Detection of Reduced Stiffness in Lateral Load Resisting Systems due to Seismic Damage

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ABSTRACT:

Optimality criterion approaches have been shown by the authors to be a promising means for damage detection in the absence of error. The work presented expands on these previous studies and establishes the effects of modeling error on the accuracy of the damage detection method. When supplied with appropriately located static measurements, the algorithm is able to accurately detect areas of stiffness reduction of two dimensional structures in the presence of modeling error. Using finite element models and measured static displacements, changes in the stiffness parameters are found. These changes in stiffness indicate locations of potential damage.

KEYWORDS: Damage detection, optimality criterion, static response structural health monitoring

1. INTRODUCTION

Seismic events, even small ones, can be detrimental to civil structures and their occupants. Even well designed structures can experience damage when construction is faulty or unexpected loads are experienced. Old structures are especially susceptible to seismic damage as their construction may not meet the current standards for seismic design. Additionally, local damage that is present prior to a seismic event, such as fatigue or corrosion, can lead to unexpected behavior both during and following such an event. If seismic damage goes undetected for an extensive amount of time it can ultimately lead to large repair costs, or worse, catastrophic failure of the structural system. Prompt detection of damage is essential in order to assure the safety of civil structures.

Traditional damage detection relies on visual inspection of areas suspected of damage, which can often be inefficient and unreliable. Non-invasive damage detection helps to alleviate some of the disadvantages of visual inspection. By comparing the structural response of a structure both pre and post damage, areas of potential damage can be identified without the unnecessary destruction of nonstructural elements. Therefore, damage can be detected early and accurately, reducing repair costs.

A new damage detection algorithm is presented that utilizes static response data to accurately detect areas of stiffness reduction. Using Optimality Criterion optimization, areas of stiffness reduction can be detected using minimal computing effort. Displacement measurements are compared both pre and post damage to accurately identify elements that have sustained damage. Previous work by the authors has shown that the algorithm can accurately detect stiffness reductions in the absence of modeling error. The work presented here helps to establish the effect modeling error has on the damage detection algorithm. Through presented examples the efficiency of the algorithm will also be illustrated.

2. THEORY

For the purpose of comparison, the damage detection problems will be solved using two different optimization methods. The first method employed is the Optimality Criterion (OC) algorithm. The proposed algorithm has been shown to be an effective structural optimization tool, for example [1], and is known for its ability to solve nonlinear optimization problems efficiently using only first order gradients (i.e. without calculation of a Hessian matrix). The second method employed is the Conjugate Gradient (CG) method. The reason for the two different methods is to provide a means of comparing the efficiency of the OC algorithm with another method that is widely accepted and recognized.

2.1 OC Algorithm

The OC algorithm is a constrained nonlinear optimization method derived from the Karush-Kuhn-Tucker conditions for optimality. The derivation and theoretical background of the OC algorithm can be found in [2]. The damage detection problem is hinged on the linear force displacement relation:

$$\{P\} = [K]\{u\} \tag{2.1}$$

where P is a defined load vector, K is the global structural stiffness matrix, and u is a vector of nodal displacements. For damage detection, the load vector is fully defined by the loads applied during the testing procedure. For practical reasons, the displacement vector can only be partially defined by measured displacements resulting from the applied loads. The linear relation of Equation 2.1 is used to develop constraints that help detect damage. The OC algorithm recreates a stiffness matrix that closely mimics the change in displacement measurements of the partially defined displacement vector before and after damage. The design variables become the stiffness parameters that comprise the stiffness matrix of the identification model, moment of inertia for bending elements or cross sectional area for truss elements. The problem used to detect damage becomes:

$$\begin{aligned} &Minimize \quad \Phi = \frac{1}{2}\sum_{r=1}^{p}\{u_r\}^T[K]\{u_r\} \\ &s.t. \quad g_j = u_{q,r} - u_{q,r}^m = 0 \\ &x_i^L \le x_i \le x_i^U \quad for\ i = 1,2,\ldots,n \end{aligned} \tag{2.2}$$

where $\{u_r\}$ is the calculated displacement vector of the identification model for load case r, $u_{q,r}$ is the calculated displacement of the identification model for load case r at degree of freedom q, $u_{q,r}^m$ is the measured displacement of the analytical model for load case r at degree of freedom q, and p is the total number of load cases. The terms x_i^u and x_i^l are the upper and lower bounds on the i^{th} design variable, respectively. Typically the upper limit is given by the undamaged stiffness terms and the lower bound is zero in order to assure that all stiffness parameters remain positive, real. The objective function, Φ, represents the strain energy of the identification model and is chosen based on the principle of minimum potential energy.

