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# A SIMPLIFIED NON-LINEAR SPRING APPROACH FOR DYNAMIC SOIL-STRUCTURE INTERACTION

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**ABSTRACT:**

The linear behavior of the soil is only valid for very small level of strain. Therefore, the deformations induced by a seismic motion in the soil can easily reach the limit of its linear elastic domain. Consequently, it is very important to develop methods for taking into account the non-linearity of the soil in the problem of the dynamic soil-structure interaction, especially for moderate to strong motions able to induce damage on the superstructure. With the aim of making a simplified dynamic soil-structure interaction analysis, instead of a complete costly numerical modelling which includes the soil and the superstructure, it is possible to model the soil by springs which have equivalent characteristics (Winkler's springs approach). Thus, in order to take into account the non linear behavior of the soil, the use of springs with bilinear elastoplastic behavior is investigated. For each simplified model, two sets of springs are used: one for vertical displacements and the other for the horizontal one. The investigated structure is a two-story reinforced concrete frame placed on a rigid shallow foundation. Plastic-hinge beam-column elements are used to represent the behavior of reinforced concrete structural elements. The values of the springs' parameters are obtained by matching the dynamic non-linear structural response of the proposed simplified approach and a complete 2D finite element analysis using a realistic constitutive model for the soil. In order to obtain the optimal set of springs' parameters for a given input motion, an optimization procedure based on the value of the induced structural damage is applied. It is shown that for a given optimal set of springs' parameters, the accuracy of the simplified method is strongly correlated with the frequency content of the input. Thus, an applicability domain can be established for a set of parameters in terms of the soil classification and frequency contents of the motion.

**KEYWORDS:** Non-linear Dynamic Soil-Structure Interaction, Winkler Model, Concrete inelastic frames, Simplified model, Damage

## 1. INTRODUCTION

The deformations induced by a seismic motion in the soil can reach the limit of its linear elastic behavior and thus it is necessary to take into account its non linear behavior in the dynamic soil-structure interaction problem (DSSI). Consequently, it is very important to develop methods considering the non-linearity of the soil in the DSSI, especially for moderate to strong motions, able to induce damage on the superstructure. Direct, substructure and simplified methods are ways to investigating the SSI. The simplest representation of the effect of the soil on the seismic response of the structure was suggested in 1867 by E. Winkler (Kerr, 1984). The Winkler spring model was extensively used for shallow foundations under static loading and, since 1960, for dynamic loadings (Betbeder-Matibet, 2003). The concept of the method is to schematize the relation between soil and foundation by a bed of independent springs acting in the vertical direction. The fundamental associated problem is to determine the stiffness of the springs that replace the soil below the foundation (Dutta & Ray, 2002). Considerable studies were done to find the stiffness of these springs. Biot (1937) solved the problem for an infinite beam with a concentrated load lying on a continuous elastic soil. Terzaghi (1955) proposed some recommendations of the stiffness for a rigid plate placed on the ground (Daloglu & Vallabhan, 2000). Vesic (1961) tried to develop a value for stiffness of the springs. He proposed an equation for this value to be used in the Winkler model. We can also find some recommendations for the values of stiffness of the springs to be used on seismic codes, for example, the values advised by ATC 40 based on the works of Gazetas (1991) (ATC40, 1996). The model of coupled Winkler springs was proposed to avoid the independency of the springs that can not reproduce the stress concentration on

the edges of foundations (Betbeder-Matibet, 2003). Chen and Lai proposed an elastoplastic model of the Winkler springs, which allows the uplift of the foundation at the seismic response of the pillars of bridges (Chen & Lai, 2003). A simplified model was proposed by Kocak for the three-dimensional analysis of the SSI for layered soil with Winkler springs (Kocak & Mengi, 2000).
In this work, a simplified method is used to evaluate the nonlinear behavior of soil over the dynamic soil-structure interaction problem. In order to compute the spring parameters, we match the results of a FE direct model with the proposed simplified approach by an optimization procedure. The sensitivity of the parameters of the springs is also investigated. The results are compared for different earthquakes in order to evaluate the effectiveness of the computed parameters.

