COMPLEX MODE SUPERPOSITION METHOD CONSIDERING THE EFFECT OF MULTIPLE FOLD EIGENVALUE IN SEISMIC DESIGN

Ruifang Yu[1,2], Xiyuan Zhou[2]

[1]*Institute of Geophysics, China Earthquake Administration, Mingzu Daxue South Road No. 5, Haidian District, Beijing, 100081, P.R. China.*
[2]*Beijing Laboratory of Earthquake Engineering and Structural Retrofit, Beijing University of Technology, Chaoyang District, Pingleyuan No. 100, Beijing 100022, P.R.China.)*
Email: yrfang126@126.com, zhouxy@bjut.edu.cn

ABSTRACT:

When structures possess multiple fold eigenvalues, the orthogonality among different modes no longer exists in most cases. Therefore, in this paper, transfer function method independent of orthogonal relation is adopted to analyze the dynamic response based on theory of linear algebra and complex variable function, the dynamic response analysis method in time domain is derived, which is suitable for both non-classically and classically damped linear system with multiple fold eigenvalues. In addition, the structural response spectrum is introduced successfully and the CCQC algorithm is deduced, which can consider the effect of multi-fold-eigenvalues. The applicability of the deduced formula is verified through Newmark integration computation of liner structure subjected to prescribed earthquake motion. Meanwhile, the results show that, for the structure with multiple fold eigenvalues, the calculation errors will be fairly large if simply neglecting the effect of multiple fold eigenvalues. Finally, it is pointed out that the method derived in this paper is suitable for computing seismic responses of MDOF classical or non-classical damping linear system with or without multiple eigenvalues.

KEYWORDS: Complex mode superposition, multiple fold eigenvalue, transfer function method

1. INTRODUCTION

Mode superposition analysis method is widely used in seismic design of structures to simplify the dynamic analyses through decoupling the vibration equation based on orthogonal property and in result the concerned complex MDOF system can be turned into linear superposition of independent dynamic responses of a series of SDOF systems subjected to identical ground motion. Based on stationary random process theory and comparative analyses of time history, the square root of the sum of squares (SRSS) and complete quadratic combination (CQC) methods for classical damping linear system have proved to be an effective means for prediction of maximum response of structures subjected to earthquake excitation (Caughey, 1960). Recently the dynamic analysis of non-classically damped linear systems has been paid more attention because it is noticed that there are many structures whose damping are non-uniform, for instance, soil-structure interacting system and structures equipped with supplemental linear viscous dampers such as oil dampers. For the non-classical damping system, the traditional modal decomposed methods can not make the motion equation decoupled; so, many researchers make effort to discuss the modal decamped method based on complex modes (Igusa, *et el.* 1984, Skinner, *et el*, 1993). Zhou and Yu (2004) derived the complex complete quadratic combination (CCQC) method for the non-classically damped linear system, which is completely in real form, and the complex square root of the sum of squares (CSRSS) method if correlations among modal responses are ignored. However, for either classical damping system or non-classical damping system, multiple fold eigenvalue problems, which is frequently emerged in branch system or symmetrical structure of large scale, is not yet attracted sufficient attentions in earthquake engineering community.

When structures possess multi-fold-eigenvalues, the orthogonality among different modes no longer exists in most cases. Therefore, in this paper, transfer function method (Chen and Zhu, 1990; Zhen, 2002), the theory of line independent of orthogonal relation is adopted to analyze the dynamic response. By using comprehensively the theory of linear algebra and complex variable function, the dynamic response analysis method in time domain is derived, which is suitable for both non-classically and classically damped linear system with multiple fold eigenvalues. The new algorithm not only has explicit physical meaning, but also enables to consider the effect of multiple fold eigenvalues. In addition, the response spectrum is introduced successfully and CCQC method is proposed, which can consider the effect of multi-fold-eigenvalues. The validity and correction of formula are verified through Newmark integration computation of liner structure subjected to prescribed earthquake motion. Meanwhile, the results show that, for the structure with multiple fold eigenvalues, the calculation errors will be fairly large if simply neglecting the effect of multiple fold eigenvalues. Finally, it is pointed out that the method derived in this paper is suitable for computing seismic responses of MDOF classical or non-classical damping linear system with or without multiple fold eigenvalues, .

