AN INTEGRATED METHODOLOGY FOR DAMAGE IDENTIFICATION IN EXISTING BUILDINGS USING OPTIMAL SENSOR PLACEMENT TECHNIQUES

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ABSTRACT :

In this study, a complete methodology for vibration based damage identification in important buildings in seismically active areas is investigated. In the first phase of the study, the buildings are instrumented with sensors and their modal parameters are determined using system identification techniques. These modal properties belong to the damaged state of the structure. In the second step, a detailed finite element (FE) model of the structure is developed. This initial FE model is then updated iteratively by minimizing the differences between the modal parameters obtained from the possibly damaged structure and the modal parameters obtained from the FE model with respect to the unknown model parameters which are the stiffness reduction factors. This is an ongoing study. The system identification and the model updating part of this study is published separately. In this paper, the focus will be on the optimal sensor placement (OSP) techniques and their aplication in civil engineering structures. This paper addresses the application of six different optimal sensor placement (OSP) techniques on building type structures with flexible joints. Different techniques that are implemented are the Effective Independence Method (EFI), Optimal Driving Point (ODP) based method, Non-Optimal Driving Point (NODP) based method, the Effective Independence Driving Point Residue Method and the Singular Value Decomposition (SVD) based method. The techniques are compared using the determinant, trace and the condition number of the Fisher Information Matrix. The results show that the Effective Independence (EFI) Method is the best and results in a sensor configuration possessing a smaller estimate error covariance matrix yielding better state estimates than the other methods. This study also shows that the SVD based and the EFI methods give the same final sensor configurations and are essentially identical except for the fact that SVD based method brings a criteria for deleting more number of sensors at one iteration. Next, the robustness of each sensor placement technique to the presence of noise in the measurements is investigated by the Modal Assurance Criteria Values (MAC) between the mode shapes obtained from the FE model and the noisy measurements. The best correlations are obtained from the EFI method. The results obtained from this study show that in structures with complex damage patterns, the sensor configurations must be based on OSP techniques. Within this context, EFI method used for on-orbit modal identification and correlation of LSS is promising for widespread use in civil structures.

KEYWORDS: Structural health monitoring, optimal sensor placement, system identification, finite element model updating, inverse problems in mechanics, Fisher Information Matrix, Singular Value Decomposition, covariance matrix, fractional eigenvalue distribution, system identification.

1. INTRODUCTION

The post-earthquake damage identifications in existing buildings in Turkey after the catastrophic 1999 Kocaeli and Duzce earthquakes have largely relied on visual inspections or the so-called 'walk down evaluations'. This is also the common practice for many seismically active areas in the world after catastrophic earthquakes. The implementation of these rapid assessment methods may be easier, cheaper and quicker for damage identification in existing residential buildings in big Metropolitan Cities due to the excessively large size of the building stock, scarcity of time, qualified manpower and financial resources. However, in important buildings such as hospitals, schools, fire and police stations, implementation of more objective and scientific methods is imperative. In this study, a complete methodology for vibration based damage identification in important buildings in seismically active areas is investigated. The fundamental idea of this methodology is that damage results in changes in the dynamic properties of structures. In the first phase of the study, the buildings are instrumented with sensors and their modal parameters (frequencies and mode shapes) are determined using system identification techniques. These modal properties belong to the damaged state of the structure. Special care is given to optimal sensor placement (OSP). The second step of the study is the identification of damage by the finite element model updating technique. For this purpose, a detailed Finite Element (FE) model of the structure is developed first. The unknown parameters of the FE model updating problem are the reduction factors of the stiffnesses of the finite elements. This initial FE model is then updated iteratively by minimizing the differences between the modal parameters obtained from the possibly damaged structure and the modal parameters obtained from the FE model with respect to the unknown model parameters which are the stiffness reduction factors. The system identification and the model updating applications on building type structures can be found in different papers of the author (Gundes Bakir *et al.* (2007, 2008) and Reynders *et al.* (2007)). It is shown that using this methodology, damage is detected, located and quantified accurately based on completely objective scientific criteria rather than the current subjective methods based on engineering judgement. In this paper on the other hand, the focus will be on the optimal sensor placement techniques and their aplication in civil engineering structures.