## 2. PROPOSED APPROACHES

In order to investigate the effect of non-linear soil behavior on seismic response of the structure, a comparative dynamic analysis is carried out. First, a complete FE model including soil and structural non-linear behavior is used to asses the effect of non-linear DSSI on the structural response. Secondly, a two-step approach is carried out solving a non-linear 1D wave propagation problem for a soil column. The obtained free field motion is imposed as ground motion to spring supported structural model. The two approaches are sketched in Fig 1.

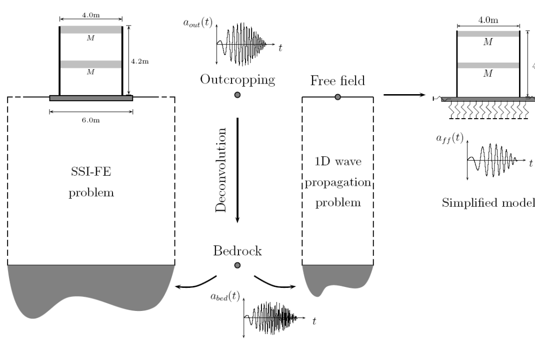


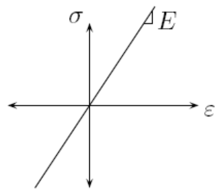
Figure 1 : Summary of proposed approaches

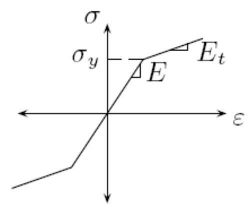
The investigated soil deposit is a homogenous dry dense sand. The bedrock is supposed to be placed at the depth of 30m.

### *2.1. Structural model*

The beam-column model used for the superstructure has nonlinear hysteretic bending moment-end rotation characteristics. The model consists of a linear component and an ideally elastoplastic component, then each beam can have only bilinear hysteresis loops at the ends, as a result of the nature of the model. For this model, the initial slope on the moment-curvature diagram is determined from the sum of the stiffness of both the components while the second slope is determined by the stiffness of only the linear component of the beam. Plastic hinges that yield at constant moment form the elastic-plastic component. The moment in the elastic component continues to increase, simulating strain hardening (Saez, 2007). The mass of the building is assumed to be uniformly distributed along each beam, and columns are massless. The total mass of the building is 40.75 tones. The foundation is shallow of 6m length. The Young modulus of the elastic material of the foundation is large enough to simulate a rigid foundation. The values of the mechanical properties of the reinforced concrete, the size of the sections as well as the distribution of reinforcement are in accordance with the earthquake codes. With these values, the fundamental period ($T_0$) of the building is 0.24s. A viscous damping, $\beta = 0.02$, is considered for all computations.
Two different behaviors of the springs are supposed. Firstly, elastic springs are modelled by traditional 1D bar elements. The parameter which controls the behavior of each bar is the modulus of elasticity, E (Fig. 2a). Secondly, a bi-linear model for elastoplastic behavior is used in order to add hysteretical damping to springs (Fig. 2b). In this case, the behavior is controlled by three parameters: the modulus of elasticity $E$, the hardening

modulus $E_t$ and the yield stress, $\sigma_y$ (Fig. 2b). The stress-strain relationship for a dynamic loading into an elastoplastic spring is shown in Fig. 2c.



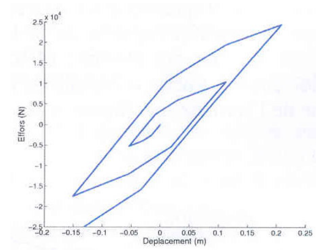




Fig.2a: Elastic behavior of spring Fig.2b: Elastoplastic behavior of spring Fig.2c: Stress-Strain behavior