2. ESTABLISHMENT AND EXTENSION OF TRANSFER FUNCTION MATRIX FOR LINEAR SYSTEM

As it has been known, for a discrete system, having N degrees of freedom, the equations of motion in terms of nodal displacements are expressed as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{C}\dot{\boldsymbol{x}}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{f}(t) \tag{2.1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the $N\times N$ mass, damping and stiffness matrices, $\boldsymbol{x}$ is $N\times 1$ nodal displacement vector which describes the dynamic response of the structure , N is an arbitrarily large integer, and $\boldsymbol{f}(t)$ is $N\times 1$ nodal load vector.

If damping matrix does not satisfy the decoupling condition based on real modes, that is, when system is not the classical damping system, the Eq.(2.1) has to be solve by state space method. Let $\boldsymbol{y}^T=[\dot{\boldsymbol{x}} \quad \boldsymbol{x}]^T$, Eq. (2.1) can be rewritten into a group of linear differential equations of one order and the corresponding eigenvalue problem can be defined as non-zero solution of following equation:

$$(s\boldsymbol{A}+\boldsymbol{B})\Phi=0 \tag{2.2}$$

in which

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{M}\\ \boldsymbol{M} & \boldsymbol{C}\end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix}-\boldsymbol{M} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{K}\end{bmatrix}$$

Obviously, it is equivalent to the eigenvalue problem as follow

$$(s^2\boldsymbol{M}+s\boldsymbol{C}+\boldsymbol{K})\phi=0 \tag{2.3}$$

and $\Phi=[\lambda\phi \quad \phi]$.

When structures possess multi-fold-eigenvalues, the orthogonality among different modes no longer exists in most cases, so transfer function method is used to analyze this case.

Using Laplace transfer, Eq.(2.1) can become into the equation in the complex-field (shortened ‘s’ field) based on the parameter $s=-\alpha+i\beta$, i.e.

$$(\boldsymbol{M}s^2+\boldsymbol{C}s+\boldsymbol{K})\boldsymbol{X}(s)=\boldsymbol{F}(s) \tag{2.4}$$

that is

$$\boldsymbol{Z}(s)\boldsymbol{X}(s)=\boldsymbol{F}(s) \tag{2.5}$$

in which $\boldsymbol{Z}(s)$ is the impedance matrix in s-field of system, which is nonsingular and symmetric matrix for the restraint system and has inverse matrix, hence we can get

$$\boldsymbol{X}(s)=(\boldsymbol{Z}(s))^{-1}\boldsymbol{F}(s)=\boldsymbol{H}(s)\boldsymbol{F}(s) \tag{2.6}$$

where

$$\boldsymbol{H}(s)=\left(\boldsymbol{Z}(s)\right)^{-1}=\frac{\mathrm{adj}\left(\boldsymbol{Z}(s)\right)}{\det\left(\boldsymbol{Z}(s)\right)}=\frac{\boldsymbol{J}(s)}{D(s)} \tag{2.7}$$

is called as transfer function matrix. When external excitation and initial condition of system are definitive, the dynamic response in s-field for every generalized coordinates of system will depend on matrix $\boldsymbol{H}(s)$, whose property reflects the dynamic performance of system. In Eq.(2.7), $D(s)=\det\left(\boldsymbol{Z}(s)\right)$ is the determinant of matrix $\boldsymbol{Z}(s)$, which can be denoted as the $2N$-order polynomial with real coefficients concerning parameter s, that is

$$D(s)=\sum_{r=0}^{2N} b_r s^r = b_0+b_1 s+b_2 s^2+\cdots+b_{2N}s^{2N} \tag{2.8}$$

In addition, $\boldsymbol{J}(s)=\mathrm{adj}\left(\boldsymbol{Z}(s)\right)$ is the companion matrix of matrix $\boldsymbol{Z}(s)$, which is a $N\times N$ symmetric matrix and the element $J_{ij}(s)$ can be written as $2(N-1)$ order polynomial with parameter s, that is

$$J_{ij}(s)=\sum_{k=0}^{2N-2} a_k s^k = a_0+a_1 s+a_2 s^2+\cdots+a_{2N-2}s^{2N-2} \tag{2.9}$$