The sensor location problem is a key issue for on-orbit modal identification and correlation of large space structures (LSS). The subject has been extensively studied in aerospace industry but very little has been done in civil engineering up to date for the finite element model updating and identification of civil structures. The practice in civil engineering applications today is that experts investigate the mode shapes obtained from the FE model and judge the locations of the sensors in a rather subjective way. However, when the damage pattern is very complex and there are too many design variables, this subjective choice substantially affects the reliability of the identifications and the results of the finite element model updating procedure. This paper addresses the application of different optimal sensor placement (OSP) techniques for the FE model updating problem of building type structures with flexible joints. For this purpose, a toolbox OPTISEP (OPTImal SEnsor Placement) is developed in MATLAB which can compute the optimum sensor locations according to seven alternative optimal sensor placement techniques currently used in NASA for large space structures. Different techniques that are incorporated in the toolbox are the Effective Independence Method (EFI) developed by Kammer (1991), Optimal Driving Point (ODP) based method (Imamovic,1998), Non-Optimal Driving Point (NODP) based method (Imamovic,1998), Effective Independence

Driving Point Residue Method, the Singular Value Decomposition based method developed by Kim *et al.* (Kim and Park, 1997) and Sensor Set Expansion Technique (Kammer,2005). In the rest of the paper, these techniques will be explained first and will be applied on a numerical example subsequently.

2. OPTIMAL SENSOR PLACEMENT TECHNIQUES

In this section, different OSP techniques such as the Effective Independence Method (EFI), Optimal Driving Point (ODP) based method, Non-Optimal Driving Point (NODP) based method, Effective Independence Driving Point Residue Method and the Singular Value Decomposition based method will be explained in detail.

2.1. The Effective Independence Method (EFI)

For the purpose of an accurate and reliable test-analysis correlation, measurements must be obtained at the locations of the structure which provide linearly independent test-mode partitions. Spatial independence means that if the sensor output equation is given by:

$$\mathbf{u}_s = \boldsymbol{\varphi}_s \mathbf{q} \tag{1}$$

the sensors can be sampled and an estimate can be calculated for the target states $\hat{\mathbf{q}}$ by solving Eq.(1) resulting in:

$$\hat{\mathbf{q}} = [\boldsymbol{\varphi}_s^{\mathrm{T}} \boldsymbol{\varphi}_s]^{-1} \boldsymbol{\varphi}_s^{\mathrm{T}} \mathbf{u}_s \tag{2}$$

where $\mathbf{u_s}$ is the output from the sensors, $\boldsymbol{\varphi}_s$ is the matrix of the target modes partitioned to the sensor locations obtained from the FE model, and $\mathbf{q}$ is the vector of target modal coordinates. The best estimate in placing *m* sensors within the *s* candidate locations implies that the covariance matrix of the estimate errors will be a minimum. Within this context, the output $\mathbf{u_s}$ must be modified as:

$$\mathbf{u}_s = H(\mathbf{q}) + N = \boldsymbol{\varphi}_s \mathbf{q} + N \tag{3}$$

where *H* is the process measurement and the vector *N* represents the stationary Gaussian white noise variance $\boldsymbol{\psi}_{\mathbf{o}}^2$. The covariance matrix of the estimate error can be expressed as:

$$P = E[(\mathbf{q}-\hat{\mathbf{q}})(\mathbf{q}-\hat{\mathbf{q}})^T] = \left[\left(\frac{\partial H}{\partial \mathbf{q}}\right)^T \left[\boldsymbol{\psi}_0^2\right]^{-1} \left(\frac{\partial H}{\partial \mathbf{q}}\right)\right]^{-1} \tag{4}$$

where *E* denotes the expected value. The covariance matrix can be simplified as:

$$P = \left[\boldsymbol{\varphi}_s^{\mathrm{T}} (\boldsymbol{\psi}_o{}^2) \boldsymbol{\varphi}_s\right]^{-1} = Q^{-1} \tag{5}$$