### *2.2. Soil constitutive model*

The ECP's elastoplastic multi-mechanism model (Aubry et al., 1982; Hujeux, 1985; Aubry & Modaressi, 1992), commonly called Hujeux model is used to represent the soil behavior. This model can take into account the soil behavior in a large range of deformations. The model is written in terms of effective stress. The representation of all irreversible phenomena is made by four coupled elementary plastic mechanisms: three plane-strain deviatoric plastic deformation mechanisms in three orthogonal planes and an isotropic one. The model uses a Coulomb type failure criterion and the critical state concept. The evolution of hardening is based on the plastic strain (deviatoric and volumetric strain for the deviatoric mechanisms and volumetric strain for the isotropic one). To take into account the cyclic behavior, a kinematical hardening based on the state variables at the last load reversal is used. The soil behavior is decomposed into pseudo-elastic, hysteretic and mobilized domains. The model's parameters of the soil are obtained using the methodology suggested by Lopez-Caballero et al. (Lopez-Caballero, Modaressi & Elmi, 2003; Lopez-Caballero, Modaressi & Modaressi, 2007).

### *2.3. Input earthquake motion*

The used seismic input motions are the acceleration records of Friuli earthquake - San-Rocco site (Italy, 1976), $T_m = 0.46$, Superstition Hills earthquake - Supers. Mountain site (USA, 1987), $T_m = 0.38$, Kozani earthquake (Greece, 1995), $T_m = 0.28$, Aegion earthquake (Greece, 1995), $T_m = 0.56$, Umbria earthquake (Italy, 1997), $T_m = 0.14$, San Fernando earthquake (USA, near Pacoima dam, 1971), $T_m = 0.48$ and Lefkas earthquake (Greece, 2003), $T_m = 0.69$. The frequency content was characterized with the mean period ($T_m$) (Rathje, Abrahamson & Bray, 1998). The earthquakes are scaled to different maximum outcropping acceleration values.

### *2.4. Finite element approach (direct approach)*

The Finite Element model is composed of the structure, the soil foundation and a part of the bedrock. The considered structure is a two-story, one bay frame. The 30m thick (100m in horizontal direction) homogenous soil deposit is modeled by 4 node linear elements. At the bottom, a layer of 5m of elastic bedrock is added to the model. Plane strain condition is assumed for the soil deposit and the bedrock.
For the bedrock's boundary condition, paraxial elements simulating deformable unbounded bedrock have been used. The incident waves, defined at the outcropping bedrock are introduced into the base of the model after deconvolution. In the analysis, the lateral limits of the problem are considered to be far enough, periodic conditions are verified. Then, only vertically propagating shear waves are studied resulting in the free field response. The obtained movement at the bedrock is composed of the incident waves and the reflected signal. The computations are carried-out in the time domain. The simulations are performed with the Finite Element tool GEFDYN (Aubry, Chouvet, Modaressi & Modaressi, 1985; Aubry & Modaressi, 1996).

### *2.5. Spring supported approach*

The first step is to solve a non-linear one-dimensional wave propagation problem for a soil column. The mesh consists of one column of solid elements obeying the same constitutive model as in the direct approach. The same boundary conditions have been imposed. The incident waves, defined at the outcropping bedrock are introduced into the base of the model after deconvolution. In a second step, the obtained free field motion is imposed as the ground motion to the spring supported structural model. This two-step approach takes into account the effect of non-linearity behavior of both soil and structure and takes into account the SSI effects approximately.

## 3. SOIL COLUMN ANALYSIS AND RESULTS

In order to define the input motion for the spring supported approach, a free field dynamic analysis of the soil profile with soil properties already described, is performed. The response of the free field soil profile is analysed for the seven earthquake records as outcropping input. The Fig.3a shows the obtained values and a tendency curve for the PGA (Peak Ground Acceleration) with respect to maximum acceleration on the bedrock ($a_{\max,bedrock}$). It is noted that an amplification of the ground response for moderate range of $a_{\max,bedrock}$ is obtained. When the amplitude of the earthquake increases, the soil responds in plastic domain and introduces damping in the system. That is, a part of the energy is dissipated in the ground and therefore, the soil weakening attenuates the seismic motion compared to a purely elastic soil.

