ADAPTIVE ELEMENT FREE GALERKIN METHOD APPLIED TO SEISMIC RESPONSE ANALYSIS OF LIQUEFIABLE SOIL

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ABSTRACT :

A numerical method is presented to analyze the seismic response of liquefiable ground. The method is based on the element free Galerkin method and h-refinement procedure. The Zienkiewicz and Zhu (Z-Z) error estimator using the T-Belytschko (TB) stress recovery scheme is incorporated into the method for the a posteriori error estimation. The effective cyclic elasto-plastic constitutive model is used to describe the non-linear behavior of saturated soils. The efficacy of the method is demonstrated by comparing the results of an embankment on liquefiable ground computed by the proposed method and the finite element method.

KEYWORDS: adaptive element-free Galerkin method, liquefaction, error estimation

1. INTRODUCTION

Many structures were damaged during the 1964 Niigata earthquake and 1995 Hyogoken-Nanbu earthquake caused by large deformation of liquefied soils. Although the finite element method (FEM) has been widely used in engineering computations, the results obtained by the FEM strongly depend on mesh configuration when dealing with large deformation caused by liquefaction. Recently developed meshless method could overcome these disadvantages for meshless method does not use any element. On this meaning, meshless method is an affective alternative, and it has been used for liquefaction simulation and produced good results (Sato T and Matsumaru T, 2003; TANG Xiao-Wei et al, 2002). Among many meshless methods, element free Galerkin method (EFGM) proposed by Belytschko et al. (1994) is the most promising and well-developed method. The EFGM differs from the FEM by using the moving-least-square (MLS) interpolation (Lancaster and Salkauskas, 1981). On the other hand, as a type of numerical approximation method, errors are inevitable in analytical results obtained by EFGM. Evidently, uniformly increasing nodes during discretization minimizes error, but producing a heavy calculation burden. Hence, an automatic adaptive refinement procedure which only increases nodes in the region with large error should be incorporated into the meshless method.

In this paper, an automatic adaptive refinement procedure is added to the EFGM to solve the liquefaction problem under earthquake loading. The adaptive refinement employs the a posteriori error estimator and the h-refinement fission procedure. Shape functions are established by moving least-square approximants and the weight function is chosen to be a quartic spline function. The seismic response of an embankment resting on saturated sand was analyzed, and the nodes with large error were well refined.

2. MOVING LEAST SQUARE APPROXIMATION

In the EFGM, variables are approximated by the MLS interpolants. The MLS approximant (Lancaster and Salkauskas, 1981) $u^h(\bar{\mathbf{x}})$ is posed as follows

$$u^h(\bar{\boldsymbol{x}}) = \sum_{j=0}^{m} p_j(\bar{\boldsymbol{x}}) a_j(\bar{\boldsymbol{x}}) = \boldsymbol{p}^{\mathrm{T}}(\bar{\boldsymbol{x}}) \boldsymbol{a}(\bar{\boldsymbol{x}}) \tag{2.1}$$

where $p_j(\bar{\boldsymbol{x}})$ is monomial in the space co-ordinates, $\bar{\boldsymbol{x}}^{\mathrm{T}}=[x,y]$, and $a_j(\bar{\boldsymbol{x}})$ its coefficient.

At each point $\bar{\boldsymbol{x}}$, $a_j(\bar{\boldsymbol{x}})$ is so chosen as to minimize the weighted residual L_2-norm:

$$J=\sum_{I=1}^{n} w(\bar{\boldsymbol{x}}-\bar{\boldsymbol{x}}_I)[\boldsymbol{p}^{\mathrm{T}}(\bar{\boldsymbol{x}})\boldsymbol{a}(\bar{\boldsymbol{x}})-\boldsymbol{u}_I]^2 \tag{2.2}$$

where n is the number of nodes in the neighborhood of $\bar{\boldsymbol{x}}$ for which the weight function $w(\bar{\boldsymbol{x}}-\bar{\boldsymbol{x}}_I)\neq 0$, and u_I refers to the nodal index of u at $\bar{\boldsymbol{x}}=\bar{\boldsymbol{x}}_I$. The minimum of J with respect to $\boldsymbol{a}(\bar{\boldsymbol{x}})$ gives