Where Q is the Fisher Information Matrix. The best state estimate $\hat{\mathbf{q}}$ can be obtained by maximizing Q which results in the minimization of the covariance matrix. For simplicity, it is assumed that the measurement noise is uncorrelated and possesses identical statistical properties of each sensor. The Fisher Information Matrix can be simplified as:

$$Q = \frac{1}{\boldsymbol{\psi}_{\mathbf{o}}^2} \boldsymbol{\varphi}_{\mathbf{s}}^{\mathbf{T}} \boldsymbol{\varphi}_s = \frac{1}{\boldsymbol{\psi}_{\mathbf{o}}{}^2} A_o \tag{6}$$

Thus, the problem reduces to the fact that in order to maximize Q, a suitable norm of A_o must be maximized. In the rest of the paper, A_o will be referred to as the Fisher information matrix. First, the following eigenvalue problem is solved:

$$[A_o - \lambda \mathbf{I}]\boldsymbol{\psi} = 0 \tag{7}$$

The following relations hold for this problem:

$$\boldsymbol{\psi}^{\mathbf{T}} A_o \boldsymbol{\psi} = \lambda \tag{8}$$

$$\boldsymbol{\psi}^{T} \boldsymbol{\psi} = \mathbf{I} \tag{9}$$

A matrix G can be formed, each column of which can be summed to the eigenvalue of A_o and which can be expressed as:

$$G = [\boldsymbol{\varphi}_{\mathbf{s}} \boldsymbol{\psi}] \otimes [\boldsymbol{\varphi}_{\mathbf{s}} \boldsymbol{\psi}] \tag{10}$$

where the symbol $\otimes$ represents a term-by-term matrix multiplication. The *i*th entry within a column of the *G* matrix represents the contribution of the *i*th sensor to the associated eigenvalue. In order to bring each direction within the absolute identification space to equal importance, matrix *G* must be multiplied by the matrix of eigenvalues λ from right as shown:

$$F_E = [\boldsymbol{\varphi}_{\mathbf{s}} \boldsymbol{\psi}] \otimes [\boldsymbol{\varphi}_{\mathbf{s}} \boldsymbol{\psi}] \lambda^{-1} \tag{11}$$

Matrix F_E is named as the fractional eigenvalue distribution. The *i*th term in the *j*th column of the matrix F_E represents the fractional contribution of the *i*th sensor location of the *j*th eigenvalue. Addition of the terms within each row of F_E results in the column vector E_D which is called the 'Effective Independence Distribution'. Alternatively, the independence distribution vector E_D can be expressed as the diagonal of the matrix *E* which is given below:

$$E = \boldsymbol{\varphi}_{\mathbf{s}} \boldsymbol{\psi} \lambda^{-1} \boldsymbol{\psi}^{\mathbf{T}} \boldsymbol{\varphi}_{\mathbf{s}}^{\mathbf{T}} = \boldsymbol{\varphi}_{\mathbf{s}} A_o^{-1} \boldsymbol{\varphi}_{\mathbf{s}}^{\mathbf{T}} \tag{12}$$

Using the orthonormality of the eigenvector $\boldsymbol{\psi}$, *E* can be expressed as:

$$E = \boldsymbol{\varphi}_{\mathbf{s}} \left[\boldsymbol{\varphi}_{\mathbf{s}}{}^{\mathbf{T}} \boldsymbol{\varphi}_{\mathbf{s}}\right]^{-1} \boldsymbol{\varphi}_{\mathbf{s}}^{\mathbf{T}} \tag{13}$$

The diagonal entries of the matrix E can get values between 0 and 1. If E_{ii} = 0, the modes are not observable from the corresponding sensor. If E_{ii} = 1, the corresponding sensor location is critical to the identification of target modes. The above equations mean that the diagonal entries of the E matrix represent the contributions to the rank of $\boldsymbol{\varphi}_s$. The diagonal element that has the smallest value in the prediction matrix represents the degree of freedom and the corresponding sensor that has the smallest contribution to the identification of $\boldsymbol{\varphi}_s$. This sensor location can be eliminated from the initial set of candidate sensors until the number of sensors in the initial candidate set equals the fixed sensor number M being used. The final sensor configuration is obtained such that the trace and the determinant of the Fisher Information Matrix are maximized and the condition number of the Fisher Information Matrix is minimized.

