A NUMERICAL MODEL FOR THE NONLINEAR SEISMIC ANALYSIS OF THREE-DIMENSIONAL R.C. FRAMES

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ABSTRACT:

A lumped plasticity model for the nonlinear dynamic analysis of three-dimensional reinforced concrete (r.c.) frames subjected to bi-directional ground motion is proposed. The frame members are idealized by means of two parallel elements, one elastic-perfectly plastic and the other linearly elastic, assuming a bilinear moment-curvature law. An interaction surface axial force-biaxial bending moment is considered for the end sections of each frame member. The nonlinear dynamic analysis is performed using a step-by-step procedure based on a two-parameter implicit integration scheme and an initial-stress like iterative procedure. At each step, the elastic-plastic solution is obtained adopting the Haar-Kàrmàn principle. A single-storey r.c. three-dimensional frame subjected to a bi-directional artificial ground motion, whose response spectrum matches on average that adopted by Eurocode 8 for a medium risk seismic region and a medium soil, is assumed as test structure for the numerical investigation. The sensitivity of the model to changes in input parameters is investigated. Moreover, a refined fibre model is considered to validate the proposed numerical model.

KEYWORDS: Three-dimensional structure, nonlinear seismic analysis, bi-directional motion, axial force-biaxial flexure interaction, lumped plasticity model

1. INTRODUCTION

In conventional earthquake design it is accepted that the structure could undergo inelastic deformations under strong ground motions. However, these deformations must be limited in order to avoid local or global collapse as well as to limit the structural damage. A nonlinear dynamic analysis to predict the three-dimensional structural response is needed for irregular distributions (in plan and/or in elevation) of mass, stiffness and strength and/or simultaneous horizontal bi-directional ground motion (Magliulo and Ramasco, 2007).
Many nonlinear three-dimensional models of reinforced concrete (r.c.) slender frame members have been proposed in literature. They can be classified according to the level of discretization and modeling. Finite element models require the availability of triaxial concrete stress-strain models (e.g. Kwon and Spacone, 2002), providing an accurate representation of the nonlinear response, but the computational effort restricts their application to structural sub-assemblages. In the last few years, fibre models, able to follow the detailed stress-strain response at a large number of points over several cross-sections of a frame member, have become popular due to their combination with adaptive nonlinear analysis, applying automatic mesh refinement when and where necessary (Izzudin *et al.*, 2002). This approach represents a good balance between reliability of the results and computational efficiency. A representation of the key features of the structural behaviour can be obtained using r.c. member-type models. Specifically, the multisurface plasticity model (Powell and Chen, 1986) and the nine-spring model (Lai *et al.*, 1984) represent classical examples of lumped plasticity models. Recently, a model based on lumped damage mechanics and theory of fracture has also been proposed (Marante and Flórez-López, 2003). Finally, distributed plasticity models in which member inelasticity is monitored at several sections along the frame member have also been proposed in literature (e.g. Sfakianakis and Fardis, 1991).
The aim of this work is the formulation of a lumped plasticity model for r.c. frame members with axial force-biaxial flexure interaction, which represents a suitable compromise between simplicity and accuracy. A finite element code for the nonlinear dynamic analysis of three-dimensional r.c. framed structures subjected to bi-directional ground motion is prepared, investigating the sensitivity of the proposed model to variations in input parameters. Moreover, the model is tested using results from a refined fibre model (SeismoSoft, 2008).

2. THREE-DIMENSIONAL R.C. FRAME MODELING

2.1. Discretization in space and time

The spatial framed structure shown in Figure 1a is discretized as an n-degree-of-freedom system (six for each joint), with translational nodal masses m_k. The dynamic equilibrium equation can be expressed as:

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{f}[\mathbf{u}(t)]-\mathbf{p}(t)=\mathbf{0} \qquad (2.1.1)$$

where $\mathbf{M}$ is the mass diagonal matrix, $\mathbf{C}$ is the viscous damping matrix in the classical Rayleigh form, $\mathbf{f}$ is the elastic-plastic reaction vector (i.e., the vector collecting bending moments with equilibrating shear forces, elastic axial forces and torsional moments shown in Fig. 1a) and $\mathbf{p}$ represents the external load vector including the inertial forces corresponding to the horizontal ground accelerations along the global axes X and Y:

$$\ddot{u}_g(t)=\ddot{\psi}_g(t)\cos\gamma-\ddot{\zeta}_g(t)\mathrm{sen}\gamma \quad ; \quad \ddot{v}_g(t)=\ddot{\psi}_g(t)\mathrm{sen}\gamma+\ddot{\zeta}_g(t)\cos\gamma \qquad (2.1.2a,b)$$

referring to a biaxial ground motion rotated at an angle γ (i.e., $\ddot{\psi}_g$ and $\ddot{\xi}_g$ ground accelerations shown in Fig. 1a).