$$\boldsymbol{a}(\bar{\boldsymbol{x}})=\boldsymbol{A}^{-1}(\bar{\boldsymbol{x}})\boldsymbol{B}(\bar{\boldsymbol{x}})\boldsymbol{u} \tag{2.3}$$

where $\boldsymbol{A}=\boldsymbol{p}^{\mathrm{T}}w(\bar{\boldsymbol{x}})\boldsymbol{p}$ and $\boldsymbol{B}=\boldsymbol{p}^{\mathrm{T}}w(\bar{\boldsymbol{x}})$ .Therefore, the approximation is obtained as

$$u^h(\bar{\boldsymbol{x}})=\sum_{I=1}^{n}\sum_{j=0}^{m} p_j(\bar{\boldsymbol{x}})(\boldsymbol{A}^{-1}(\bar{\boldsymbol{x}})\boldsymbol{B}(\bar{\boldsymbol{x}}))_{jI}u_I=\sum_{I=1}^{n}\phi_I(\bar{\boldsymbol{x}})\boldsymbol{u}_I \tag{2.4}$$

where the shape function $\phi_I(\bar{\boldsymbol{x}})$ is defined by

$$\phi_I(\bar{\boldsymbol{x}})=\sum_{j=0}^{m} p_j(\bar{\boldsymbol{x}})(\boldsymbol{A}^{-1}(\bar{\boldsymbol{x}})\boldsymbol{B}(\bar{\boldsymbol{x}}))_{jI}=\boldsymbol{p}^{\mathrm{T}}\boldsymbol{A}^{-1}\boldsymbol{B}_I \tag{2.5}$$

The weight function $w_I(\bar{\boldsymbol{x}})=w(\bar{\boldsymbol{x}}-\bar{\boldsymbol{x}}_I)$ plays an important role in EFGM. The weight function should be non-zero only over a small neighborhood of $\bar{x}_I$ .Usually, $w_I(\bar{\boldsymbol{x}})=w(\bar{\boldsymbol{x}}-\bar{\boldsymbol{x}}_I)=w_I(d_I)$, where $d_I=\left\|\bar{\boldsymbol{x}}-\bar{\boldsymbol{x}}_I\right\|$ is the distance between the two points $\bar{\boldsymbol{x}}_I$ and $\bar{\boldsymbol{x}}$. Quartic spline weight function is considered in the present formulation:

$$w(d_I)=\begin{cases}1-6\left(\dfrac{d_I}{d_{mI}}\right)^2+8\left(\dfrac{d_I}{d_{mI}}\right)^3-3\left(\dfrac{d_I}{d_{mI}}\right)^4, & (0\le d_I<d_{mI})\\ 0, & (d_{mI}\le d_I)\end{cases} \tag{2.6}$$

where d_{mI} is the size of the compact support of $\bar{x}_I$.

3. WEAK FORM AND NUMERICAL IMPLEMENTATION

3.1. Governing Equations and Variational Formulations

Based on Biot's two-phase mixture theory, the equation of motion for the total mixture can be simplified to

$$\sigma_{ij,j}+\rho b_i-\rho\ddot{u}_i=0 \tag{3.1}$$

where σ_{ij} is the Cauchy total stress, b_i is the body force acceleration, ρ is the density of mixture and $\ddot{u}_i$ is the skeleton acceleration. Substituting the equation of fluid motion into the equation of mass conservation, and assuming the distribution of porosity to be smooth enough in the soils, the final form of continuity equation is

$$\rho_f \ddot{\varepsilon}_{ii} - \frac{\partial^2 p}{\partial x_i^2} - \frac{\gamma_f}{k}(\dot{\varepsilon}_{ii} - \frac{n}{K_f}\dot{p}) = 0 \tag{3.2}$$

where n is the porosity ratio, ρ_f and γ_f are the density and the unit weight of fluid respectively, k is the permibility coefficient, K_f is the bulk modulus of the fluid phase, p is the excess pore pressure, x_i the coordinate and ε_{ii} the strain. The u-p approximation (Zienkiewicz et al, 1982 and 1984; Akai K and Tamura T, 1978) is valid for low frequency problem of dynamic analysis.

