DEVELOPMENT OF A PERIOD DATABASE FOR BUILDINGS IN MONTREAL USING AMBIENT VIBRATIONS

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ABSTRACT:

The fundamental period formulae in the National Building Code of Canada 2005 (NBCC 2005) are based on a database comprised of structures in California whose fundamental periods were measured during several earthquakes. Since each earthquake produced ground motions of different amplitudes at each site, and since the fundamental period varies with the excitation amplitude, this database is somewhat inconsistent. Further, the empirical formulae fit these data quite poorly, and past research has suggested that care be exercised before using the California period data in other parts of the world. This paper will discuss an ongoing research project which aims to develop a period database for the city of Montreal, Canada, using ambient vibration measurements and the Frequency Domain Decomposition method. This database will represent a consistent data set for the low-amplitude fundamental periods of buildings in Montreal, which will be used to evaluate the NBCC 2005 formulae, and develop improved period equations. This database will also be used for seismic vulnerability studies in Montreal and as a pre-damage benchmark for the measured buildings.

KEYWORDS: Fundamental period, building code, ambient vibrations, frequency domain decomposition

1. INTRODUCTION

Structural dynamics principles indicate that the fundamental period plays a prominent role in anticipating the forces to which a structure will be subjected during earthquake ground motions. In the 2005 edition of the National Building Code of Canada (NBCC 2005), seismic hazard is represented using uniform hazard spectra (UHS), and soil amplification effects, as well as higher mode effects, are accounted for using period-dependent relations, thus highlighting the importance of accurately determining the fundamental period of a structure during the design stage. Most building codes, and more specifically the NBCC 2005, include simple empirical formulae based on building geometry for the evaluation of the fundamental sway period of structures as a necessary step in the determination of horizontal seismic design forces. This is important for new designs since this period can not be accurately determined before the structure is designed. Since spectral acceleration ordinates in UHS tend to decrease with increasing period, building codes aim to provide equations that tend to underestimate the fundamental period, thereby leading to conservative design forces.

The period formulae in the NBCC 2005 are based on a database comprised of structures in California whose fundamental periods were measured during several earthquakes, from the 1971 San Fernando to the 1994 Northridge earthquakes. However, there is large variability in the data, and these equations fit the data rather poorly. Further, the fundamental period of a structure tends to increase with increasing excitation amplitude (Udwadia and Trifunac 1974); therefore, since each of the earthquakes produced ground motions of significantly different amplitudes at each location, it seems that this data set is somewhat inconsistent. Moreover, Goel and Chopra (1997; 1998) warned that caution should be exercised before using these data in other parts of the world; the reason being that different soil conditions, seismicity, and design and construction practices may affect the measured fundamental periods.

In this study, ambient vibration measurements and the Frequency Domain Decomposition method (FDD) are used to obtain the dynamic characteristics of a large number of buildings in Montreal. The primary objective is to provide a consistent data set for the low-amplitude fundamental periods of buildings in Montreal, which will be used to evaluate current period equations, and develop conservative period equations for design purposes. Since codes aim to underestimate the fundamental period, and since the fundamental period tends to increase with increasing excitation amplitude, it seems reasonable, for code purposes, to use periods obtained from ambient vibration studies, in which the excitation is generally quite weak. Better estimates of the actual periods expected during the design ground motion could then come from magnitude-period elongation relations, the development of which is left for future studies. There are also a few secondary objectives to developing such a period database. It will also be used, in conjunction with microzonation studies, to evaluate the seismic vulnerability of the targeted structures. Also, the database will provide a valuable pre-damage benchmark for these structures. In the case of significant ground motion (or any other important event that could lead to structural damage), comparing the results of ambient vibration studies before and after the event may provide insight into whether significant (and perhaps not visible) damage has occurred. Finally, comparing this data with the results from ambient vibration measurements for similar buildings in other parts of the world may provide insight into the importance of local factors such as seismicity, geology, and design and construction practices.

