer function shows a big sharp peak at a frequency of 14.16 Hz, which corresponds to the oil column natural frequency. The oil column is extremely important factor in the behavior of transfer function phase: Figure 10 shows how the inversion of phase in the transfer function occurs exactly in correspondence with the oil column resonant frequency.

CONCLUSION

Two numerical models of the TREES Lab at Eucentre seismic shake table were established using the Matlab system-modeling package Simulink. The main difference between the two models is in the servovalve actuator block system, which is the most critical part of the shake table because it is the part where mechanics, hydraulics and electronics are fully involved and interact.

In the first model a linear approximation of the servovalve actuator system was used; the second model instead used a realistic detailed nonlinear system. The rest of the shake table system was modeled in the same way.

A comprehensive set of tests, random and periodic, were conducted on the real shake table, in order to determine experimentally all of the parameters involved in the two models.

The particularity of this study lies in the two approaches used to calculate the shake table components. In fact, in the first approach explicit identification of these components using random tests was adopted. For the second approach the shake table and its subsystems were represented by a basic simplified dynamic equilibrium equation. Periodic tests were used to identify the table system's effective horizontal stiffness, rigid mass and total dissipative force.

Finally, the two shake table numerical models were validated by comparing their transfer functions with the one obtained experimentally on the real system.

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