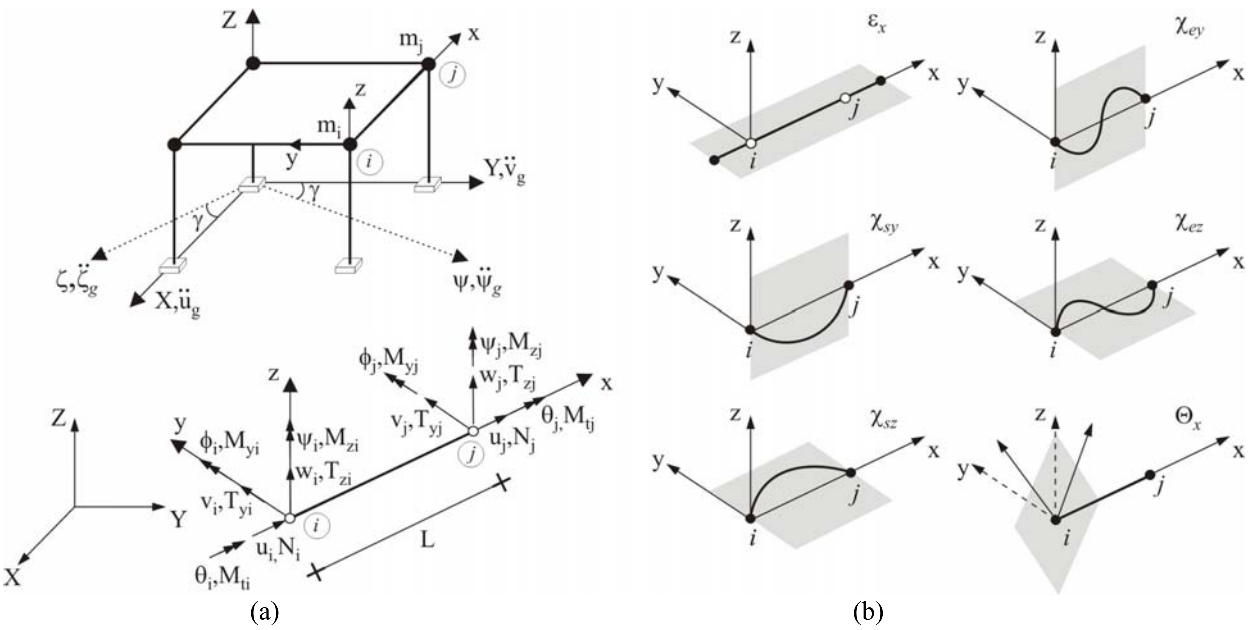


Figure 1 Structural discretization of a spatial frame (a) and natural strain modes of a beam element (b)

Time integration is carried out by using an implicit two-parameter scheme that proves to be optimal with regard to numerical instability, round-off error effects and beat phenomena between spurious solutions (Casciaro, 1975). The scheme operates by dividing the time axis in successive intervals $\Delta t = t_1 - t_0$, employing for each interval the following recurring equations:

$$\mathbf{q}_1-\mathbf{q}_0+(1/2-\alpha)\Delta t(\mathbf{s}_0-\mathbf{p}_0)+(1/2+\alpha)\Delta t(\mathbf{s}_1-\mathbf{p}_1)=\mathbf{0} \quad ; \quad \mathbf{u}_1=\mathbf{u}_0+(1/2-\beta)\Delta t\dot{\mathbf{u}}_0+(1/2+\beta)\Delta t\dot{\mathbf{u}}_1 \qquad (2.1.3a,b)$$

where $\mathbf{q}(=\mathbf{M}\dot{\mathbf{u}})$ is the moment vector and α and β depend on time step Δt and maximum and minimum vibration periods of the structure.

2.2. Elastic solution

The beam element of a three-dimensional frame (Fig. 1a) is described by the coordinates of the two nodes (i and j) associated with the end sections, in the space defined by the global Cartesian system (X, Y, Z). In the local system (x, y, z) the kinematical behaviour can be conveniently expressed as a combination of the six natural strain modes shown in Figure 1b (Argyris *et al.*, 1979):

$$\mathbf{u}_n=\{\varepsilon_x,\chi_{sy},\chi_{ey},\chi_{sz},\chi_{ez},\Theta_x\}^T=\mathbf{B}_{ne}\mathbf{u}_e \qquad (2.2.1)$$

where $\mathbf{B}_{ne}$ is a compatibility matrix (Mazza, 2007) and $\mathbf{u}_e$ is a vector collecting the local components of the end-section displacements and rotations shown in Figure 1a:

$$\mathbf{u}_e=\{u_i,v_i,w_i,\theta_i,\phi_i,\psi_i,u_j,v_j,w_j,\theta_j,\phi_j,\psi_j\}^T \quad (2.2.2)$$