The influence of the inelastic behaviour of the soil deposit on the structural response can be directly related to pseudo-spectral acceleration (PSA) at the structure base level (Saez, Lopez-Caballero & Modaressi, 2006). The comparison between outcropping and obtained structure's base normalized PSA for different acceleration levels using Friuli earthquake are shown in figure 3b. It is noted that for periods between 0.3 and 0.75s the spectral values increase compared to the outcropping spectra. This change in the frequency content is related to the resonance period of the soil (0.46 sec). Indeed, we can find the frequency of resonance between the ground and signal frequency in this range. For small periods (T <0.3s), the modifications of the spectra are not significant. For periods exceeding 0.75s, for the signal with 0.1g amplitude, we note the attenuation, against, for the 0.45g amplitudes we have the amplification compared to outcropping spectra. Concerning the fixed based fundamental period of the structure, with considering the site effect, for the small amplitude (0.1g), the spectral values are reduced, against when the signal amplitude increases, the spectral amplitudes join similar values of outcropping response spectrum.

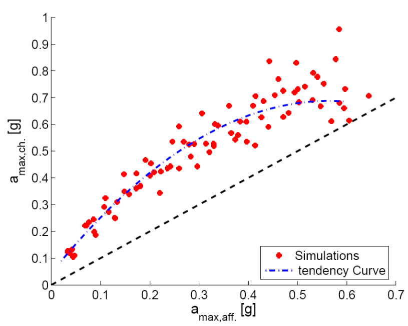



Figure 3a : Site effect

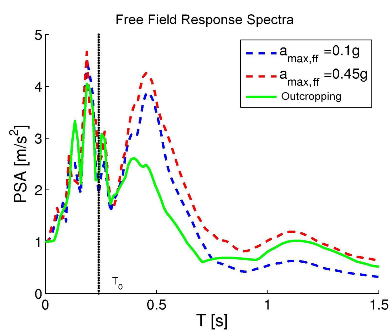



Figure 3b : Response Spectra of Friuli earthquake

## 4. NON-LINEAR SSI ANALYSIS AND RESULTS

Any damage criterion must include not only the maximum response, but also the effect of repeated cyclical loads. The damage index used in this paper to evaluate the structural damage of the structures is based on the Park damage model (Park & Ang, 1985) for reinforced concrete. The Park damage model accounts for damage due to maximum inelastic excursions, as well as damage due to the history of deformations. Both components of damage are linearly combined. Two damage indices are computed using this damage model: a local element damage index ($DI_{loc}$) for columns and beams and an overall structure damage ($DI_{ov}$). Since the inelastic behaviour is confined to plastic zones near the ends of some members, the relation between element and overall structure integrity is not direct. According to the used structural non-linear model, for each element section $i$, it is possible to compute a local index of damage (Fig.4a):

$$DI_{loc,i} = \frac{\Psi_{m,i}}{\Psi_u} + \frac{\lambda_p}{\Psi_u M_y} \int dE_i \tag{4.1}$$

where $\Psi_{m,i}$ is the maximum curvature reached during the load history, $\Psi_u$ is the ultimate curvature capacity of the section, $M_y$ is the yield moment end, $E_i$ is the energy dissipated in the section. $\lambda_p$ is a model constant parameter. For nominal strength deterioration of reinforced concrete sections, a value of 0.1 has been suggested by the same authors. The value of $M_y$ is computed for a simple fixed beam with the used structural non-linear model. Finally, the $\Psi_u$ value corresponds to the most plastified section at the end of the pushover test. The overall damage index is computed using weighting factors based on dissipated hysteretic energy at each component section.

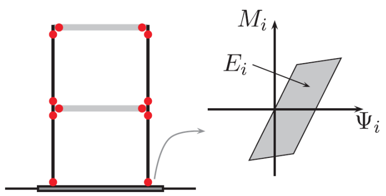



Figure 4a: Damage index models

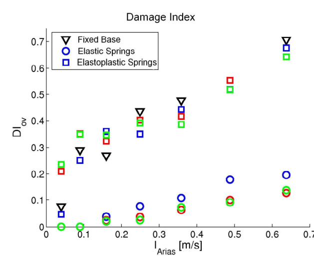



Figure 4b: Damage index for different models

| Case | Model | $E(MPa)$ | $\sigma_y(KPa)$ | $E_t(KPa)$ |
|---|---|---|---|---|
| 1 | elasto. | 500 | 50 | 10 |
| 2 | elasto. | 100 | 50 | 10 |
| 3 | elasto. | 50 | 50 | 10 |
| 1 | elastic | 500 | - | - |
| 2 | elastic | 100 | - | - |
| 3 | elastic | 50 | - | - |