Then, the element $H_{ij}(s)$ of transfer function matrix can be denoted as

$$H_{ij}(s)=\frac{J_{ij}(s)}{D(s)}=\frac{a_0+a_1 s+a_2 s^2+\cdots+a_{2N-2}s^{2N-2}}{b_0+b_1 s+b_2 s^2+\cdots+b_{2N}s^{2N}} \tag{2.10}$$

It can be named as the transfer function in s-field of system.

Because $D(s)$ is the polynomial with real coefficients, through equality $D(s)=0$, we can get $2N$ roots in the complex-field. If suppose z distinct roots $s_1,s_2,\cdots,s_z$, the overlapped number of every multi-fold-root are respectively $k_1,k_2,\cdots,k_z$, and $k_1+k_2+\cdots+k_z=2N$. Then Eq. (10) can be expressed as

$$H_{ij}(s)=\frac{J_{ij}(s)}{\prod_{r=1}^{z}(s-s_r)^{k_r}}=\sum_{r=1}^{z}\left(\frac{p_{ij}^{r1}}{(s-s_r)}+\frac{p_{ij}^{r2}}{(s-s_r)^2}+\cdots+\frac{p_{ij}^{rk_r}}{(s-s_r)^{k_r}}\right) \tag{2.11}$$

in which, p_{ij}^{r1}, p_{ij}^{r2}, …, $p_{ij}^{rk_r}$ are undetermined constant. We will discuss these constant according to the two cases as follow.

2.1 Single Eigenvalue s_r

When eigenvalue s_r is the single-root, according to the Heaviside expansion theorem, Eq.(2.11) can be written as

$$H_{ij}(s)=\sum_{\substack{l=1\\ l\neq r}}^{z}\left(\frac{p_{ij}^{l1}}{(s-s_l)}+\frac{p_{ij}^{l2}}{(s-s_l)^2}+\cdots+\frac{p_{ij}^{lk_l}}{(s-s_l)^{k_l}}\right)+\frac{p_{ij}^{r}}{s-s_r} \tag{2.12}$$

Multiple by $(s-s_r)$ in both side of Eq. (2.12), and let $s\to s_r$, then the every parameter, besides the p_{ij}^{r}, in the right side of equality are all zeros, and we can obtain

$$p_{ij}^{r}=\lim_{s\to s_r}(s-s_r)H_{ij}(s)=\lim_{s\to s_r}(s-s_r)\frac{J_{ij}(s)}{(s-s_r)\prod_{\substack{l=1\\ l\neq r}}^{z}(s-s_l)^{k_l}}=\frac{J_{ij}(s_r)}{\prod_{\substack{l=1\\ l\neq r}}^{z}(s_r-s_l)^{k_l}} \tag{2.13}$$

2.2. Multiple fold Eigenvalue s_r

When eigenvalue s_r is the multi-fold-root, the order number of pole corresponding to the transfer function

$H_{ij}(s)$ is greater than 2. Let us define the undetermined constants p_{ij}^{r1}, p_{ij}^{r2}, …, $p_{ij}^{rk_r}$. When overlapped number of eigenvalue s_r is the k_r, we can easily approve that the relational expression followed by is valid, that is

$$J_{ij}(s_r) = J_{ij}^{'}(s_r) = \cdots = J_{ij}^{(k_r-2)}(s_r) = 0 \tag{2.14}$$

$$J_{ij}^{k_r-1}(s_r) = (k_r - 1)!G(s_r) \tag{2.15}$$

Multiple by $(s-s_r)^{k_r}$ in both side of Eq. (2.11), and let

$$V(s) = \frac{1}{\prod_{\substack{l=1 \\ l\neq r}}^{z}(s-s_l)^{k_l}} \tag{2.16}$$

then

$$J_{ij}(s)V(s) = \sum_{r=1}^{z}\left[p_{ij}^{r1}(s-s_r)^{k_r-1} + p_{ij}^{r2}(s-s_r)^{k_r-2} + \cdots + p_{ij}^{rk_r}\right] \tag{2.17}$$

