STATIC PUSHOVER ANALYSIS BASED ON AN ENERGY–EQUIVALENT SDOF SYSTEM

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ABSTRACT : In this paper a new enhanced Nonlinear Static Procedure (NSP) is presented and evaluated. The steps of the proposed methodology are quite similar to those of the well-known Coefficient Method (FEMA 356/440). However, the determination of the characteristics of the equivalent single degree of freedom (E-SDOF) system is based on a different philosophy. Specifically, the E-SDOF system is determined by equating the external work of the lateral loads acting on the MDOF system under consideration to the strain energy of the E-SDOF system. After a brief outline of the method, a series of applications to planar regular frames is presented. Considering the results obtained by nonlinear time-history analysis as the reference solution, a comparison between the proposed and the conventional NSPs is conducted, which shows that the proposed method gives, in general, much better results.

KEYWORDS: Nonlinear Static Procedure, pushover analysis, coefficient method, strain energy, equivalent SDOF, nonlinear dynamic analysis

1. INTRODUCTION

The objective of this paper is the presentation and evaluation of a new enhanced Nonlinear Static Procedure (NSP) for the approximate estimation of the seismic response of structures. The steps of the proposed methodology are quite similar to those of the well-known Coefficient Method (FEMA 356/440). However, the determination of the characteristics of the equivalent single degree of freedom (E-SDOF) system is based on a different philosophy. Specifically, the definition of the E-SDOF system is based on the equalization of the external work of the lateral loads acting on the multi degree of freedom (MDOF) system under consideration to the strain energy of the E-SDOF system.

Firstly, the theoretical background and the assumptions of the proposed methodology are presented and briefly discussed. Taking into account the basic assumptions and applying well-known principles of structural dynamics, some fundamental conclusions are derived and, on their basis, an alternative, energy-equivalent SDOF system is established, which can be used for a more realistic estimation of the target displacement as well as of any other response quantities of interest such as storey drifts, internal forces, etc.

Secondly, both steps needed for the implementation of the proposed methodology along with the necessary equations are systematically presented.

Finally, the accuracy of the proposed methodology is evaluated by an extensive parametric study. In particular, the methodology is applied to a series of 3-, 6-, 9- and 12-storey R/C planar regular frames. For each frame two sets of pushover analyses are conducted: i) one based on the proposed methodology and ii) a second based on the conventional FEMA 356/440 procedure. Each set of analyses comprises 12 different response spectra corresponding to real strong earthquake motions. The storey displacements are compared with those obtained by nonlinear time-history analysis, which is considered as the reference solution. The paper closes with comments on results and conclusions.

2. ELASTIC RESPONSE OF MDOF SYSTEM

2.1. Response of SDOF Systems

It is well known that the response of a MDOF system with N degrees of freedom to earthquake ground motion $\ddot{u}_{g(t)}$ is governed by the following equations (Anastasiadis 2004, Chopra 2007):

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{Ku} = -\mathbf{M}\boldsymbol{\delta}\,\ddot{u}_{g(t)} \qquad (2.1)$$

where $\mathbf{u}$ is the vector of N displacements (translations or rotations) of the N degrees of freedom relative to the ground, $\mathbf{M}$ is the NxN diagonal mass matrix, $\mathbf{C}$ and $\mathbf{K}$ are the NxN symmetric damping and stiffness matrices respectively and $\boldsymbol{\delta}$ is the influence vector that describes the influence of support displacements on the structural displacements. The vector $\mathbf{u}$ and the vector of modal forces (or moments) $\mathbf{F_s} = \mathbf{Ku}$ can be decomposed to their modal components as follows:

$$\mathbf{u} = \sum_{i=1}^{N} \mathbf{u_i} = \sum_{i=1}^{N} \boldsymbol{\varphi}_i q_i \qquad (2.2)$$

$$\mathbf{F_s} = \sum_{i=1}^{N} \mathbf{F_{si}} = \sum_{i=1}^{N} \mathbf{Ku_i} = \sum_{i=1}^{N} \mathbf{K}\boldsymbol{\varphi}_i q_i = \sum_{i=1}^{N} \omega_i^2 q_i\,\mathbf{M}\boldsymbol{\varphi}_i \qquad (2.3)$$