2.2 CG Algorithm

The conjugate gradient method is a common unconstrained optimization method that has been employed for many different applications. The method applied here utilizes only first order gradients. The theory can be found in any introductory optimization textbook and is not repeated here (the reader is directed to [3] for theory). The objective function used in the CG algorithm is the norm of the Displacement Output Error Function developed by Sanayei and Imbaro, who showed it to be effective for damage detection problems. To form the objective function, Equation 2.1 can be partitioned into measured and unmeasured degrees of freedom, giving:

$$\begin{Bmatrix} P_m \\ P_u \end{Bmatrix} = \begin{bmatrix} K_m & K_{mu} \\ K_{um} & K_u \end{bmatrix} \begin{Bmatrix} u_m \\ u_u \end{Bmatrix} \qquad \textbf{(2.3)}$$

where *m* indicates measured degrees of freedom and *u* indicates unmeasured degrees of freedom. Again, the design variables are the stiffness parameters of each element in the identification model. After condensing out unmeasured degrees of freedom and rearranging, the Displacement Output Error Function becomes [4]:

$$Error = \left(K_m - K_{mu}K_u^{-1}K_{um}\right)^{-1}\left(F_m - K_{mu}K_u^{-1}F_u\right) - u_m \qquad \textbf{(2.4)}$$

The objective function is formed by calculating the norm of Equation 2.4, becoming:

$$\begin{aligned} &Minimize \quad \Phi = \left\| \left(K_m - K_{mu}K_u^{-1}K_{um}\right)^{-1}\left(F_m - K_{mu}K_u^{-1}F_u\right) - u_m \right\| \\ &x_i^L \le x_i \le x_i^U \qquad for\ i = 1,2,\ldots,n \end{aligned} \qquad \textbf{(2.5)}$$

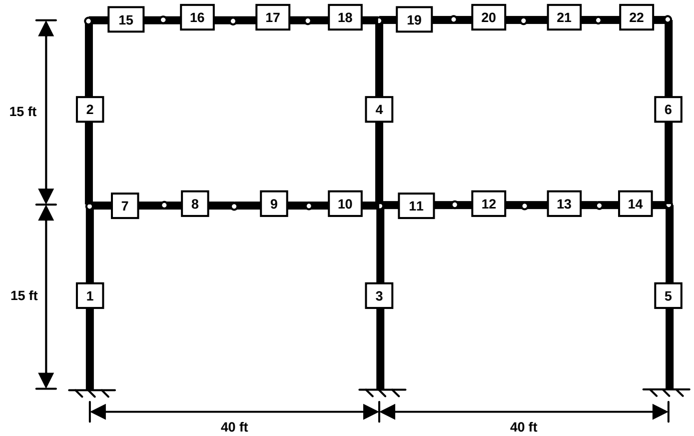
where x_i^u and x_i^l are the upper and lower bounds on the i^{th} design variable, respectively.

3. DAMAGE DETECTION PROBLEMS

The OC algorithm is used as the primary damage detection algorithm. To measure the pros and cons and to provide perspective on the efficiency of the algorithm, the damage detection problem will be solved using the CG algorithm as well.

3.1 Two Bay, Two Story Moment Frame

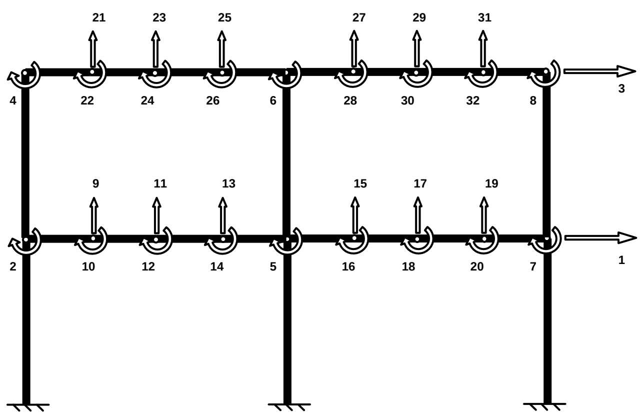
Before an algorithm can be implemented using real structural data, it must first demonstrate the ability to identify stiffness reductions when the response data is free of noise. The OC algorithm will be tested on the two bay, two story moment frame shown in Figure 1. To gain a sense of the efficiency of the algorithm, both modeling error and measurement error have been omitted. This is accomplished by using the analytical model as the identification model as well. The model is composed of 22 bending elements, creating a total of 32 degrees of freedom. Axial stiffness is ignored and so deflections are resisted through flexural stiffness of the elements, in turn the design variables for the problem are the moment of inertia of each element. The damage state that will be detected is also shown in Figure 1.