In terms of natural strain modes, the expression for the beam element strain energy becomes:

$$\Phi_n[\mathbf{u}_n]=\frac{L}{2}\left\{\int_0^1 EA\varepsilon_x^2 d\xi+\int_0^1 EI_y(\chi_{sy}+\chi_{ey}-2\chi_{ey}\xi)^2 d\xi+\int_0^1 EI_z(\chi_{sz}+\chi_{ez}-2\chi_{ez}\xi)^2 d\xi+\int_0^1 GJ_t\Theta_x^2 d\xi\right\} \quad (2.2.3)$$

where EA is the axial stiffness, EI_y and EI_z represent the flexural stiffness around the local axes y and z and GJ_t is the torsional stiffness, while the shear deformations are neglected (i.e., $GA^*_y\rightarrow\infty$ and $GA^*_z\rightarrow\infty$). This formulation allows the element stiffness matrix to be expressed in a diagonal form (Argyris *et al.*, 1979).

2.3. Elastic-plastic solution

Each frame member is idealized by means of a two-component model, constituted of two parallel elements one elastic-perfectly plastic and the other linearly elastic, assuming a bilinear moment-curvature law; axial and torsional strains are assumed to be fully elastic (i.e., $N=N_E$ and $M_t=M_{tE}$ are assumed). Moment-curvature relationships for the plastic hinges lumped at the end cross sections (i and j), in which inelastic deformations are assumed to occur, are determined by the correspondence to the elastic axial force (N_E) due to static and dynamic (seismic) loads, referring to a three-dimensional axial force-biaxial bending moment elastic domain.
At each step of the analysis the elastic-plastic response of a generic frame member, once the initial state and the incremental load in the step are known, can be obtained as a solution of the Haar-Kàrmàn principle, adopting an initial-stress like iterative procedure (Casciaro, 1975). Among the stress states verifying dynamic equilibrium conditions (Eq. 2.1.1), the step-end elastic-plastic solution $\mathbf{m}=\{m_{yi}, m_{zi}, m_{yj}, m_{zj}\}^T$ has the minimum distance from the elastic solution $\mathbf{m_E}=\{m_{Eyi}, m_{Ezi}, m_{Eyj}, m_{Ezj}\}^T$, minimizing the complementary energy:

$$\Pi_c[\mathbf{m}]=\frac{L}{6EI_y}\left\{(m_{yi}-m_{Eyi})^2+(m_{yj}-m_{Eyj})^2-(m_{yi}-m_{Eyi})(m_{yj}-m_{Eyj})\right\}+\frac{L}{6EI_z}\left\{(m_{zi}-m_{Ezi})^2+\right.$$
$$\left.+(m_{zj}-m_{Ezj})^2-(m_{zi}-m_{Ezi})(m_{zj}-m_{Ezj})\right\}+\frac{L}{2EA}(N-N_E)^2+\frac{L}{2GJ_t}(M_t-M_{tE})^2=\min. \quad \mathbf{(2.3.1)}$$

under the constraints represented by the plastic admissibility conditions:

$$-M_y\le m_{yi}\le M_y \quad ; \quad -M_z\le m_{zi}\le M_z \quad ; \quad -M_y\le m_{yj}\le M_y \quad ; \quad -M_z\le m_{zj}\le M_z \quad (2.3.2a,b,c,d)$$

The yielding moments at the end cross sections (i and j), corresponding to biaxial bending, are evaluated with respect to the principal axes y ($M_y(N_E)$) and z ($M_z(N_E)$), according to the expression:

$$(M_y/M_{py})^\alpha+(M_z/M_{pz})^\alpha-1=0 \quad (2.3.3)$$

where $M_{py}(N_E)$ and $M_{pz}(N_E)$ represent the yielding moments corresponding to single bending and the exponent α is variable depending on the elastic axial force N_E (Eurocode 2, 2004: $1\le\alpha\le2$). More specifically, if the elastic solution lies in the elastic domain then $\mathbf{m}=\mathbf{m_E}$; otherwise, the elastic-plastic solution ($\mathbf{m}$) is determined by the following three step sequence (Mazza, 2007):

$$m_{yi}^{(0)}=\max\left\{-M_{pyi}^{(0)},\min\left\{M_{pyi}^{(0)},m_{Eyi}\right\}\right\} \quad ; \quad m_{zi}^{(0)}=\max\left\{-M_{pzi}^{(0)},\min\left\{M_{pzi}^{(0)},m_{Ezi}\right\}\right\} \quad (2.3.4a,b)$$

$$m_{yj}^{(k)}=\max\left\{-M_{pyj}^{(k)},\min\left\{M_{pyj}^{(k)},m_{Eyj}-\frac{m_{Eyi}-m_{yi}^{(k-1)}}{2}\right\}\right\} \; ; \; m_{zj}^{(k)}=\max\left\{-M_{pzj}^{(k)},\min\left\{M_{pzj}^{(k)},m_{Ezj}-\frac{m_{Ezi}-m_{zi}^{(k-1)}}{2}\right\}\right\} \quad (2.3.5a,b)$$

$$m_{yi}^{(k)}=\max\left\{-M_{pyi}^{(k)},\min\left\{M_{pyi}^{(k)},m_{Eyi}-\frac{m_{Eyj}-m_{yj}^{(k)}}{2}\right\}\right\} \; ; \; m_{zi}^{(k)}=\max\left\{-M_{pzi}^{(k)},\min\left\{M_{pzi}^{(k)},m_{Ezi}-\frac{m_{Ezj}-m_{zj}^{(k)}}{2}\right\}\right\} \quad (2.3.6a,b)$$