where $\boldsymbol{\varphi}_i$ is the modal vector, q_i is the modal co-ordinate and ω_i^2 is the natural frequency of vibration mode i. The quantity:

$$V_i = \boldsymbol{\delta}^T\mathbf{F_{si}} = \omega_i^2 q_i \boldsymbol{\delta}^T\mathbf{M}\boldsymbol{\varphi}_i = \omega_i^2 q_i L_i \qquad (2.4)$$

where $L_i = \boldsymbol{\delta}^T\mathbf{M}\boldsymbol{\varphi}_i$, represents the sum of the modal loads corresponding to non zero terms of vector $\boldsymbol{\delta}$, i.e., in the usual case of horizontal excitation V_i is equal to the modal base shear parallel to the direction of excitation. By substituting Eqns. 2.2 and 2.3 into Eqn. 2.1, premultiplying both sides of Eqn. 2.1 by $\boldsymbol{\varphi}_i^T$ and using the orthogonality property of modes, N uncoupled equations can be derived:

$$M_i\ddot{q}_i + 2M_i\omega_i\zeta_i\dot{q}_i + M_i\omega_i^2 q_i = -L_i\ddot{u}_{g(t)} \Leftrightarrow \ddot{q}_i + 2\omega_i\zeta_i\dot{q}_i + \omega_i^2 q_i = -\nu_i\ddot{u}_{g(t)} \qquad (2.5)$$

where M_i, ζ_i and ν_i are the generalized mass, the damping ratio and the modal participation factor of vibration mode i respectively. Substituting $q_i = \nu_i D_i$ into Eqns. 2.4 and 2.5 and multiplying both sides of Eqn. 2.5 by L_i gives:

$$V_i = \omega_i^2\,\nu_i\,D_i\,L_i = \omega_i^2\,M_i^* D_i \qquad (2.6)$$

$$M_i^*\ddot{D}_i + 2M_i^*\omega_i\zeta_i\dot{D}_i + \omega_i^2 M_i^* D_i = M_i^*\ddot{D}_i + 2M_i^*\omega_i\zeta_i\dot{D}_i + V_i = -M_i^*\ddot{u}_{g(t)} \qquad (2.7)$$

where $M_i^* = \nu_i L_i$ is the active mass of vibration mode i. Eqn. 2.7 demonstrates that the linear elastic response of a MDOF system with N degrees of freedom subjected to an horizontal earthquake ground motion $\ddot{u}_{g(t)}$ can be expressed as the sum of the responses of N SDOF systems, each one corresponding to a different vibration mode having mass equal to the effective modal mass and elastic resisting force equal to the modal base shear relevant to this mode.

2.2. External Work of Modal Forces F_{si}

A MDOF system with N degrees of freedom which is subjected in the differential time interval dt to an excitation $\ddot{u}_{g(t)}$ performs the differential displacements $d\mathbf{u} = \sum_{i=1}^{N} d\mathbf{u_i} = \sum_{i=1}^{N} \boldsymbol{\varphi}_i dq_i = \sum_{i=1}^{N} \boldsymbol{\varphi}_i \nu_i dD_i$. The external work of modal forces $\mathbf{F_{si}}$ of mode i on the displacements $d\mathbf{u_i}$ can be written as:

$$dE_i = \sum_{j=1}^{N} du_{ji} F_{ji} \tag{2.8}$$

where du_{ji} and F_{ji} are the j-elements of vectors $d\mathbf{u_i}$ and $\mathbf{F_{si}}$ respectively. Eqn. 2.8 can be formulated in matrix form as follows:

$$dE_i = d\mathbf{u_i}^T \mathbf{F_{si}} \Rightarrow dE_i = \boldsymbol{\varphi}_i^T \nu_i dD_i \quad \omega_i^2 \nu_i D_i \mathbf{M}\boldsymbol{\varphi}_i \Rightarrow dE_i = \omega_i^2 \nu_i \nu_i (\boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_i) D_i dD_i \Rightarrow$$

$$dE_i = \omega_i^2 \nu_i \frac{L_i}{M_i} M_i D_i dD_i \Rightarrow dE_i = \omega_i^2 M_i^* D_i dD_i \Rightarrow dE_i = V_i dD_i \tag{2.9}$$

Eqn. 2.9 shows that the external work of modal forces $\mathbf{F_{si}}$ on the displacements $d\mathbf{u_i} = \nu_i \boldsymbol{\varphi}_i dD_i$ is equal to the work of the resisting force (or the strain energy) of the E-SDOF system on the displacement dD_i.