Eqs.3.1 and 3.2 are satisfied at the end of each time step, $t+\Delta t$. In the updated Lagrangian method, the relevant quantities, such as stress and strain, are correlated with the reference configuration at time t, and the weak formulations are derived as

$$\begin{aligned}&\int_{^{t+dt}\Omega} {}^{t+dt}\rho\, {}^{t+dt}\ddot{u}_i \delta v_i d\, {}^{t+dt}\Omega + \int_{^{t+dt}\Omega}\left(\int_{t_1}^{t_2} \dot{S}_{ij} dt\right)\delta \dot{E}_{ij} d\, {}^{t+dt}\Omega = \\ &\int_{^{t+dt}A} {}^{t+dt}T_i \delta v_i d\, {}^{t+dt}A + \int_{^{t+dt}\Omega} {}^{t+dt}\rho\, {}^{t+dt}b_i \delta v_i d\, {}^{t+dt}\Omega - \int_{^{t+dt}\Omega} {}^{t+dt}\sigma_{ij} \delta \dot{E}_{ij} d\, {}^{t+dt}\Omega\end{aligned} \tag{3.3}$$

$$\int_{^{t+dt}\Omega} {}^{t+dt}\rho_f\, {}^{t+dt}\ddot{\varepsilon}_{ii} d\, {}^{t+dt}\Omega - \int_{^{t+dt}\Omega} {}^{t+dt}p_{,ii} d\, {}^{t+dt}\Omega - \int_{^{t+dt}\Omega} \frac{\gamma_f}{k}\, {}^{t+dt}\dot{\varepsilon}_{ii} d\, {}^{t+dt}\Omega + \int_{^{t+dt}\Omega} \frac{n\gamma_f}{kK_f}\, {}^{t+dt}\dot{p} d\, {}^{t+dt}\Omega = 0 \tag{3.4}$$

where v_i is the velocity of soil skeleton, b_i the body force per unit volume, T_i the traction, Ω the integral volume, E_{ij} the Lagrange strain tensor and S_{ij} the second Piola-Kirchhoff stress tensor.

3.2. Governing Equations for Element-free Method

Using two dimensional element-free Galerkin method, employing interpolation functions to express the displacement of soil skeleton and excess pore water pressure, adding rayleigh damping, the ultimate discrete governing equations are given as

$$\boldsymbol{M}\ddot{\boldsymbol{u}}_N + \boldsymbol{C}_d \dot{\boldsymbol{u}}_N + (\boldsymbol{K}_l + \boldsymbol{K}_{nl})\Delta \boldsymbol{u}_N + \boldsymbol{K}_V \boldsymbol{p}_{dN} = \boldsymbol{F}_d - \boldsymbol{R}_{dt} \tag{3.5a}$$

$$\rho_f \boldsymbol{K}^{\mathrm{T}}{}_V \ddot{\boldsymbol{u}}_N - \frac{\gamma_w}{k}\boldsymbol{K}^{\mathrm{T}}{}_V \dot{\boldsymbol{u}}_N + \boldsymbol{K}_h \boldsymbol{p}_{dN} + \boldsymbol{K}_p \dot{\boldsymbol{p}}_{dN} = \boldsymbol{V} \tag{3.5b}$$

where $\boldsymbol{M} = \int_\Omega \rho \boldsymbol{N}^{\mathrm{T}} N d\Omega$, $\boldsymbol{K}_V = \int_\Omega \boldsymbol{B}_V d\Omega$ $\boldsymbol{C}_d = \alpha_0 \boldsymbol{M} + \alpha_1 \boldsymbol{K}$, $\boldsymbol{K}_l = \int_\Omega \boldsymbol{B}^{\mathrm{T}}(\boldsymbol{D}_{ep} + \psi)\boldsymbol{B} d\Omega$, $\boldsymbol{K}_{nl} = \int_\Omega \boldsymbol{B}^{\mathrm{T}}{}_{nl} \boldsymbol{A}\boldsymbol{B}_{nl} d\Omega$, , $\boldsymbol{F}_d = \int_\Omega \rho \boldsymbol{N}^{\mathrm{T}} \boldsymbol{b} d\Omega + \int_s \boldsymbol{N}^{\mathrm{T}} \boldsymbol{T} ds$, $\boldsymbol{K}_h = \int_\Omega \boldsymbol{B}_h{}^{\mathrm{T}} \boldsymbol{B}_h d\Omega$, $\boldsymbol{K}_p = \frac{n\gamma_w}{k\boldsymbol{K}_f}\int_\Omega \boldsymbol{N}_h^{\mathrm{T}} \boldsymbol{N}_h d\Omega$, $\boldsymbol{V} = \int_{\Gamma p} \boldsymbol{N}_h \boldsymbol{B}_h^{\mathrm{T}} \boldsymbol{n} ds$, $\boldsymbol{R}_{dt} = \int_\Omega \boldsymbol{B}_l{}^{\mathrm{T}} \boldsymbol{\sigma}_{ept} d\Omega$