2. BACKGROUND

2.1 Period Formulae

In the NBCC 2005, the following empirical formulae are suggested:

$$T = 0.085\ h^{3/4} \quad \text{for steel moment frames (steel MRF),} \quad (2.1a)$$
$$T = 0.075\ h^{3/4} \quad \text{for reinforced concrete moment frames (R/C MRF),} \quad (2.1b)$$
$$T = 0.1\ N \quad \text{for other moment frames (MRF),} \quad (2.1c)$$
$$T = 0.025\ h \quad \text{for braced frames,} \quad (2.1d)$$
$$T = 0.05\ h^{3/4} \quad \text{for shear walls (SW) and other structures,} \quad (2.1e)$$

where T, h and N represent the fundamental period (in seconds), the building height (in metres) and the number of stories, respectively. The above equations were adopted based on the recommendations of Saatcioglu and Humar (2003), with the exception of Eqn 2.1d, which was proposed by Tremblay (2005). Saatcioglu and Humar (2003) adapted the MRF and SW formulae for SI units, from those proposed in previous American building codes (BSSC 1997). Eqns 2.1a, b, and e were based respectively on 17 steel MRF, 14 R/C MRF, and 9 R/C SW buildings, whose periods were measured during the 1971 San Fernando earthquake. The form of these equations, with the exponent of ¾ applied to the building height, was initially developed using Rayleigh's method, and the coefficients were adjusted based on constrained regression analyses for the different building types (Goel and Chopra 1997; 1998). Recognizing that these equations were based on limited data, which exhibited significant scatter, and that period data from more recent California earthquakes were available, Goel and Chopra (1997; 1998) evaluated the existing formulae using an expanded database of 106 buildings, classified according to their lateral structural system.

For steel and R/C MRF buildings, they performed a second set of regression analyses using this expanded database. The candidate equations were all of the form:

$$T = \alpha\ h^{\beta}, \quad (2.2)$$

where α and β were the parameters to be determined from regression analyses. Two types of analyses are particularly relevant to this discussion: a *constrained* regression analysis was performed, using $\beta = ¾$, to find the best fit using an equation similar to Eqns 2.1a and 2.1b, as well as an *unconstrained* regression analysis to find the best fit among all candidate equations (Eqn 2.2). Figure 1 shows the measured periods (in the expanded database) plotted against building height for steel and R/C MRF buildings. The two solid lines on each graph represent the NBCC 2005 formula and the best fit curve obtained from the constrained regression, and the dashed line represents the best fit from the unconstrained regression.

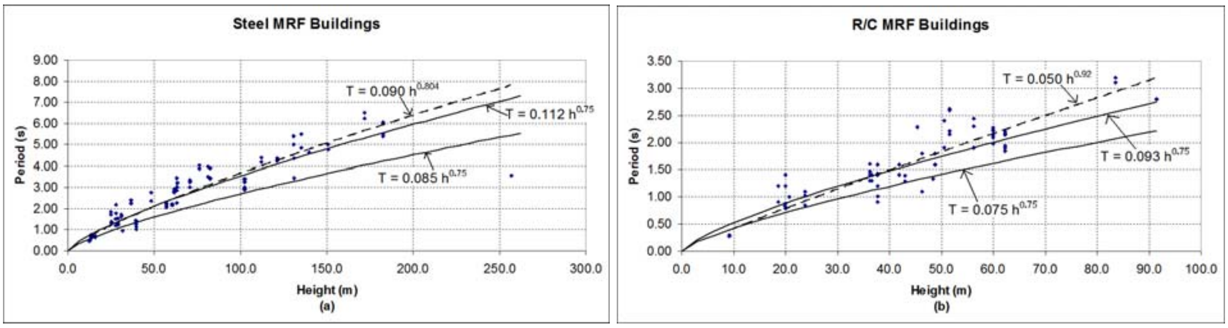

Figure 1: Measured periods and empirical formulae for (a) steel and (b) R/C MRF buildings (source: Goel and Chopra 1997)