The solution at the node i (j) is evaluated taking into account the elastic-plastic response at the node j (i). In particular, Equations (2.3.5a,b) and (2.3.6a,b) are solved iteratively until the difference, in two consecutive loops (k-1 and k), between the yielding moments corresponding to biaxial bending, obtained by the return mapping by closest-point projection to the elastic domain, is less than a prefixed error at the end sections i and j.

Finally, the elastic-plastic response of the structure is obtained by the Equations (2.1.3a,b), iteratively solved in terms of the velocity at the step end ($\dot{\mathbf{u}}_1^{(j)}$), by referring to the following residual iteration scheme:

$$\mathbf{r}^{(j)}=\mathbf{q}_1^{(j)}-\mathbf{q}_0+(1/2-\alpha)\Delta t\{\mathbf{s}_0-\mathbf{p}_0\}+(1/2+\alpha)\Delta t\{\mathbf{s}_1^{(j)}-\mathbf{p}_1\} \quad ; \quad \dot{\mathbf{u}}_1^{(j+1)}=\dot{\mathbf{u}}_1^{(j)}-\mathbf{H}\mathbf{r}^{(j)} \qquad (2.3.7a,b)$$

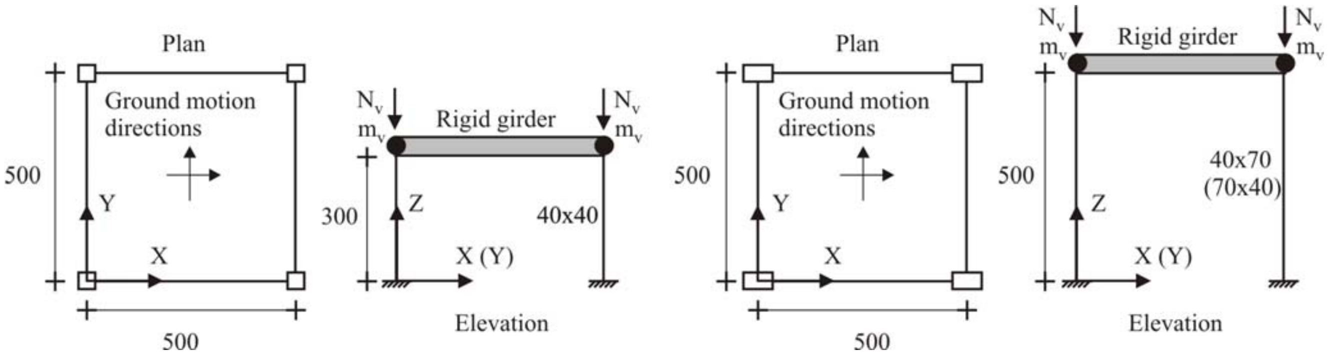
in which the index j is referred to the generic iteration loop, $\mathbf{s}=\mathbf{f}[\mathbf{u}]+\mathbf{C}\dot{\mathbf{u}}$ and $\mathbf{H}$ is the iteration matrix

$$\mathbf{H}=\left\{\mathbf{M}+(1/2+\alpha)(1/2+\beta)\Delta t^2\mathbf{K_E}+(1/2+\alpha)\Delta t\mathbf{C}\right\}^{-1} \qquad (2.3.8)$$

where $\mathbf{K_E}$ is the elastic stiffness matrix. The iteration loops are stopped when an appropriate measure of the equilibrium error ($\mathbf{r}^{(j)}$) becomes less than a prefixed tolerance.

3. TEST STRUCTURES

A single-storey r.c. three-dimensional frame with symmetric plan is assumed as test structure for the numerical investigation. Two cases are examined, referring to columns with square (Fig. 2a: A_c=40cm×40cm) and rectangular (Fig. 2b: A_c=40cm×70cm) cross sections and supposing rigid girders with elastic behaviour. More specifically, the test structures shown in Figure 2 are designed according to the provisions of Eurocode 8 (2003), assuming: high ductility class (behaviour factor, q=5); medium-risk seismic region (peak ground acceleration, PGA=0.25g) and medium soil class (class C, subsoil parameter S=1.15). The gravity load N_v, corresponding to a normalized axial load $\nu_v(=N_v/(A_c f_c))$ of the column equal to 0.2, is applied in each joint of the test structure where a lumped mass $m_v(=N_v/g)$ is considered. A cylindrical compressive strength (f_c) of 25 N/mm^2 for the concrete and yield (f_{sy}) and ultimate (f_{su}) strengths of 450 N/mm^2 and 540 N/mm^2 for the steel are considered. The longitudinal reinforcement ratio $\rho_s(=A_s/A_c)$ of each column is assumed equal to 2%, corresponding to 16ϕ16 mm (A_s= 32cm^2) and 28ϕ16 mm (A_s= 56cm^2), respectively for square and rectangular cross sections (Fig. 3).