2.2. Optimum Driving Point (ODP) Based Method

The Optimum Driving Point (ODP) technique is designed to detect positions which are close to or on the nodal lines of any mode within a predetermined frequency range. In order to identify the nodal points of mode shapes, modal constants for all chosen modes at each degree of freedom are multiplied and the result is a coefficient called the ODP parameter which can be expressed as follows:

$$ODP(i) = \prod_{r=1}^{m} \left\| \phi_{i,r} \right\| \tag{14}$$

The sensors which have an ODP parameter equal to 0 are either on a nodal line or close to a nodal point of the modes. These locations should not be selected as an excitation or sensor location. The technique selects the sensors with the largest ODP parameter in order to prevent the choice of sensors placed on nodal lines of a vibration mode.

2.3. Non-optimal driving point based method

The methodology is iterative in nature and is based on an algorithm that selects the candidate sensor position having the smallest target mode shape displacement as shown:

$$NODP(i) = \min_{r} \left(\left\| \phi_{i,r} \right\| \right) \tag{15}$$

2.4. EFI-DPR technique

The main difference between EFI method and the EFI-DPR method (Imamovic, 1998) is that when selecting the least contributing DOF to the rank of the truncated mode shape matrix, the parameter $ADDOFV_i$ defined in Eq. (16) for all DOF is accounted for so that the DOFs with low response are deleted first.

$$ADDOFV_i = \sum_{j=1}^{N} \frac{\phi_{ij}^2}{\omega_j} \tag{16}$$

In the EFI-DPR technique, the E_D vector in the EFI Method changes in the following way:

$$E_{D_i} = [\phi\psi]^2 \lambda^{-1} \{1\}_i \, ADDOFV_i \tag{17}$$

2.5. The SVD based method

A Singular Value Decomposition (SVD) based method is explained by Kim and Park (1997). This technique which is well explained in this reference is similar to the effective independence technique and will not be explained here again for the purposes of brevity. The only difference of the technique from the Effective Independence technique is the fact that it brings a criterion for deleting more number of sensors at one iteration.

3. NUMERICAL EXAMPLE: OPTIMAL SENSOR PLACEMENT FOR THE JOINT STIFFNESS IDENTIFICATION OF A MULTISTOREY BUILDING

The building type that is going to be studied in the present study is a typical existing building in the region to the north of the Marmara Sea in North West Turkey. The building is a four storey structure. Due to the complex nature of the damage, 7 modes are selected for FE model updating problem. For the initial candidate set of sensor locations 82 degrees of freedom are selected. The choice of the initial set of sensor locations is based on engineering judgement, detailed analysis of mode shapes and accessibility of these locations. Thus the problem is to select 20 sensor locations that would give the best estimates of the 7 target modes out of the initial candidate set of 82 sensors. The sensor number 20 is physically a realistic number for a four storey structure based on the economic considerations and the common practice of measuring more degrees of freedom than the number of target modes.

The goodness of the sensor configuration estimated by the different methods can be compared in terms of the trace, determinant and the condition number of the Fisher Information Matrix. Maximization of the determinant of the information matrix is more important than the others because it is equivalent to minimising the hypervolume of the confidence region. The determinant of the Fisher Information Matrix is a measure of the amount of information in measurements. The condition number of the Fisher Information Matrix is a measure of the sensor configuration estimations's robustness to model errors in the mode shapes obtained from the FE method representation of the structure.

First, the fractional eigenvalue distribution is illustrated in a three dimensional plot in Fig. 1. The abscissa and the ordinate of the figure represent the eigenvalues of the Fisher Information Matrix and the DOFs in the initial candidate sensor set. Each peak in the plot represents the fractional contribution of each sensor location to the corresponding eigenvalue of the Fisher Information Matrix. The figure shows clearly that many sensor locations do not contribute significantly to many of the eigenvalues of the Fisher Information Matrix. The highest fractional eigenvalue contributions are observed for the 70th to 80th degrees of freedoms and these sensors contribute mostly to the lowest eigenvalues. Fig.2 shows the Effective Independence value sorted in descending order. The largest value is 0.58 and it is apparent that a majority of sensors have insignificant contributions to the identification of mode shapes. The optimal sensor locations obtained using different techniques is shown in Fig. 3. Fig.4a shows the determinant values of the Fisher Information matrix calculated using different optimal sensor placement techniques. It is apparent from the results that the best OSP techniques are the EFI, SVD-OSP and EFI-DPR techniques with EFI performing slightly better than EFI-DPR. The sensor configurations obtained using these two methods give more information regarding the mode shapes compared to the ODP and NODP based methods. The figure also shows that NODP method gives the worst sensor configuration.