3. INELASTIC RESPONSE OF MDOF SYSTEM

3.1. Response of SDOF Systems

In the inelastic range of behavior some basic assumptions have to be made. A major assumption is that the response of a MDOF system can be expressed as superposition of the responses of appropriate SDOF systems just like in the linear range. Each SDOF system corresponds to a vibration "mode" i with "modal" vector $\boldsymbol{\varphi}_i$. The displacements $\mathbf{u_i}$ and the inelastic resisting forces $\mathbf{F_{si}}$ are supposed to be proportional to $\boldsymbol{\varphi}_i$ and $\mathbf{M}\boldsymbol{\varphi}_i$ respectively. Furthermore, "modal" vectors $\boldsymbol{\varphi}_i$ are supposed to be constant, despite of the successive development of plastic hinges. The response of the MDOF system is governed by the following equations (Anastasiadis 2004, Chopra 2007):

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{F_s} = -\mathbf{M}\boldsymbol{\delta}\, \ddot{u}_{g(t)} \tag{3.1}$$

The only difference between Eqns. 2.1 and 3.1 is that the resisting forces (or moments) $\mathbf{F_s}$ can't be expressed as linear functions of the displacements $\mathbf{u}$, because the terms of stiffness matrix $\mathbf{K}$ do not remain constant during the loading process. However, due to the aforementioned assumptions, they can be expressed as the sum of "modal" contributions as follows:

$$\mathbf{F_s} = \sum_{i=1}^{N} \mathbf{F_{si}} = \sum_{i=1}^{N} \alpha_i \mathbf{M}\boldsymbol{\varphi}_i \tag{3.2}$$

where α_i is an hysteretic function that depends on the "modal" co-ordinate q_i and the history of excitation. The quantity:

$$V_i = \boldsymbol{\delta}^T \mathbf{F_{si}} = \alpha_i L_i \tag{3.3}$$

represents, just like in the linear range, the sum of “modal” loads corresponding to non zero terms of vector **δ**, i.e., in the usual case of horizontal excitation V_i is equal to the “modal” base shear parallel to the direction of excitation. By substituting Eqns. 2.2 and 3.2 into Eqn. 3.1, premultiplying both sides of Eqn. 3.1 by $\boldsymbol{\varphi}_i^T$ and using the orthogonality property of “modes”, N uncoupled equations can be derived:

$$\ddot{q}_i + 2\omega_i\zeta_i\dot{q}_i + \alpha_i = -\nu_i\ddot{u}_{g(t)} \tag{3.4}$$

Substituting $q_i = \nu_i D_i$ into Eqn. 3.4 and multiplying both sides by L_i gives:

$$L_i\nu_i\ddot{D}_i + L_i 2\omega_i\zeta_i\nu_i\dot{D}_i + L_i\alpha_i = -L_i\nu_i\ddot{u}_{g(t)} \Leftrightarrow M_i^*\ddot{D}_i + 2M_i^*\omega_i\zeta_i\dot{D}_i + V_i = -M_i^*\ddot{u}_{g(t)} \tag{3.5}$$

Eqn. 3.5 shows that, due to the aforementioned assumptions, the nonlinear response of a MDOF system with N degrees of freedom subjected to an horizontal earthquake ground motion $\ddot{u}_{g(t)}$ can be expressed as the sum of the responses of N SDOF systems, each one corresponding to a vibration “mode” having mass equal to the effective “modal” mass and inelastic resisting force equal to the “modal” base shear relevant to this “mode”.