Member	Stiffness Reduction
7-10	33%
21-22	47%

Figure 1 Analytical Model of Two Bay, Two Story Moment Frame

The measurement data is simulated by using the damaged stiffness parameters in the analytical finite element model. To detect damage, the algorithm is initialized using a set of stiffness parameters that closely mimics the structural response in the healthy state. Inherently it is assumed that an appropriate identification method has been performed to determine these suitable stiffness parameters. To detect damage, 12 load cases are used with 14 total displacement measurements. The structural degrees of freedom and measurement locations are shown in Figure 2. Each load case applies a 10 kip load to the degree of freedom listed. Each load case applies only a single load.



Load Case	DOF Loaded	DOF Measured
1	9	11
2	11	11
3	13	1, 11
4	15	17
5	17	17
6	19	17
7	21	23
8	23	23
9	25	3, 23
10	27	29
11	29	29
12	31	29

Figure 2 Moment Frame Degrees of Freedom and Load Case Data for Damage Detection

Each displacement measurement is used as a constraint in the OC algorithm and as a portion of the objective function in the CG algorithm. After initializing each algorithm with the suitable set of healthy stiffness parameters, each algorithm is allowed to iterate until an optimal solution is found.

The results of the OC algorithm are shown in Figure 3. Six iterations were required to find an optimal solution. It is seen that each constraints were satisfied signifying a feasible solution was found. A sharp increase in both the objective and constraints is apparent during the first iteration as the algorithm attempts to find an optimal search path, typical of many OC problems. After the first iteration the objective function is minimized. The stiffness reductions were detected correctly by the algorithm. Again, data error has been omitted; therefore exact detection of stiffness reductions is an expected and promising result.

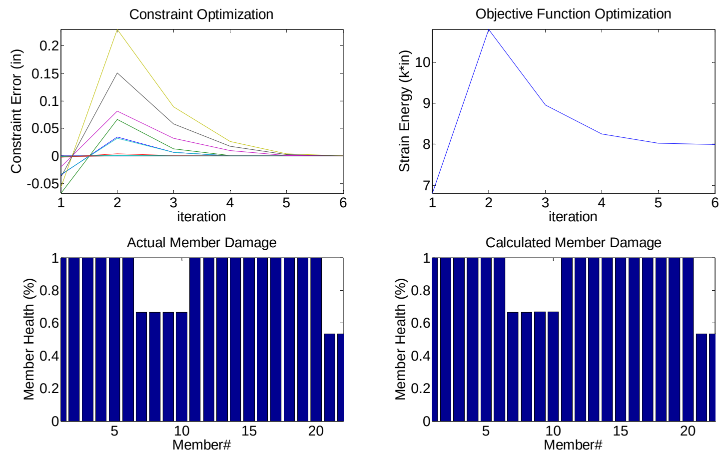


Figure 3 OC Algorithm Damage Detection Results

Figure 4 shows the optimization results for the CG algorithm. The CG algorithm is an unconstrained optimization method but the constraint plot is presented to illustrate the search path of the algorithm. To be clear, the constraints are not used in the actual algorithm. The figure shows that the objective experienced a continual decrease, however the constraints took a much less direct path to the optimum solution than the OC algorithm. Ultimately the CG algorithm required 96 iterations to reach an optimal solution and the stiffness reductions were detected to a high degree of accuracy.