where $\boldsymbol{N}$ 、$\boldsymbol{N}_h$ are moving least square shape functions of $\boldsymbol{u}$ and $\boldsymbol{p}$ respectively, $\boldsymbol{B}_l$ is the geometric matrix, $\boldsymbol{B}_h$ the distribution tensor of pore pressure variation, $\boldsymbol{\sigma}_{ept}$ the effective stress tensor, $\boldsymbol{b}$ the body force tensor, $\boldsymbol{T}$ the traction tensor and $\boldsymbol{n}$ is the unit normal to surface, $\boldsymbol{A}$ 、$\boldsymbol{B}_{nl}$ 、$\boldsymbol{B}_v$ are symmetry matrix、non-linear strain matrix and volumetric strain matrix respectively.

The Newmark method is used to solve the dynamic equations (3.5a) and (3.5b), the parameters β and are set respectively as 0.3025 and 0.6 to ensure numerical stability. For saturated soil, a cyclic elasto-plastic model proposed by Oka is employed, in which a new stress-dilatancy relationship and cumulative strain-dependent characteristics of the plastic shear modulus have been incorporated. The constitutive model is fit for analyzing elasto-plasticity and liquefaction of saturated soil and its details are described by OKA F *et al* (1999).

4. ERROR ESTIMATION AND ADAPTIVE REFINEMENT

4.1. Definition and Evaluation of Error

The error of the EFGM solution can be defined as the difference between the exact and the EFGM solution.

$$e_u = \boldsymbol{u} - \boldsymbol{u}^h \qquad e_\sigma = \boldsymbol{\sigma} - \boldsymbol{\sigma}^h \tag{4.1}$$

In Eq. (4.1), $\boldsymbol{\sigma}^h$ is the EFGM stress. In the error estimate process, the rather accurate values instead of the exact solution are used to calculate errors because the exact solution is not easy or impossible to obtain. In this study, we use recovery stress $\boldsymbol{\sigma}^p$ to replace the exact stress $\boldsymbol{\sigma}$.

The direct definitions of error described in Eq. (4.1) are not convenient for use in the process of error estimation. In this study, we used the energy norm of stress to measure error. The difference between $\boldsymbol{\sigma}^p$ and $\boldsymbol{\sigma}^h$ will then be used to calculate the estimate error norm, the approximation of the error in the energy norm for the whole solution domain can be calculated as

$$\|E\| = \left\{ \frac{1}{2} \int_\Omega \boldsymbol{\sigma}^{eT}(x) \boldsymbol{D}^{-1} \boldsymbol{\sigma}^e(x) d\Omega \right\}^{1/2} = \left\{ \frac{1}{2} \int_\Omega (\boldsymbol{\sigma}^p - \boldsymbol{\sigma}^h)^{\mathrm{T}} \boldsymbol{D}^{-1} (\boldsymbol{\sigma}^p - \boldsymbol{\sigma}^h) d\Omega \right\}^{1/2} \tag{4.2}$$

The local error norm at a point $x = x_i$ can be measured as follows

$$\|e(\boldsymbol{x}_i)\| = \left\{ \frac{1}{2} \boldsymbol{\sigma}^{eT}(x_i) \boldsymbol{D}^{-1} \boldsymbol{\sigma}^e(x_i) \right\}^{1/2} = \left\{ \frac{1}{2} (\boldsymbol{\sigma}^p(x_i) - \boldsymbol{\sigma}^h(x_i))^T \boldsymbol{D}^{-1} (\boldsymbol{\sigma}^p(x_i) - \boldsymbol{\sigma}^h(x_i)) \right\}^{1/2} \tag{4.3}$$