(a) Columns with square cross section (b) Columns with rectangular cross section

Figure 2 R.c. framed structures (dimensions in cm)

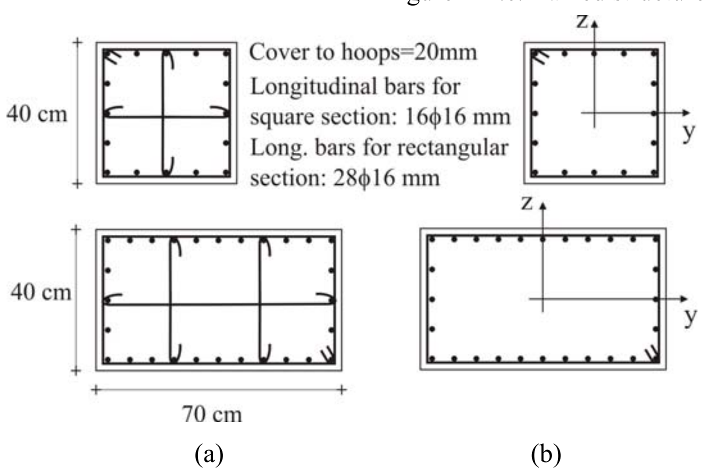


Figure 3 Details of longitudinal and transverse reinforcements of a column: (a) critical end zones; (b) central zone

Table 1 First yielding axial loads and single bending moments for the critical end zones of a column

	Square section	Rectangular section
N_t	1437 kN	2514 kN
N_c	5860 kN	10317 kN
N_y	2400 kN	4200 kN
N_z	2400 kN	4900 kN
M'_{py}	185 kN·m	358 kN·m
M'_{pz}	185 kN·m	507 kN·m
$M'_{py,max}$	380 kN·m	723 kN·m
$M'_{pz,max}$	380 kN·m	1136 kN·m

At the critical end regions of the columns, whose length is assumed according to Eurocode 8 (2004), the spacing of hoops and cross ties is assumed to be equal to 10 cm, ensuring that the distance between consecutive longitudinal bars engaged by hoops or cross ties does not exceed 20 cm (Fig. 3a). Along the central region of the columns only hoops with a spacing of 20 cm are considered (Fig. 3b). The monotonic constitutive law proposed by Mander *et al.* (1988) is assumed for the concrete, with the ultimate concrete strain modified according to Montejo and Kowalsky (2000). Stress-strain curves for square and rectangular cross sections of the columns, distinguishing between unconfined and confined concrete, are considered constructing the elastic domain axial load-biaxial bending moments. More specifically, in Table 1 the parameters corresponding to the first yielding of the longitudinal reinforcement ($\varepsilon_{sy}=f_{sy}/E_s=0.00218$), for the square and rectangular cross sections of the critical end regions, are reported: tensile axial load (N_t); single bending moments for an axial load equal to zero (M'_{py} and M'_{pz}, with respect to y and z axes); maximum single bending moments ($M'_{py,max}$ and $M'_{pz,max}$, with respect to y and z axes) and corresponding axial loads (i.e., N_y and N_z); moreover, the compression axial load (N_c) corresponds to a concrete deformation equal to $\varepsilon_{c0}=0.002$ ($\cong\varepsilon_{sy}$).

3. NUMERICAL RESULTS

In order to study the behaviour of the test structures, whose properties are illustrated in the previous section, a computer code has been prepared for the nonlinear dynamic analysis of r.c. three-dimensional frames subjected to biaxial ground motion in the horizontal direction. In particular, all the results have been obtained using as seismic input an artificial ground motion (labelled as EC8.C) along the global axes X and Y (Fig. 2). This ground motion, having a duration of 12 seconds, is generated by using the computer code SIMQKE (Gasparini and Vanmarcke, 1976), so as to be stationary in frequency in the range of vibration periods 0.05s÷2s, with a value of PGA close to that of the target EC8 spectrum for medium soil and medium risk seismic region (i.e., PGA=1.15×0.25g=0.29g). In the Rayleigh hypothesis, the damping matrix is assumed as a linear combination of the mass matrix and the elastic stiffness matrix, assuming a viscous damping ratio equal to 5% with reference to the first (translational) and third (torsional) vibration periods of the structure.
Firstly, a sensitivity study was performed to define the necessary accuracy of the input parameters for the application of the proposed lumped plasticity model (LPM). The sensitivity of the model was considered in terms of displacement time histories at the top of the columns, along the global axes X and Y. More specifically, in Figure 4 the curves for square (Fig. 4a) and rectangular (Fig. 4b) cross sections, assuming three values for the flexural stiffness of the columns, to account for the different effect of the axial force on the cracking ($EI=rEI_g$, where the coefficient $r\leq1$ is adopted for reducing the geometric flexural stiffness EI_g) are reported.