Let $s \to s_r$ in Eq.(2.17) and use the relation in Eq.(2.14), we can get

$$p_{ij}^{rk_r} = 0 \tag{2.18}$$

Use Leibnitz's rule and calculate the first derivative in both side Eq.(2.17), and let $s \to s_r$ and consider the relation of Eq.(2.14), we can get

$$p_{ij}^{rk_r-1} = 0 \tag{2.19}$$

By this procedure， the undetermined coefficients can be obtained. That is

$$p_{ij}^{r2} = p_{ij}^{r3} = \cdots = p_{ij}^{rk_r} = 0 \tag{2.20}$$

$$p_{ij}^{r1} = \frac{1}{(k_r-1)!}\frac{J_{ij}^{(k_r-1)}(s_r)}{\prod_{\substack{l=1 \\ l\neq r}}^{z}(s_r-s_l)^{k_l}} \tag{2.21}$$

Substitute the Eq.(2.21) into Eq.(2.11), and using symbol p_{ij}^{r} to replace the symbol p_{ij}^{r1}, then we can get

$$H_{ij}(s) = \frac{J_{ij}(s)}{\prod_{r=1}^{z}(s-s_r)^{k_r}} = \sum_{r=1}^{z}\frac{p_{ij}^{r}}{s-s_r} \tag{2.22}$$

From complex variable function, the expression of undetermined coefficient p_{ij}^{r} is the residue in pole s_r for function $H_{ij}(s)$, that is

$$p_{ij}^{r} = Res\left[H_{ij}(s), s_r\right] \tag{2.23}$$

The preceding analysis shows that, if the z eigenvalues are different, the every element in transfer function matrix can be expressed as the sum of z simple fractions according to the z different eigenvalues. The numerator of every simple fraction can be expressed by the residue corresponding to eigenvalue (pole).

Since the every element in transfer function matrix can be expressed as the sum of z simple fractions according to the different eigenvalues, then transfer function matrix is also expanded as the sum of simple fractions according to the different eigenvalues, that is

$$\boldsymbol{H}(s) = \sum_{r=1}^{z}\frac{\boldsymbol{P}^{r}}{s-s_r} \tag{2.24}$$

in which, $\boldsymbol{P}^{r}$ is the residue matrix corresponding to eignvalue (pole) s_r, which can be determined by Eq.(2.23).

3. DYNAMIC RESPONSES IN TIME DOMAIN FOR LINEAR SYSTEM

In practice, the eigenvalues normally occur in complex conjugate pairs for the damped system, but for highly damped systems, an even number of them can be real (Inman and Andry jr., 1980), which means the characteristics equation of the system comprises over-critical damping (Clough and Penzien, 1993). We discuss the case in paper by Yu and Zhou (2006), so the case will not be handled in this article.

Based on the analysis as above, we suppose $2m$ different complex eigenvalues, then Eq.(2.22) can be rewritten as

$$H_{ij}(s)=\sum_{k=1}^{m}\left(\frac{p_{ij}^{k}}{s-s_k}+\frac{\bar{p}_{ij}^{k}}{s-\bar{s}_k}\right) \tag{3.1}$$

in which s_k and $\bar{s}_k$ represent a pair of conjugate complex eigenvalues, p_{ij}^k and $\bar{p}_{ij}^k$ are the corresponding residues which can be calculated by the Eq.(2.13) and Eq.(2.21) according to the practice cases. In fact, if the overlapped number of eigenvalue is equal to 1, the Eq. (2.21) will become to Eq. (2.13). Similarly, the transfer function matrix may be expressed by a pair of conjugate eigenvalue, that is

$$\boldsymbol{H}(s)=\sum_{k=1}^{m}\left(\frac{\boldsymbol{P}^{k}}{s-s_k}+\frac{\bar{\boldsymbol{P}}^{k}}{s-\bar{s}_k}\right) \tag{3.2}$$

in which, $\boldsymbol{P}^k$ and $\bar{\boldsymbol{P}}^k$ are the a pair of residue matrices in complex pole s_k and conjugate pole $\bar{s}_k$.