Table 2.1: Results of regression analyses

Structural system	Constrained regression			Unconstrained regression		
	Equation (best fit)	s_e	R^2	Equation (best fit)	s_e	R^2
Steel MRF	$T = 0.0112\ h^{0.75}$	0.236	0.857	$T = 0.090\ h^{0.804}$	0.233	0.876
R/C MRF	$T = 0.093\ h^{0.75}$	0.229	0.769	$T = 0.050\ h^{0.920}$	0.209	0.858
R/C SW	$T = 0.048\ h^{0.75}$	0.424	0.438	$T = 0.016\ h^{1.076}$	0.392	0.653

It is evident that the data exhibit significant scatter, and that the code equations fit the data rather poorly. Table 2.1 shows the results of the regression analyses. For steel and R/C MRF buildings, comparing the standard error of estimate (s_e) and the coefficient of determination (R^2) - which are both measures of the goodness of fit of a linear regression model - for the best fit curves from the constrained and unconstrained regression analyses, it is clear that the fit can be improved, slightly for steel and significantly for R/C MRF buildings, by increasing the exponent in Eqn 2.2. Goel and Chopra (1997) rounded the exponents and suggested the following formulae to determine the fundamental period of R/C and steel MRF buildings:

$$T = 0.0055\ h^{0.90} \quad \text{for R/C MRF buildings, and} \qquad (2.3a)$$
$$T = 0.0011\ h^{0.80} \quad \text{for steel MRF buildings.} \qquad (2.3b)$$

For R/C shear wall buildings, they found that Eqn 2.1e was grossly inadequate. The scatter in the data is evident in Figure 2, which also shows the NBCC 2005 equation, the best fit curve from a constrained regression (β = ¾), and the best fit curve from an unconstrained regression. As a result, the best fit curves from the constrained and unconstrained analyses both fit the data quite poorly, as evidenced by the low values of R^2 in Table 2.1. These analyses, as presented here, were not directly discussed by the authors; rather, they considered the scatter in the data and suggested that the building height alone was not sufficient to determine the fundamental sway period of such structures. They proposed a more complex formula, based on Dunkerley's method (Jacobsen and Ayre 1958), considering both shear and flexural deformations, and calibrated using regression analysis on the measured data. According to Goel and Chopra, this equation reduced the standard error of estimate to 0.143. But the increase in accuracy came at the expense of simplicity; the new formula requiring the estimation of parameters that may be unknown at the beginning of the design process. For simplicity, the NBCC 2005 did not adopt the suggestion of Goel and Chopra and retained a simple formula based on height alone (Eqn 2.1e).

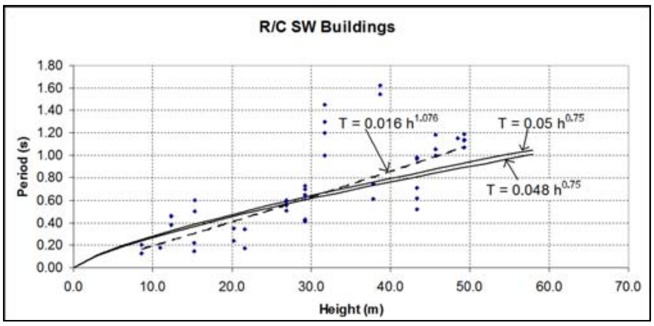

Figure 2: Measured periods and empirical formulae for R/C SW buildings (Goel and Chopra 1998)

Eqn 2.1c, for other MRF, has been included in building codes since at least the 1960s. In the NBCC, it was initially used for all MRF buildings (NBCC 1970). This formula is being slowly phased out of the building code as more research is performed on particular structural systems; however it remains for other types of systems.