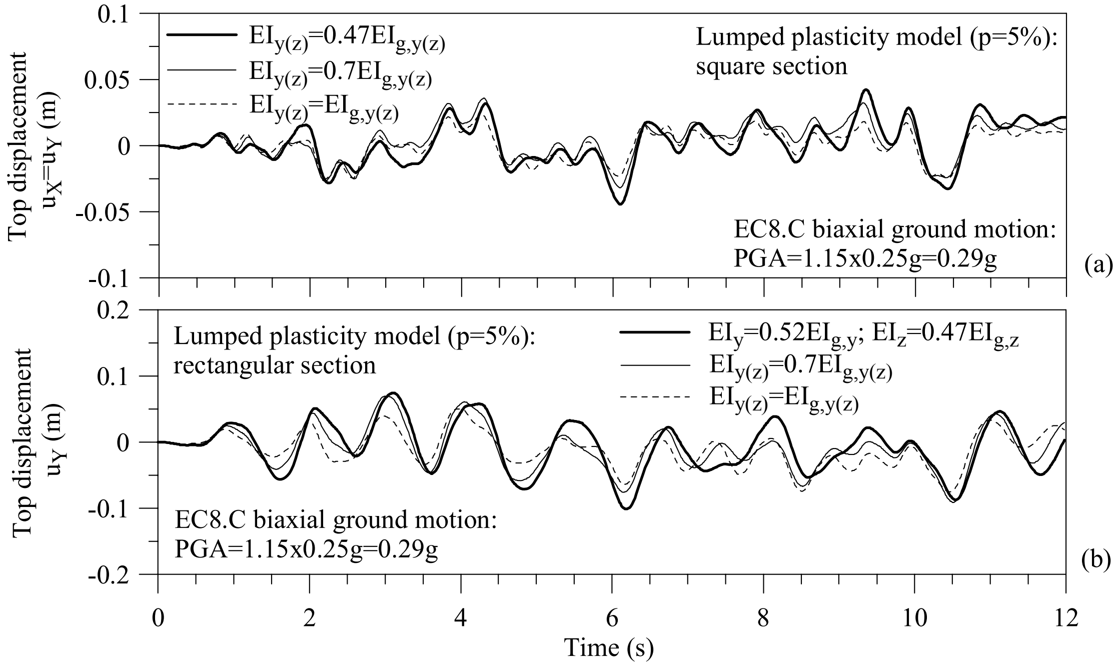


Figure 4 Effect of change in the flexural stiffness (EI) on the response of the lumped plasticity model (LPM)

The secant stiffness of the equivalent bilinear moment-curvature law for the gravity loads only (N_v), connecting the origin with the yield point (i.e., $EI_{y(z)}=0.47EI_{g,y(z)}$ for square cross section, $EI_y=0.52EI_{g,y}$ and $EI_z=0.47EI_{g,z}$ for rectangular cross section), is considered as reference value. As can be observed, the response is sensitive to the choice of the flexural stiffness EI in terms of both maximum displacement and waveform and periodicity of the time history. These effects proved to be more evident in the case of columns with rectangular cross section (Fig. 4b), characterized by a high seismic response (i.e., having a maximum drift angle of 2%).
Response sensitivity of the LPM for the effect of change in hardening ratio (p) of the bilinear moment-curvature law, expressed as a percentage of the geometric flexural stiffness EI_g, is also investigated. Specifically, curves of top displacement for rectangular cross section are reported in Figure 5, assuming three values of the hardening ratio (i.e., p=0, corresponding to an elastic-perfectly plastic behavior, 2.5% and 5%). As expected, the effect is limited mainly to maximum values, for seismic intensities strong enough that yielding and plastic deformations occurred. Analogous results, which are omitted for sake of brevity, are also obtained for square section.

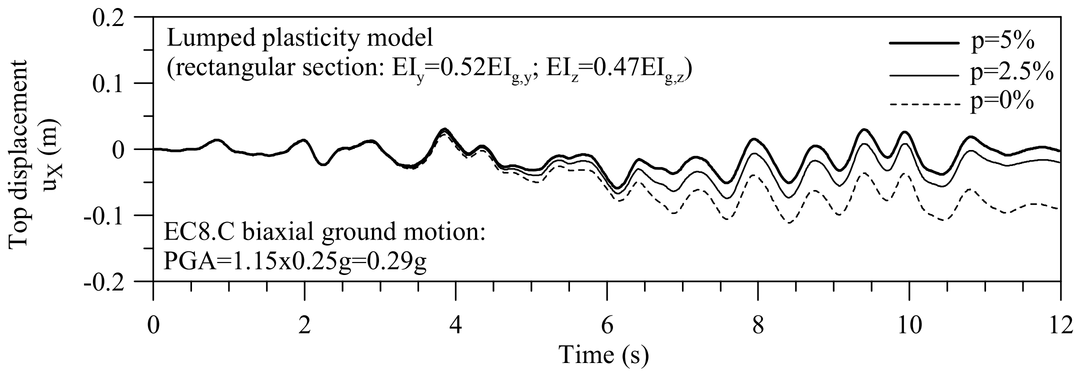


Figure 5 Effect of change in the hardening ratio (p) on the response of the lumped plasticity model (LPM)