Substitute the Eq.(3.2) into Eq.(2.6), the following equation can be obtained

$$\boldsymbol{X}(s)=\boldsymbol{H}(s)\boldsymbol{F}(s)=\left[\sum_{k=1}^{m}\left(\frac{\boldsymbol{P}^{k}}{s-s_k}+\frac{\bar{\boldsymbol{P}}^{k}}{s-\bar{s}_k}\right)\right]\boldsymbol{F}(s) \tag{3.3}$$

The inverse transformation of Laplace is used in both Eq.(3.3), we can get

$$\boldsymbol{x}(t)=\sum_{k=1}^{m}\left(\boldsymbol{P}^{k}\int_{0}^{t}e^{s_k(t-\tau)}f(\tau)\,d\tau+\bar{\boldsymbol{P}}^{k}\int_{0}^{t}e^{\bar{s}_r(t-\tau)}f(\tau)\,d\tau\right) \tag{3.4}$$

Suppose:

$$s_k=-\alpha_k+i\beta_k\,,\quad \bar{s}_k=-\alpha_k-i\beta_k \tag{3.5}$$

where $\alpha_k=\zeta_k\omega_k$ and $\beta_k=\omega_{Dk}=\omega_k\sqrt{1-\zeta_k^2}$ are damping coefficient and damped frequency of the k-th mode respectively, and the free vibration frequency ω_k and the corresponding critical damping ratio ζ_k can be deduced from the general orthogonality relations. Separate real and imaginary parts of residue matrix and combine the contributions of a pair of conjugate values, then the structure response showed by Eq.(3.4) can be obtained

$$\boldsymbol{x}(t)=2\sum_{k=1}^{m}\boldsymbol{u}_k\int_{0}^{t}e^{-\alpha_k(t-\tau)}\cos\beta_k(t-\tau)\,\ddot{y}_g(\tau)\,d\tau-2\sum_{k=1}^{m}\boldsymbol{v}_k\int_{0}^{t}e^{-\alpha_k(t-\tau)}\sin\beta_k(t-\tau)\,\ddot{y}_g(\tau)\,d\tau \tag{3.6}$$

where $\boldsymbol{u}_k=\mathrm{Re}\left(\boldsymbol{P}^k\right)\boldsymbol{MI}$ and $\boldsymbol{v}_k=\mathrm{Im}\left(\boldsymbol{P}^k\right)\boldsymbol{MI}$ are the $N\times 1$ vector, in which $\mathrm{Re}\left(\boldsymbol{P}^k\right)$ and $\mathrm{Im}\left(\boldsymbol{P}^k\right)$ represent the real and imaginary parts of residue matrix $\boldsymbol{P}^k$, respectively.

If the Duhamel integration for $\cos\beta_k(t)$ is substituted by sine Duhamel integration (Zhou and Yu, 2004), the Eq.(3.6) can be written as

$$\boldsymbol{x}(t)=\sum_{k=1}^{m}\left\{\boldsymbol{A}_k q_k(t)+\boldsymbol{B}_k\dot{q}_k(t)\right\} \tag{3.7}$$

in which

$$\boldsymbol{A}_k=-2\left(\zeta_k\omega_k\boldsymbol{u}_k-\omega_{Dk}\boldsymbol{v}_k\right) \tag{3.8}$$

$$\boldsymbol{B}_k=-2\boldsymbol{u}_k \tag{3.9}$$

$q_k(t)$ can be expressed as solution of the following equation.

$$\ddot{q}_k(t)+2\zeta_k\omega_k\dot{q}_k(t)+\omega_k^2 q_k(t)=-\ddot{y}_g(t) \tag{3.10}$$