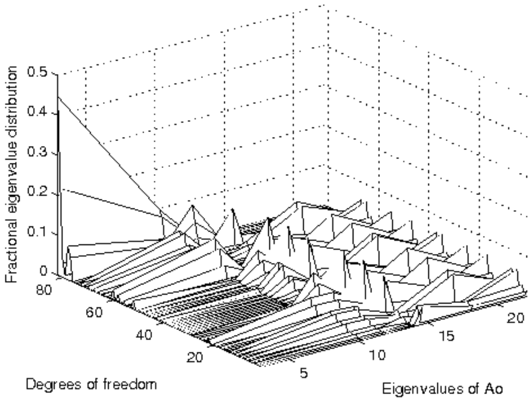

Figure 1: Fractional eigenvalue distribution for initial candidate sensor set and 7 target modes.

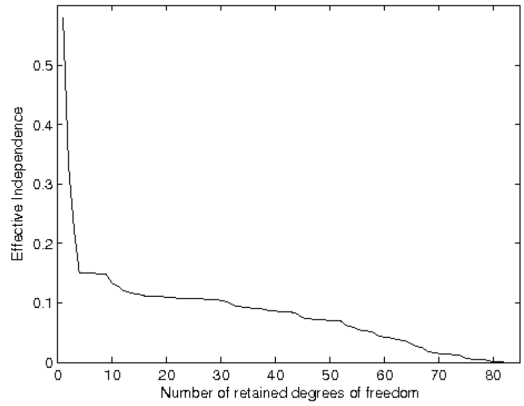

Figure 2: Effective independence distribution for 82-DOF DOF candidate sensor set and 7 target modes.

Fig.4b shows the trace value of the Fisher Information matrix calculated using different optimal sensor placement techniques. The figure shows that the EFI-DPR and EFI techniques result in a sensor configuration possessing a smaller estimate error covariance matrix yielding better state estimates than the ODP and especially NODP techniques. EFI-DPR slightly outperforms the EFI. Although the performance of the ODP is relatively close to the EFI and EFI-DPR techniques, NODP certainly underperforms and gives the worst sensor configuration.

Table 1 shows the condition number of the Fisher Information Matrix obtained at the end of the iterations for each OSP technique. It is apparent that the best condition numbers are obtained from the EFI and the OSP-SVD methods.

Next, different OSP techniques are compared in terms of the MAC values. All the analytical modes obtained from the FE model are correlated with all the simulated measured modes and the results are placed in a matrix. In this paper, the measurements are assumed to be obtained from the undamaged reference structure. The aim is to investigate the robustness of each sensor placement technique to the presence of noise in the measurements.

Figs. 5, 6, 7 and 8 show the MAC values between the mode shapes obtained from the FE model and the simulated noisy measurements in the presence of moderate noise based on an optimal sensor configuration determined by the Optimal Driving Point based, Non-Optimal Driving Point based, Effective Independence and Effective Independence Driving Point Residue Methods, respectively. The SVD based technique is not included in the analysis because it gives exactly the same configuration with the EFI method. It is apparent from the comparison of the figures that all methods give very high MAC values for the first, fourth, fifth and sixth modes. For the second mode, the best predictions are obtained using the sensor configuration of the EFI technique. For the third mode, EFI-DPR and ODP perform the best. For the third mode, EFI method gives a MAC value of almost 68% which is a value higher than the prediction of EFI-DPR for the second mode. For the seventh mode, the worst results are obtained by the ODP method giving a MAC value 34%. Overall, the EFI method gives the best MAC values in the presence of moderate noise.