The exact energy norm $\|u\|$ is not known in advance, during the adaptive refinement, it will be approximated by $\|u_h\|$, the energy norm of the EFGM solution. That is

$$\|u\| \approx \|u_h\| = \left\{ \frac{1}{2} \int_\Omega \left(\boldsymbol{\sigma}^h\right)^{\mathrm{T}} \mathbf{D}^{-1} \left(\boldsymbol{\sigma}^h\right) d\Omega \right\}^{1/2} \tag{4.4}$$

The estimated average relative error of the problem η, can be calculated as

$$\eta = \frac{\|E\|}{\|E\| + \|u_h\|} \tag{4.5}$$

The local error indicator for i-th node is defined in a similar way as

$$\eta_i = \frac{\|e(x_i)\|}{\|e(x_i)\| + \|u_h\|_i} \tag{4.6}$$

As the central idea of the error estimator builds around the construction of a smoothed or projected stress field as an approximation of the exact stress, one can expect that the accuracy of the recovered stress will affect the efficiency of the error estimator. In this study, the T-Belytschko's stress recovery scheme(TB scheme) is used which is a popular recovery stress scheme first suggested by Tabbara et al (1994) and then subsequently used by Chung and Belytschko (1998) for error estimation in the EFGM. The recovery stress field $\boldsymbol{\sigma}^{TB}$ can be written

$$\boldsymbol{\sigma}^{TB}(x)=\sum_{i=1}^{n_{TB}}\Phi_i^{TB}(x)\boldsymbol{\sigma}^h(x_i) \tag{4.7}$$

In Eq. (4.7), $\boldsymbol{\sigma}^h(x_i)$ is the EFGM stress evaluated at node i, Φ_i^{TB} is a set of recovery MLS shape functions and $\boldsymbol{n}_{TB}$ is the number of nodal points whose support includes the point $\boldsymbol{x}$.

4.2. H-version Mesh Refinement

In the refinement process, an acceptable relative error limit η_{tar} must first be given. If the relative error, η_i of the i-th node exceeds this limit, then the new adaptive nodes will be generated (LIU Xi, 2000). The refinement scheme is shown in Fig 1. The refinement scheme is that, if $d_{\mathrm{Im}}*0.5>l$ （where d_{Im} is the minimum distance between the refined node and its surrounding nodes, l is the minimum permeable distance between nodes.）, the following nodes will be generated for the refined node:
$(x_I, y_I+\frac{d_{\mathrm{Im}}}{2})$, $(x_I, y_I-\frac{d_{\mathrm{Im}}}{2})$, $(x_I+\frac{d_{\mathrm{Im}}}{2}, y_I)$, $(x_I-\frac{d_{\mathrm{Im}}}{2}, y_I)$, $(x_I+\frac{d_{\mathrm{Im}}}{2}, y_I+\frac{d_{\mathrm{Im}}}{2})$, $(x_I-\frac{d_{\mathrm{Im}}}{2}, y_I+\frac{d_{\mathrm{Im}}}{2})$,
$(x_I+\frac{d_{\mathrm{Im}}}{2}, y_I-\frac{d_{\mathrm{Im}}}{2})$, $(x_I-\frac{d_{\mathrm{Im}}}{2}, y_I-\frac{d_{\mathrm{Im}}}{2})$

Figure 1 The *h*-refinement strategy of a node in two dimension

The new node falls outside the domain is ineffective. The parameters of refined node are transferred to the new adding nodes, and the variables of the new nodes are interpolated from values of the old nodes. The next calculation step is based on the new node discretization.

5. NUMERICAL ANALYSIS

Seismic analysis of an embankment by the *h*-adaptive EFGM is introduced. The embankment constructed on saturated sand is shaken by an earthquake in x-direction. The dimension and material distribution of the embankment and soil foundation is shown in Fig.2, the material 1 and 2 are saturated sands, material 3 is filling soil without pore water, and the parameters are shown in table 1. The parameter definitions are those introduced by Oka et al. (1999). Initial stresses of the nodes are calculated for gravity. Input earthquake acceleration is shown in Fig.3, the maximum value is 7.22m/sec^2.