4. CALCULATION METHOD BASED ON RESPONSE SPECTRA

The deviation or mean square response of $\boldsymbol{x}(t)$ in Eq. (3.10) is:

$$E\left[\boldsymbol{x}^2(t)\right]=\sum_{i=1}^{z}\sum_{j=1}^{z}\left[\bar{\boldsymbol{A}}_i\bar{\boldsymbol{A}}_j < q_i(t)q_j(t)> +\bar{\boldsymbol{B}}_i\bar{\boldsymbol{B}}_j < \dot{q}_i(t)\dot{q}_j(t)> +2\bar{\boldsymbol{B}}_i\bar{\boldsymbol{A}}_j < \dot{q}_i(t)q_j(t)>\right] \tag{4.1}$$

in which the symbol $< >$ represents operation of calculation of average. Let us calculate covariances $< q_i(t)q_j(t)>$ 、 $< \dot{q}_i(t)\dot{q}_j(t)>$ and $< \dot{q}_i(t)q_j(t)>$, and substitute into Eq.(4.1), we can get

$$E[\boldsymbol{x}^2(t)]=\sum_{i=1}^{z}\sum_{j=1}^{z}[\bar{\boldsymbol{A}}_i\bar{\boldsymbol{A}}_j\rho_{ij}^{dd}+\bar{\boldsymbol{B}}_i\bar{\boldsymbol{B}}_j\omega_i\omega_j\rho_{ij}^{vv}+2\bar{\boldsymbol{B}}_i\bar{\boldsymbol{A}}_j\omega_i\rho_{ij}^{vd}] < q_i(t)^2 >^{1/2} < q_j(t)^2 >^{1/2} \tag{4.2}$$

in which the calculation and discussion of displacement correlation coefficient ρ_{ij}^{dd}, velocity correlation coefficient ρ_{ij}^{vv} and displacement-velocity correlation coefficient ρ_{ij}^{vd} can be seen in reference (Zhou and Yu, 2004).

If we assume as usual that the maximum response $|\boldsymbol{x}(t)|_{\max}$ is proportional to the root of the mean square response, the following closed-form formula of complex mode response-spectrum superposition which considers the effect of multiple fold eigenvalues, i.e. the complex complete quadratic combination (CCQC) formula is deduced:

$$|\boldsymbol{x}(t)|_{\max}=\left[\sum_{i=1}^{z}\sum_{j=1}^{z}[\bar{\boldsymbol{A}}_i\bar{\boldsymbol{A}}_j\rho_{ij}^{dd}+\bar{\boldsymbol{B}}_i\bar{\boldsymbol{B}}_j\omega_i\omega_j\rho_{ij}^{vv}+2\bar{\boldsymbol{B}}_i\bar{\boldsymbol{A}}_j\omega_i\rho_{ij}^{vd}]|q_n(t)|_{\max}|q_m(t)|_{\max}\right]^{1/2} \tag{4.3}$$

5. NUMERICAL EXAMINATION AND APPLICATIONS

A single-sphere network shell is considered in this paper, whose diameter is 10m and height is 5m. The lattice layout of network shell is the 'sunflower' form. The material is the seamless steel pipe whose sectional dimension is expressed as: Φ35×2 (the bar in ring dimension of network shell), Φ15×2 (diagonal brace). Young's modulus is 2.1×10^{11}N/m^2 of the materials and uniformly distributed load is 200Kg/m^2. In addition, the bearing of network is the fixed hinge bearing, and the plain and three-dimensional views are figured in Fig.1 and Fig.2, respectively. The serial numbers of structural nodes are shown in Fig.1. Six freedom degrees are considered for every node. Therefore, there are 36 freedom degrees in total.

Define the damping matrix as $\boldsymbol{C}=\alpha\boldsymbol{M}+\beta\boldsymbol{K}$, in which $\boldsymbol{M}$ and $\boldsymbol{K}$ are the structural mass and stiffness matrices, respectively, and let $\alpha=0.1757$ (1/s), $\beta=0.00173$ (s), then the first two damping ratios are equal to 0.02.