Table 1: The condition number of the Fisher information matrix

Technique	*Condition number*
NODP	5.58e4
EFI	4.63
ODP	177.915
EFI-DPR	12.23
OSP-SVD	4.42

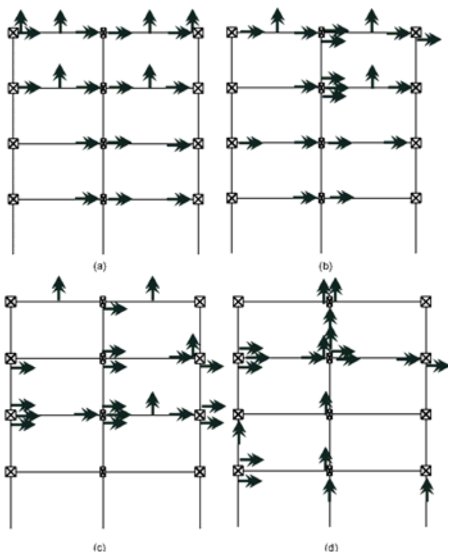

Figure 3: Selected final sensor configurations according to the (a) EFI and the SVD based methods; (b) EFI-DPR method; (c) ODP method; (d) NODP method.

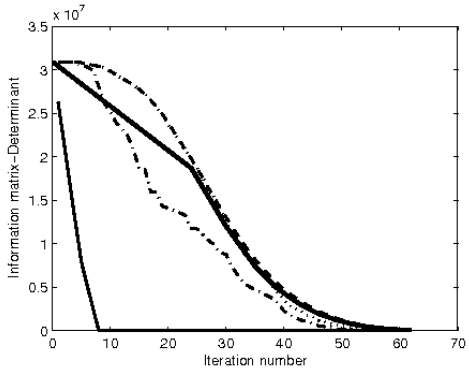

Figure 4a: The Determinant of the Fisher Information Matrix according to the different methods: NODP (solid line), EFI (dashed line), ODP (dashed-dotted line), EFI-DPR (dotted-line) and the SVD based method (thick solid line).

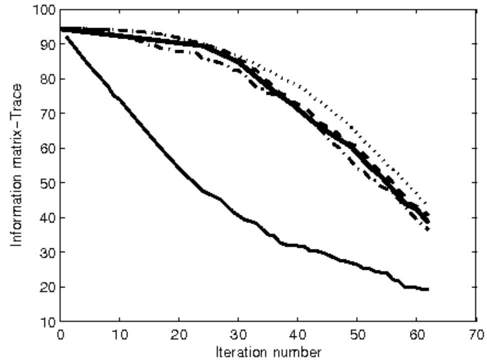

Figure 4b: The Determinant of the Fisher Information Matrix according to the different methods: NODP (solid line), EFI (dashed line), ODP (dashed-dotted line), EFI-DPR (dotted-line) and the SVD based method (thick solid line).

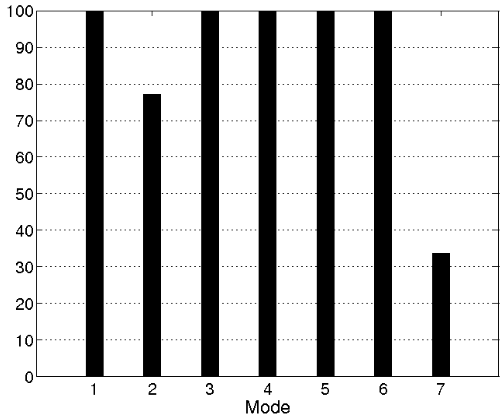

Figure 5: MAC values between the mode shapes obtained from the FE model and the simulated noisy measurements in the presence of moderate noise. The optimal sensor configuration is determined based on the Optimal Driving Point based method.

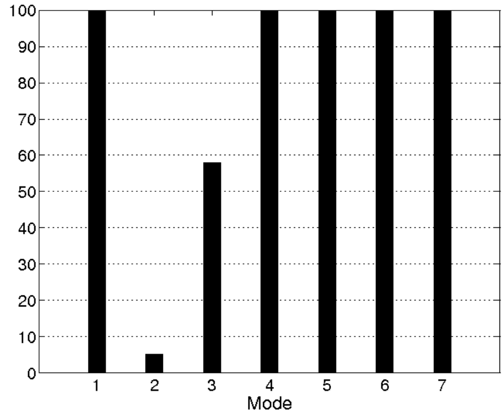

Figure 6: MAC values between the mode shapes obtained from the FE model and the simulated noisy measurements in the presence of moderate noise. The optimal sensor configuration is determined based on the Non-Optimal Driving Point based method.