3.2. External Work of “Modal” Forces F_{si}

A MDOF system with N degrees of freedom which is subjected in the differential time interval dt to an excitation $\ddot{u}_{g(t)}$ performs the differential displacements $d\mathbf{u} = \sum_{i=1}^{N} d\mathbf{u_i} = \sum_{i=1}^{N} \boldsymbol{\varphi}_i dq_i = \sum_{i=1}^{N} \boldsymbol{\varphi}_i \nu_i dD_i$. The external work of “modal” forces $\mathbf{F_{si}}$ of “mode” i on the displacements $d\mathbf{u_i}$ can be written as:

$$dE_i = \sum_{j=1}^{N} du_{ji} F_{ji} \tag{3.6}$$

where du_{ji} and F_{ji} are the j-elements of vectors $d\mathbf{u_i}$ and $\mathbf{F_{si}}$ respectively. Eqn. 3.6 can be written in matrix form as follows:

$$dE_i = d\mathbf{u_i}^T\mathbf{F_{si}} \Rightarrow dE_i = \boldsymbol{\varphi}_i^T\nu_i dD_i \quad \alpha_i \mathbf{M}\boldsymbol{\varphi}_i \Rightarrow dE_i = \alpha_i\,\nu_i\,dD_i\,(\boldsymbol{\varphi}_i^T\mathbf{M}\boldsymbol{\varphi}_i) \Rightarrow$$

$$dE_i = \alpha_i \frac{L_i}{M_i} dD_i\,M_i \Rightarrow dE_i = \alpha_i\,L_i\,dD_i \Rightarrow dE_i = V_i\,dD_i \tag{3.7}$$

Eqn. 3.7 shows that the external work of “modal” forces $\mathbf{F_{si}}$ for the displacements $d\mathbf{u_i} = \nu_i\,\boldsymbol{\varphi}_i\,dD_i$ is equal to the work of the resisting force (or the strain energy) of the SDOF system for the displacement dD_i.

4. CHARACTERISTICS OF INELASTIC SDOF SYSTEMS

An inelastic SDOF system is usually described by a bilinear force – displacement diagram V – D (figure 1), from which the most important characteristics can be derived. For the implementation of NSPs the characteristics of interest are the natural period T and the yield strength reduction factor R (Eqn. 4.1),

$$T = 2\pi\sqrt{\frac{mD_y}{V_y}} \rightarrow S_a \rightarrow R = \frac{mS_a}{V_y} \tag{4.1}$$

where S_a is the spectral acceleration. Also, the behavior of an inelastic SDOF can be described by a strain energy - displacement diagram E – D (figure 1) and the characteristics of interest can be derived from Eqns. 4.2 and 4.3 (where S_d is the spectral displacement). The E – D diagram is a 2nd degree parabolic curve in the linear range ($E = ½k\ D^2$), while in the nonlinear range is a superposition of a parabola and a line [$E = E_{el} + ½\alpha k\ (D\text{-}Dy)^2 + V_y\ (D\text{-}Dy)$]. In the special case of elastic – perfectly plastic system ($\alpha = 0$) the curve degenerates to a line with slope V_y (discontinuous line in figure 1). The two alternative ways of describing the behavior of an inelastic SDOF are absolutely equivalent.

$$E_{el} = \frac{1}{2} V_y D_y = \frac{1}{2} k\, D_y^2 \qquad (4.2)$$

$$T = 2\pi \sqrt{\frac{mDy^2}{2Eel}} \rightarrow S_a \rightarrow S_d \rightarrow R = \frac{Sd}{Dy} \qquad (4.3)$$

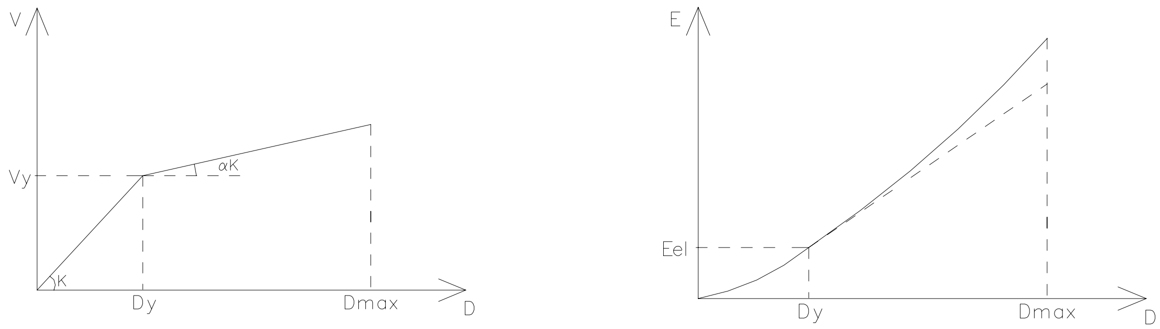


Figure 1 Force – displacement V – D and strain energy – displacement E – D curves