For braced frames, Tremblay (2005) considered previous work by Housner and Brady (1963), which suggested that for most shear buildings a simple expression with the period varying linearly with building height would yield better estimates than those with the period varying linearly with height and inversely with the square root of the length of the lateral load resisting system, $\sqrt{D_s}$, as was the norm at the time. Also, there was much confusion among practicing engineers about the interpretation of the length of the lateral load resisting system. The change was never incorporated into the NBCC, prior to 2005, therefore Tremblay (2005) undertook an analytical study to improve the period formula for braced steel frames. He first reviewed a few experimental studies where the periods of braced steel frames were reported and compared to analytical predictions, and suggested that the fundamental period of braced steel frames could be accurately predicted from analytical models. He then used a large database of braced steel frames reported in the literature, for which the fundamental periods were analytically computed, to examine the validity of previous building code formulae. Tremblay then presented an extensive parametric study to establish a conservative empirical formula for braced steel frames. In this study, all structures were designed using the equivalent static procedure of the NBCC 2005 provisions and the periods were computed using a closed-form solution based on Rayleigh's method.

This study confirmed that a linear variation of period with building height was more appropriate for braced frames, and that the implied precision of the formula involving $\sqrt{D_s}$ was not justified. Eqn 2.1d, adopted by the NBCC 2005, was found to provide a conservative estimate of the fundamental period for all steel braced frames (both concentrically and eccentrically braced). However, Tremblay (2005) cautioned that field measurements were required to validate these analytical findings.

2.2 Ambient Vibration Tests (AVT)

The modal (or dynamic) properties of a structure can be obtained by several different testing methods, such as free vibration, forced vibration, earthquake response, and ambient vibration tests (AVT). Each testing method has advantages and limitations. Since the dynamic properties, particularly the natural periods, tend to vary with the intensity of the excitation (Udwadia and Trifunac 1974), earthquake response tests represent the best means to predict the dynamic properties of a structure responding to strong ground motions. However, to develop an interesting database for Montreal, this would require permanent instrumentation of many buildings, which in turn would require a very large number of sensors, which are not currently available. Further, it would require the occurrence of significant ground motion, a rare event in the region of relatively infrequent seismic activity near Montreal. AVT, on the other hand, allow several buildings to be tested at different times, thereby offering the opportunity to create a large database with limited resources. In this study, AVT are chosen instead of free or forced response tests since the latter require large equipment to displace or artificially excite a structure.

AVT have been widely touted as a practical modal identification technique, mainly due to the easy and inexpensive setup required, as well as the fact that the modal properties are obtained under the actual operating conditions of the structure. They have been shown to yield good estimates of the natural frequencies and mode shapes under normal operating conditions, but estimates of modal damping ratios are not very reliable (Brownjohn 2003). Trifunac (1972) showed that the properties obtained from ambient and forced vibration tests compared very well when the excitation amplitudes were of the same order of magnitude, and this was confirmed by several other authors. AVT rely on low-amplitude excitation from ambient sources, such as wind and micro-tremors, to drive building motion, which is measured and analyzed to obtain the structure's dynamic properties. In this type of analysis, the input forces are not measured; therefore, to extract the modal parameters of the system, the excitation is usually assumed to have the properties of a broadband, stationary Gaussian white noise. This implies that the excitation has approximately equal energy content throughout the frequency range of interest. And therefore, the dynamic properties of the system can be inferred directly from the measured response, without any consideration of the forces responsible for the motion.

3. EXPERIMENTAL PROCEDURE

For this study, two Lennartz tri-axial LE-3D/5s velocity transducers and LEAS CitySharkII data acquisition systems (DAS) are used. The sensors have an operating range of 0.2 – 40 Hz. Each transducer measures velocity in three orthogonal directions and relays the information to the corresponding DAS. Typically, building measurements are taken on at least eight floors, and at three different locations on each floor, along a symmetry axis of rigidity. Since most of the targeted buildings have simple geometry, this axis of rigidity is usually assumed to coincide with an axis of symmetry of the building plan. Such a setup allows the identification of the sway and torsional modes. Since only two sensors are used at present, a reference sensor is kept near the top of the building, away from the centre of rigidity, where it is assumed that most low frequency modes should be excited, and a roving sensor is moved around the building to measure the different points of interest.