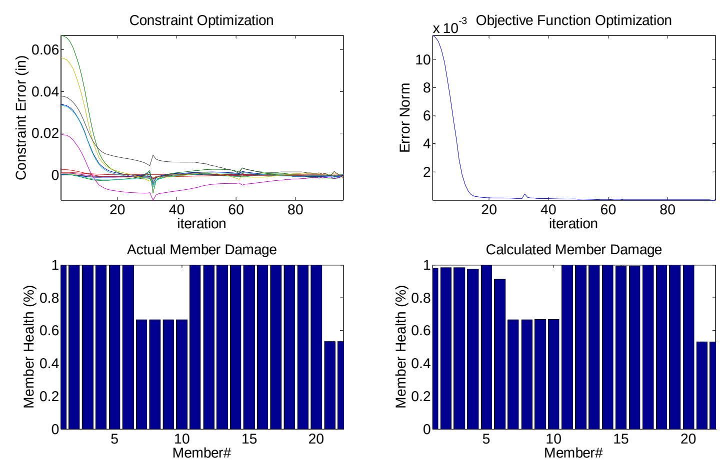


Figure 4 CG Algorithm Damage Detection Results

A brief summary of the two methods is shown in Table 1. Both algorithms were able to detect the stiffness reductions accurately. The main difference between the two methods is the computation effort required. The OC algorithm required 6 iterations and a run time of 1.25 seconds (using a Microsoft Windows based Pentium 4 at 2.40GHz). The CG algorithm, on the other hand, required 96 iterations and a run time of over 500 seconds. Both algorithms were able to accurately detect the presence of damage, the locations of damage, and the quantity of damage.

Table 1 Optimization Summary of Damage Detection Algorithms

			Damage Detection		
	Iterations	Computing time	Presence	Localization	Quantification
OC Algorithm	6	1.25 seconds	X	X	X
CG Algorithm	96	510.9 seconds	X	X	X

3.2 Continuous Beam

It has been shown that both algorithms are able to detect damage in the absence of data error, be it at grossly different efficiencies. The next step is to determine whether the OC algorithm is able to detect stiffness reductions in the presence of modeling error. To investigate this, an analytical model of a 45ft long continuous beam is used as the test structure and is shown in Figure 5. The finite element model of the beam contains 450 bending elements, resulting in a total of 899 degrees of freedom. The healthy stiffness properties of each element are: $I_x = 1000\text{in.}^4$, E = 29000ksi. Only in plane degrees of freedom are considered.

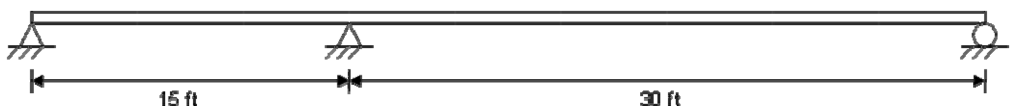


Figure 5 Analytical Model of Continuous Beam

The 899 degree of freedom analytical model is approximated using a 17 degree of freedom identification model. The identification model is created using 9 bending elements and is shown in Figure 6. Each element of the identification model represents 50 elements of the analytical model.

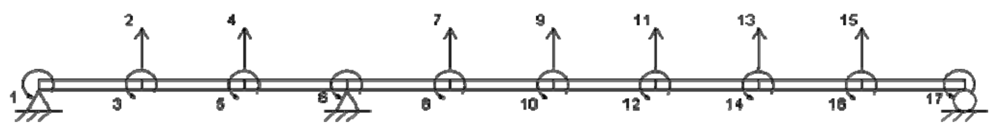


Figure 6 Nine Element Identification Model of Continuous Beam

In order to assure that modeling error is incorporated in the procedure, damage is simulated so that it does not span an entire element represented by the identification model. A maximum of five consecutive elements is damaged in the analytical model, representing only 10% of one of the identification model elements. The damage state is shown in Table 2. To detect damage, 3 load cases are used with 9 displacement measurements. The load cases and measured degrees of freedom are shown in Table 3. Each load case applies a 10 kip load to the degree of freedom listed. The load cases and measured displacements are chosen so that only vertical loads are applied and only vertical displacements are measured. This is done to utilize only measurement data and loadings that are easiest to apply on real structures, as rotational degrees of freedom are difficult to load as well as measure. It is expected that the damage detection algorithms will calculate damage in members 2 and 8 of the identification model.

Table 2 Damage State for Continuous Beam Analytical model

Member No.	Stiffness Reduction
61-65	60%
371-375	80%

Table 3 Load Case Data for Damage Detection

Load Case	DOF Loaded	DOF Measured
1	2	2, 4
2	4	2, 4
3	11	7, 9, 11, 13, 15

Again it is assumed that an appropriate identification procedure is executed prior to damage detection so each stiffness parameter can be initialized at the healthy stiffness value. The two algorithms are allowed to iterate until an optimal solution is reached. The results for the CG algorithm are shown in Figure 7. The objective function shows a smooth decrease and noticeable damage is calculated in members 1, 2, and 8. It is promising that damage was calculated in member 8 and that the presence of damage was calculated in the vicinity of members 1 and 2, however the algorithm calculated more damage in member 1 than member 2, even though member 2 is where the damage was actually concentrated. Despite the slight error in damage location the algorithm shows good, stable convergence. The most glaring drawback of the method is the rate of convergence. The algorithm is able to reduce the objective function very quickly during early iterations, but converges very slowly once it gets close to the optimal solution. This pattern is very common with CG methods.