4. CONCLUSIONS

In this study, several different techniques developed for optimal sensor placement in large space structures are implemented on multi-storey buildings. For this purpose, a toolbox OPTISEP (OPTImal SEnsor Placement) is developed in MATLAB which can compute the optimum sensor locations according to 6 alternative optimal sensor placement techniques. The optimal sensor locations are then determined using each of the sensor placement techniques in the toolbox. It is apparent that the best results are obtained from the Effective Independence Method, Effective Independence Driving point Residue Method and the Singular Value Decomposition Based Method. The results also show that the Effective Independence method and the Singular Value Decomposition based method give exactly identical results for the optimal sensor configuration. This should be anticipated as these two methods are in fact identical. The improvement that the Singular Value Decomposition based method brings on the Effective Independence Method is a criteria for deleting more number of sensors at one iteration. However, it should be stated that the Effective Independence Method overall is computationally inexpensive and with the advances in computer technology, computational time is no longer an important issue in optimal sensor placement technology for ordinary structures. If the techniques are going to be applied on large scale structures, Singular Value Decomposition based method may be preferable. Next, the methods are compared in terms of the MAC values between the mode shapes obtained from the FE model and the simulated noisy measurements. The aim is to investigate the robustness of each technique to the presence of noise in measurements. The MAC values calculated showed that for noisy measurements, the Effective Independence Method outperforms the other methods.

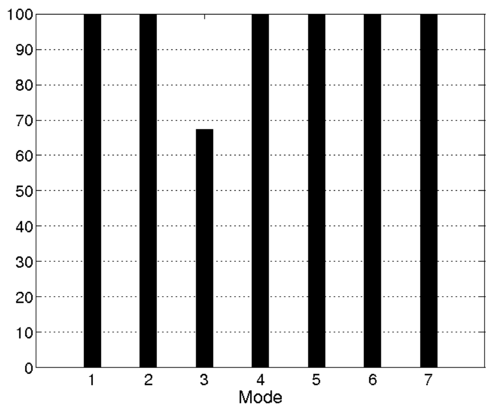

Figure 7: MAC values between the mode shapes obtained from the FE model and the simulated noisy measurements in the presence of moderate noise. The optimal sensor configuration is determined based on the Effective Independence method.

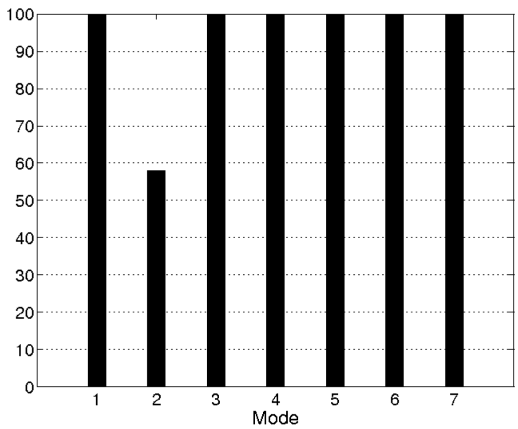

Figure 8: MAC values between the modeshapes obtained from the FE model and the simulated noisy measurements in the presence of moderate noise. The optimal sensor configuration is determined based on the Effective Independence-Driving Point Residue method.

It is apparent from the results of this study that the Effective Independence method developed initially for on-orbit modal identification and correlation of large space structures can be confidently used for the optimal sensor placement in civil structures such as buildings and bridges. The locations of sensors in civil structure measurements today are largely based on the inspection of the mode shapes from the FE model and subjective engineering judgement of the engineers. This subjective configuration of sensors can give satisfactory results in structures with simple damage patterns. However, if the damage pattern in a structure is complex, location of sensors become crucial for the system identification and damage detection by FE model updating studies. Thus, in structures with complex damage patterns, the sensor configurations must be based on optimal sensor placement techniques. Within this context, EFI method used for on-orbit modal identification and correlation of large space structures is promising for widespread use in civil structures.

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