Finally, the numerical results obtained by using a refined fibre model (FM) available in the computer program SeismoStruct (SeismoSoft, 2008) are used for testing the reliability of the proposed lumped plasticity model (LPM). To this end, time histories of displacement at the top of the columns, along the global axes X and Y (Figs. 6a and 7a,b), and base shear of the more stressed column, along the local axes y and z (Figs. 6b and 7c,d), are plotted for square (Fig. 6) and rectangular (Fig. 7) cross sections.

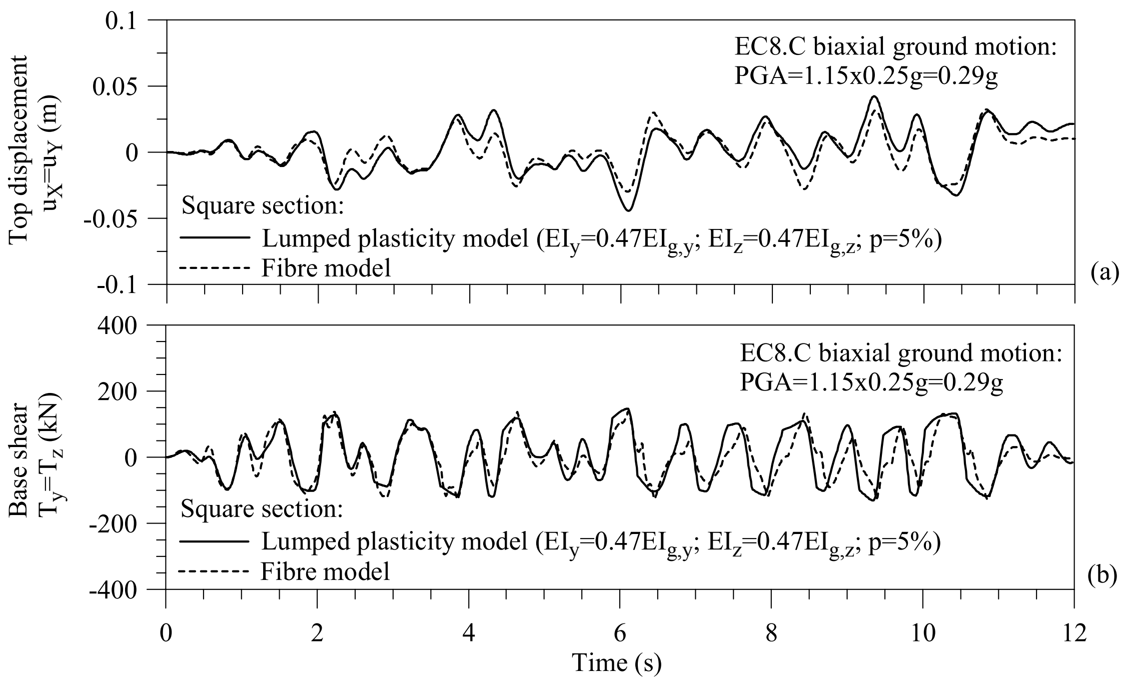


Figure 6 Comparison between LPM and FM for columns with square section: (a) top displacement along the global axis X(≡Y); (b) base shear of the more stressed column along the local axis y(≡z)

More specifically, in the FM each column is modeled with six sub-elements, two for each critical end region (with a length equal to 1/8 of the column height) and two for the central region (with a length equal to 1/4 of the column height). The concrete cross section is subdivided in 400(=20x20) and 700(=20x35) fibres, respectively, for square and rectangular shapes. The nonlinear cyclic behaviour of the concrete is modeled using the Mander model, distinguishing between unconfined and confined concrete, for both critical-end and central regions of each column, whereas the steel is modeled using the Menegotto and Pinto model. Moreover, in the LPM the flexural stiffness EI is assumed to be equal to the above mentioned secant values and a hardening ratio p=5% of the geometric flexural stiffness EI_g is considered. From the comparison of results it is evident that the LPM provides a satisfactory estimation of the seismic response in all the examined cases. However, it is worth mentioning that the LPM simulates only the essential aspects of the hysteretic behaviour.