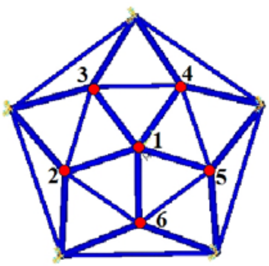

Fig. 1 plain view and node layout

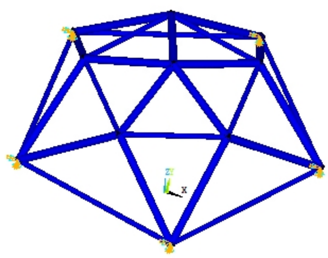
Fig. 2 Three-dimension structural view

The NS component of the El-Centro earthquake acceleration recorded on May 18, 1940 earthquake in California, which contains energy over a broad range of frequencies, is used as a ground motion input. Via mode analysis procedure and transfer function method, the dynamic response analysis is carried out in Matlab platform. The modal properties of the structure are given in Table 5.1. It can be seen that 14 pairs of multiple fold eigenvector appear for the single network shell, in which the first two modes are multiple fold. Then the modes corresponding to the multiple fold eigenvalues do not possess orthogonality. Table 5.2 shows the displacement peak value of X and Y directions, in which the values in the column 2 is the nodal displacements calculated by Newmark-β numerical method, and the values in column 3 is the results calculated by the Eq. (3.7) deduced in this paper, which coincide with the results calculated from Newmark numerical method. The comparison of two calculation methods verified the correction of Eq. (3.7). Calculation results coming from simple neglecting the effects of multiple fold eigenvalues is shown in column 4, which indicate that fairly large errors, as shown in column 5, can be produced by the simple calculation.

Table 5.1 Modal properties of the structure

Mode number	frequency	Damping ratio	Mode number	frequency	Damping ratio
1	25.4	0.0202	19	264.3	0.1067
2	25.4	0.0202	20	295.4	0.1190
3	26.4	0.0202	21	295.4	0.1190
4	26.4	0.0202	22	390.5	0.1569
5	26.7	0.0202	23	390.5	0.1569
6	28.3	0.0203	24	434.7	0.1745
7	28.3	0.0203	25	557.1	0.2233
8	30.0	0.0205	26	557.1	0.2233
9	30.0	0.0205	27	824.7	0.3302
10	31.3	0.0206	28	974.2	0.3900
11	32.3	0.0208	29	974.2	0.3900
12	32.3	0.0208	30	981.9	0.3930
13	33.4	0.0210	31	981.9	0.3930
14	37.3	0.0217	32	1162	0.4650
15	37.3	0.0217	33	1174.9	0.4702
16	58.0	0.0276	34	1174.9	0.4702
17	64.6	0.0298	35	1652.7	0.6612
18	64.6	0.0298	36	1652.7	0.6612

Table 5. 2 The maximum displacements in the X and Y direction (*10^{-3}) （unit：cm）

Node	X direction			
	Newmark $-\beta$	Eq.（3.7）	MSM*	error（%）
1	4.2805	4.2805	3.4809	18.68
2	4.9746	4.9746	4.0413	18.76
6	1.0370	1.0370	0.9490	8.49
4	4.5863	4.5863	4.1125	10.33
5	2.7632	2.7632	2.6935	2.52
6	2.1349	2.1349	1.5145	29.06
Node	Y direction			
1	4.2805	4.2805	5.4590	27.53
2	4.0519	4.0519	3.9579	2.32
6	3.4237	3.4237	4.8582	41.90
4	1.6093	1.6093	1.9321	20.06
5	5.1787	5.1787	6.9161	33.55
6	1.2342	1.2342	3.1368	154.16

* MSM: Modal superposition method

7. CONCLUSIONS

According to theoretical analysis and numerical examination in this paper, some conclusions are obtained:

1) For the linear system, the complex modal superposition method, Eq.(3.7), completely in real form, is deduced by using transfer function method, in which the effect of multiple fold eigenvalues on structural response is considered. The new algorithm is not only concise, but also convenient to be understood and grasped by the engineers. The validity of formula is verified through Newmark integration computation of liner structure subjected to prescribed earthquake motion. In addition, the response spectrum is introduced successfully and CCQC method is proposed, which can consider the effect of multi-fold-eigenvalues.

2) For the linear system with multiple fold eigenvalues, the calculation errors will be fairly large if simply neglecting the effect of multiple fold eigenvalues.

3) It is pointed out that the method derived in this paper is suitable for computing seismic responses of MDOF classical or non-classical damping linear system with or without multiple eigenvalues.

Acknowledgements: This work gets financial support from the project of DQJB06B02, IGPCEA.

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