Figure 4c: Parameters of the models

Several efforts have been made to establish a relationship between a measure of the severity of a ground motion and a level of structural damage. The parameters incorporating the amplitude and duration of the ground motion are likely to be more reliable predictor of damage than parameters that capture only the amplitude of the earthquake. Arias intensity ($I_{Arias}$) (Arias, 1970) is an earthquake severity measure that correlates well with several structural demand measures. The soil can amplify the signal and thus increase the seismic response, but at the same time, it can vary the frequency content of the signal. We can model the non-linear behavior of the soil in a more precise way, by using the elastoplastic springs with the purpose of taking into account the hysteresis damping introduced by the soil. Figure 4b shows the computed damage index of the structure for the various amplitudes of Friuli earthquake (0.1g to 0.4g). Figure 4c presents the parameters used for springs during the analyses of the structure. It can be noticed that the building with elastoplastic springs shows a damage index significantly lower than the other cases due to the amount of energy dissipated in the springs instead of the structure. Indeed, the elastoplastic springs undergo plastic deformations before the structure introducing additional damping to the system. Nevertheless, these results are not necessarily realistic because they are obtained with a set of arbitrary initial parameters. Realistic values for spring's properties will be obtained by the optimization method described in the following section.

## 5. OPTIMIZATION METHOD AND ITS APPLICATION

In order to establish a realistic set of parameters for the elastoplastic springs compatible with the type of studied soil, an optimization strategy (Gandomzadeh, 2007) is used in this paper. The strategy is based on an optimization procedure to get the results of the simplified model closer to those of the direct method. In general, the problem may be written as:

$$Min \quad f(\{Z\}) \qquad s.t. \qquad \{g(\{Z\})\} \leq \{0\} \quad and \quad \{L_b\} \leq \{Z\} \leq \{U_b\} \tag{5.1}$$

$\{Z\}$ is the vector of the variables ($\langle E, \sigma_y, E_t \rangle$); $f(.)$ is the objective function to minimize; $\{g(.)\}$ the set of restrictions, $\{L_b\}$ and $\{U_b\}$ are the lower and upper limit values for the variables of the model. In order to improve the approximate result, we must reduce the difference between the response of the simplified model ($response_{Simp.}$) and the response of the finite element model ($response_{FE}$) supposed to be the "exact" result. For every response, the following equation is used to find the corresponding normalized difference (distance in the response plane):

$$D_{i,norm.} = \frac{\sqrt{(X_{i,FE} - X_{i,Simp.})^2 + (Y_{i,FE} - Y_{i,Simp.})^2}}{\sqrt{X_{i,FE}^2 + Y_{i,FE}^2}} \tag{5.2}$$

where $X_{i,FE}$ and $Y_{i,FE}$ are the coordinates of the "exact" response, in the plane that we chose to compare the results in $X_{i,Simp.}$ and $Y_{i,Simp.}$ are the coordinates of the response of the simplified model for the case $i$ (a motion with a specific amplitude value) and $D_{i,norm.}$ is the normalized difference for this point. Then, we find the error with SRSS method:

$$e_{j,total} = \sqrt{\sum_{i=1}^{n} D_{i,norm.}^2} = f(\{Z\}) \tag{5.3}$$

where $n$ is the number of result points, $e_{j,total}$ is the total error associated to a set of spring properties $j$. Thus, we will take the total error $e_{j,total}$ as the objective function $f(\{Z\})$ for the minimization problem.
For the optimization method, the available functions on MATLAB are used (The Mathworks, 1998-2008). We define also initial values, lower and upper limits for the variables. A set of tolerances are also defined for the different convergence criteria. The optimization algorithm is type C1; the first order derivative is computed during the optimization procedure. Regarding the restrictions for the optimization problem, several criteria can be used. The maximum deformation of the springs, maximum rotation in plastic hinges of the model, maximum displacement at the top of the structure, damage index, etc.