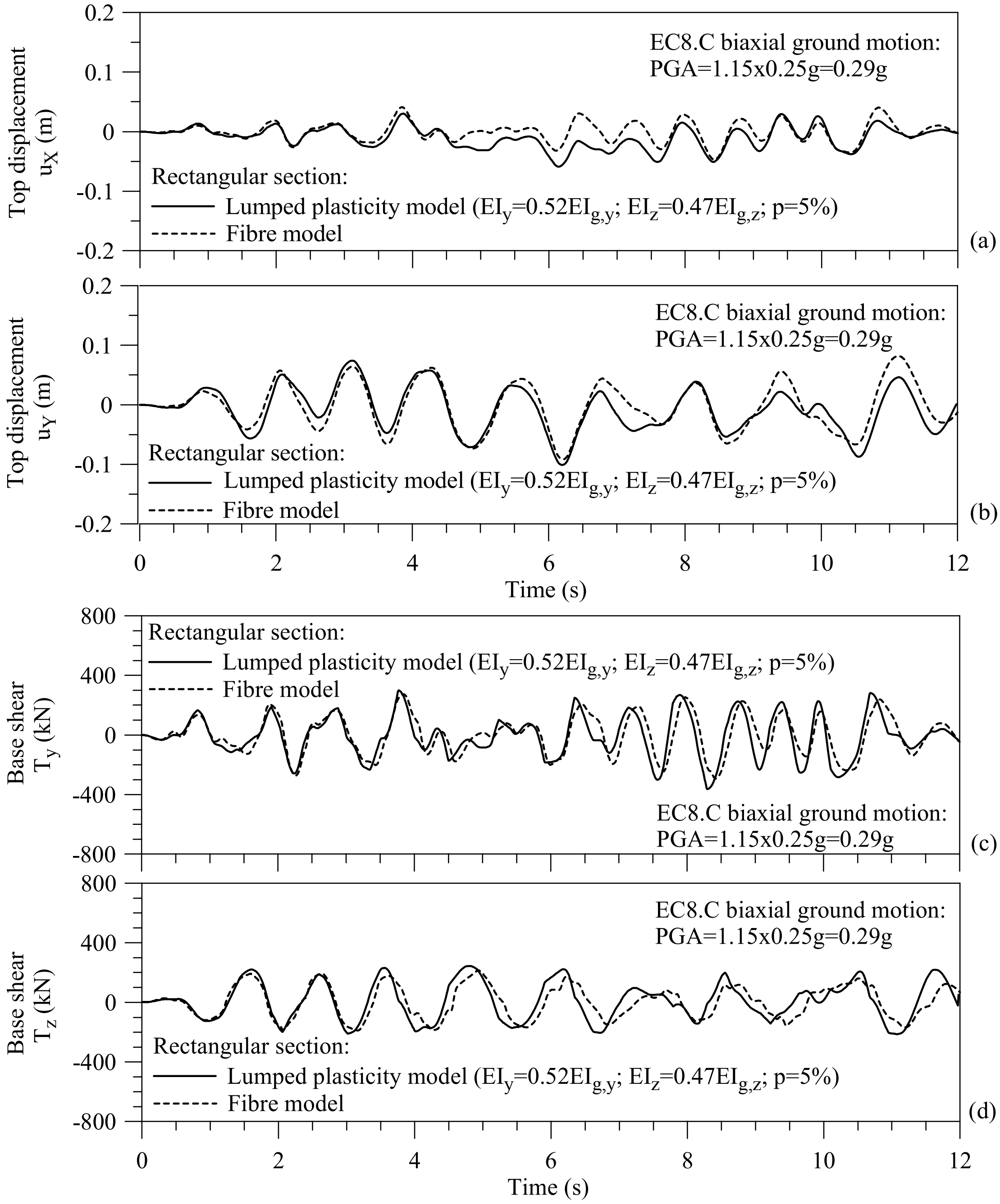


Figure 7 Comparison between LPM and FM for columns with rectangular section: (a, b) top displacements along the global axes X and Y; (c, d) base shears of the more stressed column along the local axes y and z

4. CONCLUSIONS

A lumped plasticity model (LPM) for the nonlinear dynamic analysis of three-dimensional r.c. frames subjected to bi-directional ground motion is proposed and a finite element code is prepared. The results obtained in this study permit to be drawn the following conclusions.

The nonlinear seismic response of the LPM is sensitive to the choice of different input parameters (e.g., flexural stiffness reduction factor and hardening ratio of the bilinear moment-curvature law), influencing both maximum value of the response parameters and waveform and periodicity of the corresponding time histories. From the comparison with a refined fibre model (FM), available in the computer program SeismoStruct, it can be seen that the LPM provides a satisfactory simulation of the flexural hysteretic behaviour of r.c. frame elements with axial force-biaxial bending moments interaction. The LPM is relatively simple and, therefore, can be efficiently incorporated in the nonlinear dynamic analysis of complex multi-storey r.c. framed structures.

Further studies are needed to investigate its reliability and accuracy in predicting the nonlinear seismic response, including the refinements needed to take into account the degradation of the hysteretic capacity.

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