The curves of extra pore water pressure ratio (EPWPR) at point A is given in Fig.3 for three cases, coarse nodal discretization with fixed 356 nodes that the perpendicular distance between two nodes is 1.0m, fine nodal discretization with fixed 1397 nodes that the perpendicular distance between two nodes is 0.5m and FE analysis. Reduction of the effective soil stress due to the increased excess pore water pressure ratio leads to significant loss of soil strength and stiffness. When the excess pore water pressure ratio reaches 1.0, full liquefaction occurs at about 5.0s. The curves of the two EFG cases behave similarly which indicates that nodal density has little affection on liquefaction process.

Table1 Soil parameters of the embankment

Material parameter		Material 1	Material 2	Material 3
Density	ρ (t/m^3)	1.857	1.990	1.54
Coefficient of permeability	k (m/s)	1.9E-5	2.5E-6	
Initial void ratio	e_0	0.856	0.676	0.856
Compression index	λ	0.0064	0.0250	0.0264
Swelling index	κ	0.0055	0.0025	0.0082
Initial shear modulus ratio	G_0/σ_{m0}	829	1280	829
Over consolidation ratio	**OCR**	1.0	1.0	1.0
Phase transformation stress ratio	M_m	0.91	0.91	0.91
Failure stress ratio	M_f	1.17	1.51	1.12
Hardening parameter	B_0	1600	3000	3000
Hardening parameter	B_1	16	60	0.0
Hardening parameter	C_f	0.0	0.0	
Plastic reference strain	γ^P	0.01	0.005	
Elastic reference strain	γ^E	0.03	0.05	
Dilatancy parameter	D_0	5.0	1.0	0.0
Dilatancy parameter	**n**	1.2	5.0	0.0

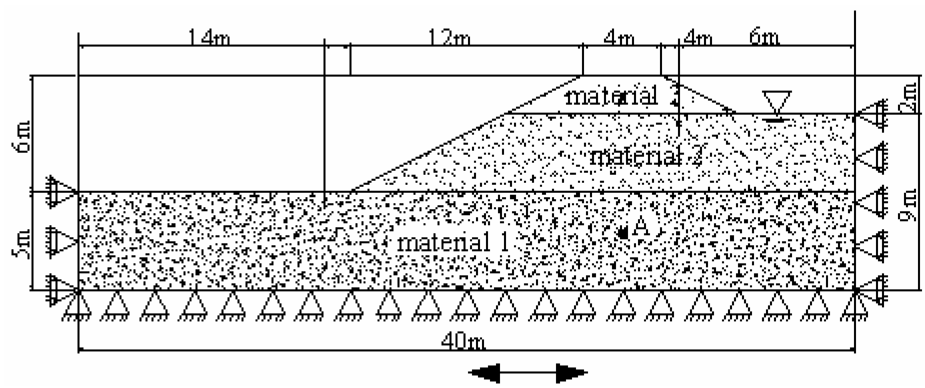


Figure 2 Embankment example

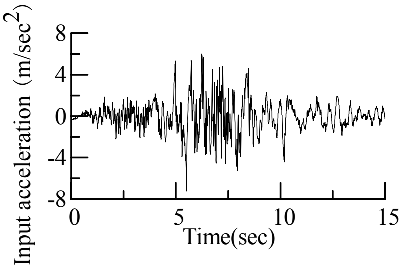


Figure 3 Input acceleration

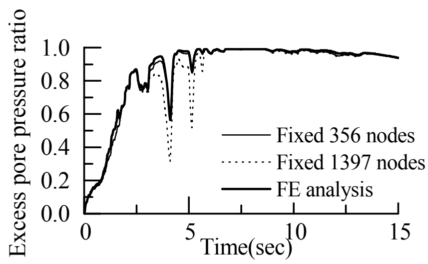
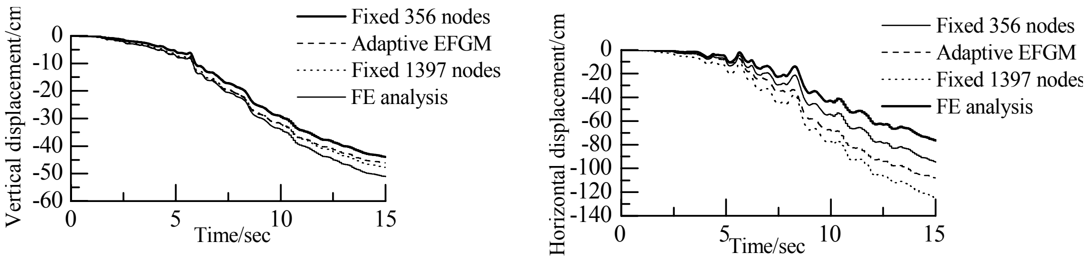