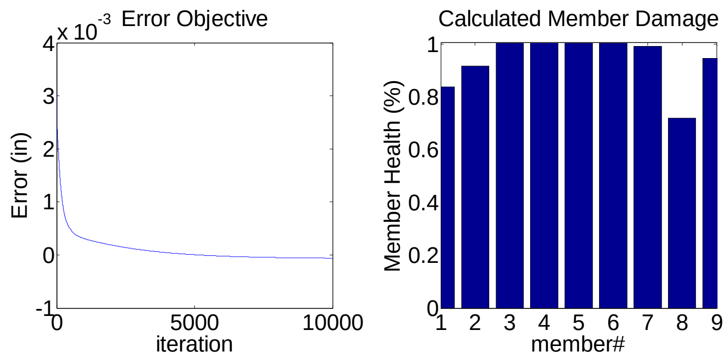


Figure 7 CG Algorithm Damage Detection Results for Continuous Beam

The OC algorithm had a much quicker convergence rate. The results of Figure 8 reveal that the algorithm required 11 iterations to obtain an optimal solution and, much like the CG algorithm, damage was detected in members 1, 2, and 8. Again, the algorithm attempts to concentrate damage near the support rather than in member 2. The convergence path was not as smooth as that given by the CG algorithm as it is observed that the constraint values oscillate before an optimal solution is reached. Ultimately all constraints are satisfied. Future work must recognize that oscillations must be controlled in order to improve the stability of the algorithm, particularly when both measurement error and modeling error are incorporated. Despite the oscillation the algorithm required only 11 iterations which is a vast improvement over the CG algorithm.

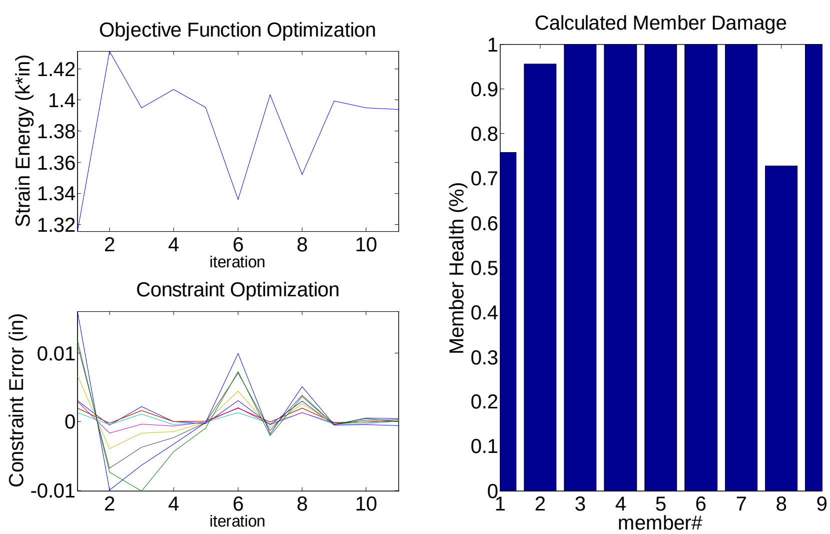


Figure 8 OC Algorithm Damage Detection Results for Continuous Beam

Quantifying damage is difficult since the identification model contains fewer elements than the analytical model. One element of the identification model encompasses 50 elements of the analytical model therefore the 30% decrease in stiffness of member 8 could represent 30% damage in 50 elements of the analytical model or it could represent highly concentrated damage in only a few elements of the analytical model. The presented examples illustrate the computational advantage that the OC algorithm has over other methods. With some improved stability, the OC algorithm can be a vital tool in damage detection methodologies.

4. CONCLUSION

A static based damage detection algorithm was presented that utilizes static deflection measurements to determine the location of reduced stiffness in finite element models. The Optimality Criterion algorithm efficiently detects areas of stiffness reduction with minimal computation effort. Results show that these stiffness reductions can be detected using a limited number of load cases with a reasonable number of measured displacements. When compared to other optimization methods it becomes clear that the OC algorithm greatly reduces the resources needed to solve such damage detection problems. Its efficiency makes the OC algorithm a valuable tool for damage detection. Further development of the algorithm is necessary however. Presented examples only show the algorithm's ability to detect damage in the presence of modeling errors. Further testing is necessary when measurement error is incorporated as well as more complex structures.

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