5. THE PROPOSED METHODOLOGY

The steps needed for the implementation of the proposed methodology are as follows:

Step 1: Create the structural model, which is, in general, a spatial frame model.

Step 2: Apply to the model a set of horizontal incremental forces (or/and moments) with distribution along the height proportional to the vector $\mathbf{M\varphi_i}$ of elastic vibration mode i and determine the strain energy – displacement curve $E_i – u_{Ni}$. The displacement u_{Ni} can be chosen to correspond to any degree of freedom, but usually the roof displacement parallel to the excitation direction is used. The strain energy E_i is equal to the work of the external forces, including forces that are perpendicular to the excitation direction and also moments around the vertical axis. In the linear range the $E_i – u_{Ni}$ diagram is a parabolic curve and if the $\boldsymbol{\varphi_i}$ vector is normalized to u_{Ni} (i.e. $\varphi_{Ni} = 1$), the strain energy is given by Eqn. 5.1:

$$E_{el,i} = \frac{1}{2}\ \mathbf{u_i}^T \mathbf{K u_i} = \frac{1}{2}\ u_{Ni}\ \boldsymbol{\varphi_i}^T \mathbf{K}\ \boldsymbol{\varphi_i}\ u_{Ni} = \frac{1}{2} k_i\ u_{Ni}^{\ 2} \qquad (5.1)$$

where k_i is the generalized stiffness of mode i. In the inelastic range the $E_i – u_{Ni}$ diagram is gradually created by superposition of lines and parabolic curves with discontinuities of curvature at the points of creation of plastic hinges.

Step 3: Divide the abscissas of the $E_i – u_{Ni}$ diagram by the quantity $\nu_i\ \varphi_{Ni} = u_{Ni}/D_i$ and determine the $E_i – D_i$ diagram of the SDOF system (figure 2). By utilizing a graphic procedure, the $E_i – D_i$ diagram can be idealized to a smoothed diagram without curvature discontinuities (like the E – D diagram of figure 1) and the characteristics of the E-SDOF system can be derived directly from Eqns. 4.2 and 4.3. However, because of the complexity of the $E_i – D_i$ diagram this approach is difficult to apply, so follow the procedure of step 4.

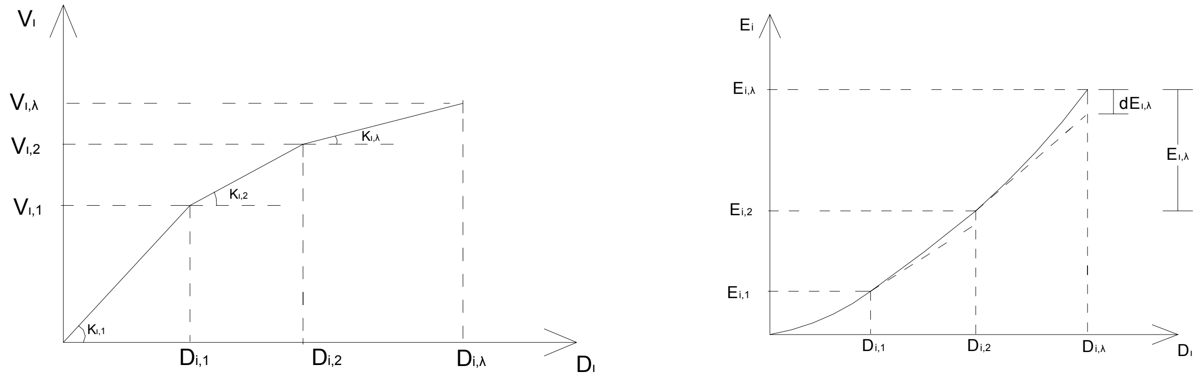


Figure 2 Force – displacement $V_i – D_i$ and strain energy – displacement $E_i – D_i$ curves