Figure 4 EPWPR response of the embankment at point A

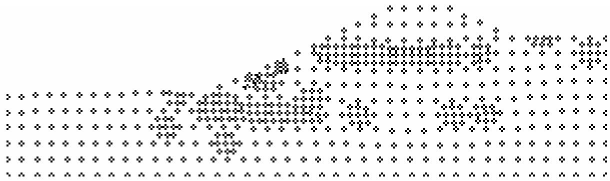
In Figs. 5(a) and (b), vertical and horizontal displacements on the top of the embankment calculated by the adaptive EFGM are compared with those three cases. From these figures, it is revealed that EFGM analysis provides a reasonable solution as FE analysis, and the adaptive refinement effectively improves accuracy.

The refined nodal discretization at times t=9.0s, 10.0s and 12.0s are shown, respectively, in Figs.6 (a)-(c). The adaptive refinement process starts at t=8.0s for the initial coarse nodal distribution with fixed 356 nodes. The relative error limit is 0.13. Adaptive refinement was done once 0.5s, nodes are increased in regions with large error step by step. One node is fissioned only once and the new adding nodes are not allowed to fission to make the contour clear, which means that these nodes will not be further refined even if the error for them exceeds the limit of relative error.

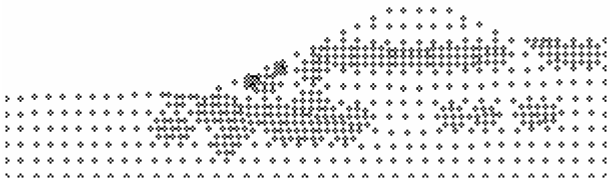


(a) Vertical displacement (b) Horizontal displacement

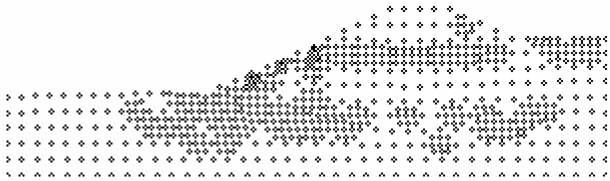
Figure 5 Vertical and horizontal displacement on the top of the embankment



(a) Refined nodal arrangement at time t=9.0s



(b) Refined nodal arrangement at time t=10.0s



(c) Refined nodal arrangement at time t=12.0s

Figure 6 Refined nodal arrangement at different times

In Fig.7, accuracies of the three different EFGM cases are compared by showing the three average relative error curves. The average relative error of the fixed 356 nodes is larger than that of the fixed 1397 nodes. In the adaptive EFGM analysis, the average relative error value decreased once the adaptive procedure started at t=8.0s. As the adaptive process proceeds, the average relative error value approaches the value for the fixed fine nodal discretization. Reduction of the average relative error confirms the efficacy of the adaptive EFGM.

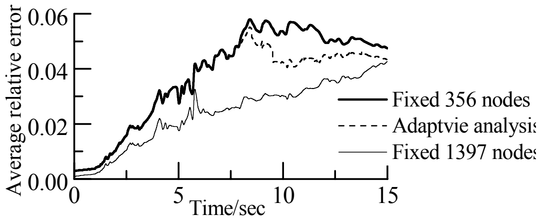


Figure 7 Comparison of average relative error

6. CONCLUSION

The adaptive element free Galerkin method is applied to nonlinear analysis of saturated soil including liquefaction phenomenon. In previous research, mesh-free method has been used to predict the behavior of saturated soil. In this study, the adaptive EFGM is applied to seismic analysis. The fission procedure belonging to *h*-refinement, indicated by the a posteriori error estimator using the T-Belytschko (TB) stress recovery scheme is incorporated into the method. The results obtained in this study show that this method produces a reasonable solution as FE analysis and improves accuracy significantly. It allows us to solve particular problems that the FEM has some difficulties in analyzing.

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