The data obtained from these tests are a set of velocity time histories for the reference and roving sensors, for each measurement setup. The records from the reference sensor are used to create a consistent data set for all points. Using the consistent data set, the Frequency Domain Decomposition (FDD) method is then used to extract the approximate modal properties from the measured responses. This method is arguably the most popular frequency domain modal identification technique currently in use. It is described in some detail in a number of papers, notably Brincker et al. (2001b), therefore only its essential aspects are discussed here.

FDD operates on the spectral density matrices of the measured responses. The spectral density of a signal is a measure of the energy content per unit frequency, and many methods exist to estimate this quantity, the most common of which uses a modified periodogram approach with a Hanning leakage reduction window (Bendat and Piersol 2000). This procedure yields the spectral density at discretely spaced frequencies. At each frequency, the spectral density matrix is assembled by considering the spectral density between each pair of channels. For example, the element of the spectral density matrix at frequency *f*, in row *i*, and column *j*, represents the spectral density at the same frequency between the records for degrees-of-freedom (DOF) *i* and *j*. The values along the main diagonal – the auto-spectral densities – are real quantities; while the off-diagonal terms – the cross-spectral densities – are generally complex. The crucial step in FDD is then to perform the singular value decomposition (SVD) of each spectral density matrix, in the following way:

$$[P] = [U]\,[S]\,[U]^H, \qquad (3.1)$$

where [P], [U] and [S] are respectively the response spectral density, singular vector, and singular value matrices, and ^{H} denotes the hermitian transformation (complex-conjugate transpose). The singular values are listed in descending order along the main diagonal of [S] and are always real, non-negative quantities. On the other hand, the singular vectors are generally comprised of complex quantities. The SVD is an approximation to the modal decomposition of the spectral density matrix (Brincker, et al. 2001b). Hence, at a resonance frequency, the singular vectors contained in [U] are an approximation to the mode shapes, and the corresponding singular values provide an estimate of the contribution of each mode to the overall energy at that frequency.

The first few singular values are plotted against frequency and the peaks are identified as resonance frequencies. The mode shapes are then estimated from the corresponding singular vectors. Figure 3 shows a plot of the first two singular values for a 6-DOF system: the blue (upper) and green (lower) lines represent the first and second singular values, respectively. Six resonance frequencies can be identified from the peaks in the first singular value. The first two, at frequencies of 0.61 and 0.98 Hz, represent the fundamental frequencies in each lateral direction. They are very well defined as they represent a large portion of the system's vibrational energy, but it is possible to estimate higher mode frequencies as well. Also, studying the second singular value helps identify closely-spaced modes (Brincker, et al. 2001b). The mode shapes are estimated as explained above and the modal damping ratios can be estimated by taking a portion of the SV curve around each peak back to the time domain and calculating the logarithmic decrement (Brincker, et al. 2001a).

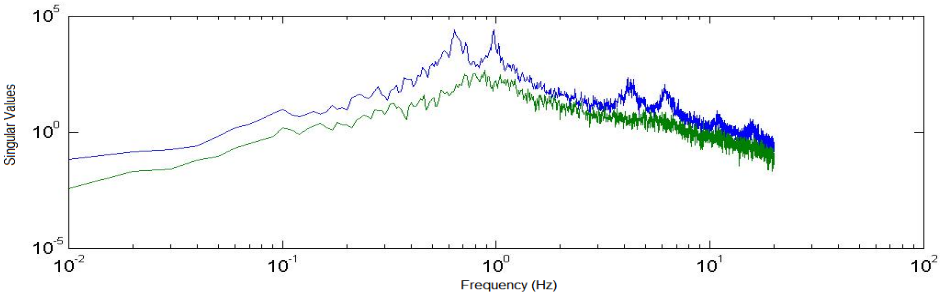

Figure 3: Plot of first two singular values for a 6-DOF system

4. PRELIMINARY RESULTS

So far, the dynamic properties of ten buildings, most of which having a lateral force resisting system of the R/C SW type, have been obtained by the method described above. Table 4.1 shows the overall dimensions of the different buildings, the type of lateral force resisting system, the fundamental period in each lateral direction, as obtained from ambient vibration tests, and the period calculated from the appropriate NBCC 2005 formula.