Step 4: Calculate the work $E_{i,\lambda}$ of the external forces (or/and moments) in each of λ discrete intervals between the successive creation of plastic hinges. $dE_{i,\lambda}$, as part of $E_{i,\lambda}$ (Eqn. 5.2), is considered to derive from Eqn. 5.3.

$$dE_{i,\lambda} = E_{i,\lambda} – V_{i,\lambda-1} (D_{i,\lambda} – D_{i,\lambda-1}) = E_{i,\lambda} – V_{i,\lambda-1}\, dD_{i,\lambda} \quad (5.2)$$

$$dE_{i,\lambda} = \frac{1}{2} k_{i,\lambda}\, dD_{i,\lambda}^{\ 2} \Rightarrow k_{i,\lambda} = 2\, dE_{i,\lambda} / dD_{i,\lambda}^{\ 2} \quad (5.3)$$

where $k_{i,\lambda}$ is the stiffness of the E-SDOF corresponding to mode i during the interval λ. The resisting force $V_{i,\lambda}$ is given by Eqn. 5.4:

$$V_{i,\lambda} = V_{i,\lambda-1} + k_{i,\lambda}\, dD_{i,\lambda} \quad (5.4)$$

For λ = 1 (i.e., when the first plastic hinge is created) the force $V_{i,1}$ is equal to the base shear parallel to the direction of excitation. By utilizing Eqns. 5.2 – 5.4 for each interval, determine the force – displacement diagram $V_i – D_i$ of mode i (figure 2).

Step 5: Idealize $V_i – D_i$ to a bilinear curve using one of the well known graphic procedures (e.g. FEMA 356, 3.3.3.2.4) and calculate the period T of the E-SDOF system corresponding to mode i from Eqn. 4.1. It is stated that the mass m is equal to the effective mass M_i^* of mode i (Eqn. 3.5).

Step 6: Calculate the target displacement and other response quantities of interest (drifts, plastic rotations, etc.) of mode i, using one of the well known procedures (e.g. FEMA 356, 3.3.3.3.2 / FEMA 440, 10.4).

Step 7: Repeat steps 2 to 6 for an adequate number of modes. Obviously, this is not necessary, because the proposed method could be applied reductively for the fundamental mode only.

Step 8: Calculate the extreme values of the response quantities, using one of the well established formulas of modal superposition (SRSS or CQC).

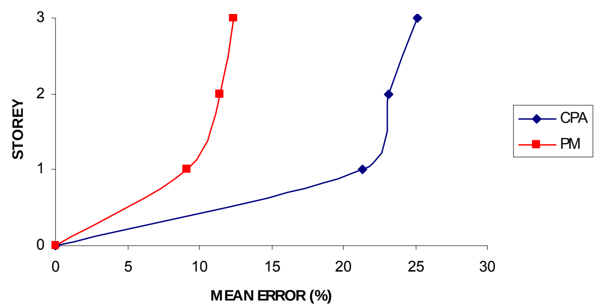
It is worth noticing that the proposed methodology can be applied without restrictions to 2D and 3D structures as well as to regular and irregular buildings. Also, it is apparent that it can be easily implemented in existing software. Finally, this approach is consistent with advanced NSPs, e.g. multi-modal pushover analysis (Chopra et al. 2001), adaptive pushover analysis (e.g., Pinho et al. 2005), etc.

6. APPLICATIONS

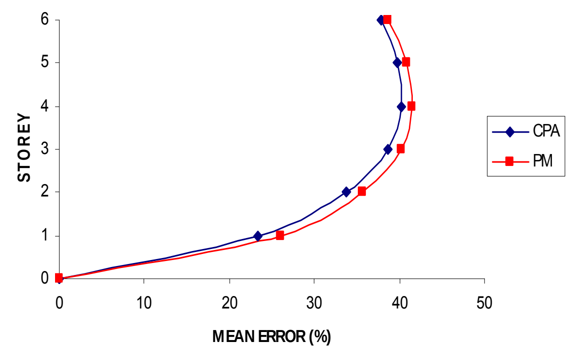
In order to evaluate the accuracy of the proposed method an extensive parametric study is carried out. In particular, the methodology is applied to a series of 3-, 6-, 9- and 12-storey R/C planar regular frames designed according to the Greek codes. For each frame two sets of pushover analyses are performed: i) one based on the proposed methodology (PM) using the fundamental mode only and ii) a second based on the conventional FEMA 356/440 procedure (CPA). Each set of analyses comprises 12 different response spectra corresponding to real strong earthquake motions recorded in Greece.