Table 4.1: Summary of building periods obtained from ambient vibration tests

Building	Height (m)	Longitudinal width L_x (m)	Transverse width L_y (m)	Lateral Force Resisting System	Fundamental period (s)		NBCC 2005 period (s)
					Longitudinal	*Transverse*	
I	50	N/A	N/A	R/C SW	1.00	1.25	0.94
II	51	33.5	30.5	R/C SW	0.67	0.69	0.95
III	62	37.5	37.5	R/C SW	0.95	1.25	1.10
IV	78	41	41	R/C SW	1.54	1.89	1.31
V	104	43	38	R/C SW	2.00	2.22	1.63
VI	121	46	46	R/C SW	1.92	2.04	1.82
VII	122	55	37	R/C SW	2.08	2.38	1.84
VIII	143	46	46	R/C SW	2.50	2.50	2.07
IX	105	77.5	29.3	Steel MRF	1.92	3.00	2.79
X	122	N/A	N/A	Masonry walls	1.20	1.75	2.80

Considering the limited amount of data, it is only possible to comment on the results for R/C SW buildings. Figure 4a shows the measured fundamental periods plotted against the building height, as well as two solid lines representing the NBCC 2005 equation for R/C SW buildings (Eqn 2.1e) and the best fit from a constrained regression (β = ¾), and a dashed line representing the best fit from an unconstrained regression of the same form as Eqn 2.2. Qualitatively, it seems that the NBCC formula provides a generally conservative estimate (lower bound value) of the fundamental period that fits the data reasonably well. However, the coefficient of determination for the best fit from the constrained regression is relatively low (0.68). For the unconstrained regression, the coefficient of determination of the best fit curve increases to 0.83. It is worth noting however that the NBCC formula should represent a conservative estimate of the building period at the excitation level of the design earthquake: this implies that the building and the underlying soil may have undergone some inelastic deformations and the fundamental period should be larger than what is measured by ambient tests.

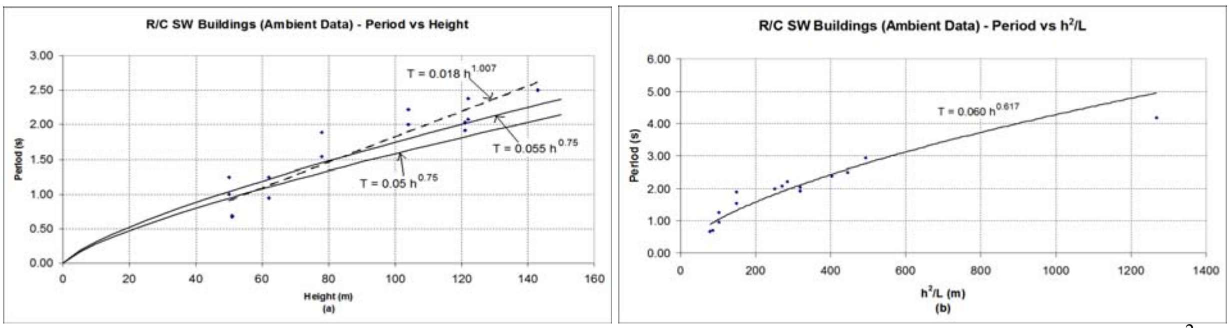

Figure 4: Plots of ambient vibration data for R/C SW buildings: (a) Period vs height; (b) Period vs h^2/L

Both of these equations, however, only account for the height, the type of material and the lateral load resisting system in predicting a structure's fundamental period. Figure 4a confirms that the height is indeed an important factor in determining the fundamental period of a structure, but many other factors also have an influence. In order to examine different equations, let us first consider the theoretical fundamental period of a continuous, prismatic, flexural cantilever beam made of linear elastic material, given by Weaver et al. (1990):

$$T = \frac{2\pi}{3.515}\sqrt{\frac{\rho A h^4}{EI}} = \frac{2\pi}{3.515}\sqrt{\frac{\rho}{E}}\sqrt{\frac{Ah^4}{I}}, \tag{4.1}$$

where ρ is the density, E is the modulus of elasticity, A is the cross-sectional area, h is the length of the beam (or the height of the building) and I is the second moment of area of the cross-section. Of course, an actual building cannot be truly represented by a continuous, prismatic, flexural beam. But, it seems reasonable to try to find simple proxies for the different variables in the above formula in an attempt to improve the code formula. If we consider that every building having a particular lateral load resisting system is made of the same material (same density and same modulus of elasticity), and if we approximate the second moment of area using the plan dimensions of the structure (rather than a more rational approach based on the actual lateral load resisting elements), then the above equation can be greatly simplified. For a building having a rectangular footprint, the above equation simplifies to:

$$T = c\, h^2/L, \tag{4.2}$$

where c is a constant and L is the length of the building in the direction under consideration. Therefore, a regression was performed on T versus h^2/L. Figure 4b shows that the results were not quite as expected. On the one hand, the best fit curve seems quite good, as evidenced by the relatively high value of the coefficient of determination (0.89). This represents a significant improvement over the NBCC 2005 formula (0.69) and a slight improvement over the best fit curve obtained from the previously discussed unconstrained regression analysis (0.83). On the other hand, it was expected that the exponent obtained would be close to unity. But, the exponent for the best fit curve is 0.62. Though an effort was made to account for more of the physical parameters of importance, it seems that the proxies used for the various parameters, or perhaps the assumption of a continuous beam, are inappropriate. For the time being, all that can be said is that the use of an equation based on a continuous beam assumption offers the potential of improving the current code formulae, without adding undue complexity. However, this analysis was performed on a relatively small data set. Hopefully, gathering more data will provide insight into which parameters play the largest role in the fundamental period of actual buildings, and aid in developing more accurate equations. One such parameter, which was not discussed in this paper, and that may have an important effect on the apparent fundamental period (from AVT), is the nature of the underlying soil.

5. CONCLUSION

In this paper, an ongoing research project involving the development of a period database for multi-storey buildings in Montreal, Canada, using ambient vibrations and the Frequency Domain Decomposition method, was discussed. It was shown that the current period database used in the National Building Code of Canada is somewhat inconsistent since it is based on building periods measured during several earthquakes in California, each of which produced ground motions of different amplitudes at each location, and since the fundamental period of a structure typically elongates with increasing ground motion intensity. Further, there is little evidence that using California period data for the seismic design of structures in Montreal is appropriate. The aim of the project is to use this new database to evaluate the current NBCC period equations, and develop improved equations for the low-amplitude fundamental periods of buildings in Montreal. These will provide a conservative estimate for design purposes. The development of relations between the excitation magnitude and the fundamental period, which would be necessary for a more accurate prediction of a structure's fundamental period during its design ground motion, is left for future studies. This approach seems to be a rational way to separate the variability in the data that arises from using simple equations based on building geometry, from the variability that comes from using period data obtained from ground motions of significantly different amplitudes. This database will also serve in vulnerability studies and may be used as a pre-damage benchmark in future damage detection studies of the targeted buildings.

Preliminary results were presented for ten buildings, eight of which were of the R/C SW type. The data from these eight structures were used to evaluate the current NBCC formula for R/C SW buildings. A theoretical equation based on a continuous beam assumption was then presented, and calibrated by regression analysis. Comparing the two equations, it seems that such an approach has the potential of improving the prediction of the fundamental period, without adding undue complexity. Admittedly, this analysis was based on very limited data, and more buildings must be surveyed before proposing any changes to the current equations.

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