The modification factor C_1 that correlates the expected maximum inelastic target displacement to the displacement calculated for linear elastic response is obtained by nonlinear dynamic analysis of the E-SDOF system for each excitation. This is considered necessary because the relevant equations given by codes are based on statistical processing of data with excessive deviation and, therefore, in case of application of NSPs using response spectra of real ground motion (as in this paper) great inaccuracies could result (Manoukas et al. 2006).

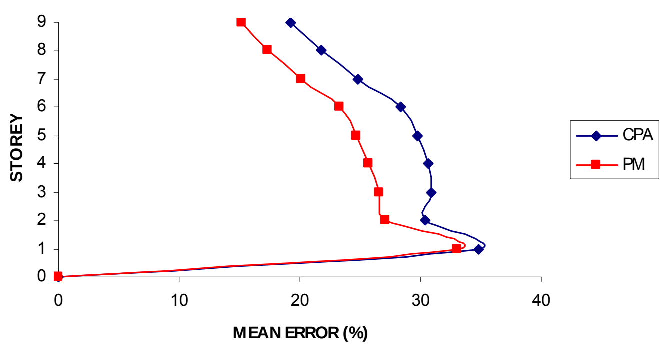
The storey displacements of the frames under consideration are compared with those obtained by nonlinear time-history analysis, which is considered as the reference solution. In figure 3 the mean errors for the 12 excitations (in relevance to the nonlinear dynamic analysis results) of storey displacements are shown. For each frame two curves are plotted: i) according to the proposed methodology (PM) and ii) according to the conventional FEMA 356/440 procedure (CPA). Notice that the positive sign (+) means that the displacements obtained by NSPs are greater than those obtained by nonlinear time-history analysis. In reverse, the negative sign (-) means that the storey displacements are underestimated by NSPs.



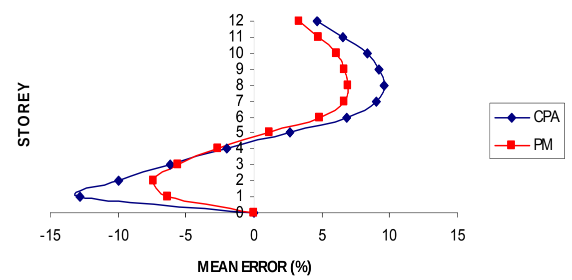
3-storey frame



6-storey frame



9-storey frame



12-storey frame

Figure 3 Mean errors (%) of storey displacements for the proposed (PM) and the conventional (CPA) NSPs

7. CONCLUSIONS

From figure 3 becomes clear that the two compared procedures give similar displacement profiles. However, the mean errors resulting from the proposed method are sufficiently smaller, except in case of the 6-storey frame. Specifically, in refer to the roof displacement, the use of the proposed method instead of the conventional pushover analysis leads to a reduction of the mean error from 25% to 12% for the 3-storey frame, from 19% to 15% for the 9-storey frame and from 5% to 3% for the 12-storey frame. In reverse, the mean error of the roof displacement of the 6-storey frame increases from 38% to 39%. Conclusively, the whole investigation shows that, in general, the proposed methodology gives much better results compared to those produced by the conventional procedure.

Similar results have been obtained from application of the proposed method to irregular planar frames (Manoukas et al. 2008). However, the generalization of such conclusions is risky. In order to obtain secure generalized conclusions excessive investigations would be necessary comprising application of the proposed method to a large variety of structures using an adequate number of earthquake ground motions. It is also clear that the achievement of a satisfactory accuracy in one response quantity (storey displacements in this paper) does not ensure analogous accuracy in other quantities of interest, e.g. drifts (Manoukas et al. 2006).

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