### *5.1. Application of optimization method*

The criterion to be considered as a response of the building is a key issue of the procedure. The criterion of maximum displacement is not a good criterion because it corresponds only to an instant during the loading and does not necessarily describe the overall state of the building. For example, the damage index may be a better criterion for comparing results because it is an indicator of the overall response of the building throughout the duration of an earthquake.

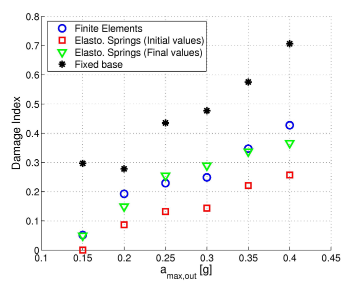


Figure 5 : Application of optimization method for damage index using Friuli earthquake

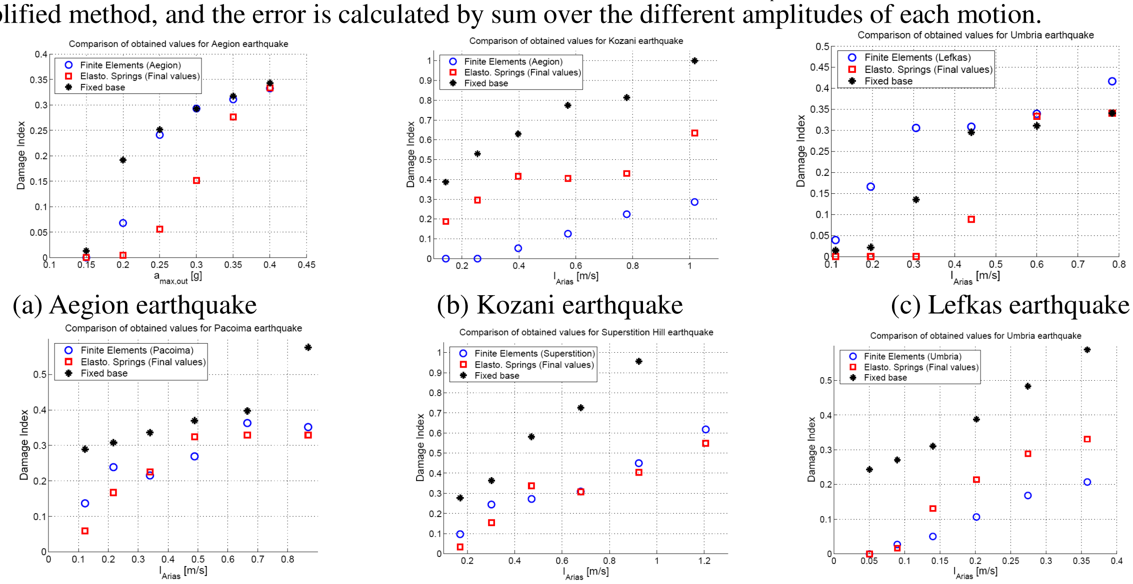
We run the optimization method with six variables, that is, three variables for horizontal springs and three variables for vertical springs. The starting points are 100MPa ($E$), 50kPa ($\sigma_y$) and 10kPa ($E_t$). The optimum properties obtained at the end of the processes are 56.6MPa ($E$), 102.8kPa ($\sigma_y$) and 10kPa ($E_t$) for vertical springs and 38.3MPa ($E$), 69.2kPa ($\sigma_y$) and 144.4kPa ($E_t$) for horizontal springs. These values are obtained after approximately 650 iterations. The related damage indexes are shown in Figure 5. The results are satisfactory but they were obtained only for Friuli earthquake and they should be verified for other earthquakes.

### *5.2. Verification of parameters*

The final parameter values obtained by optimization are used for the other earthquakes in order to control the response of the structure (Fig. 6). For the Superstition Hill and San Fernando earthquakes, the difference between the responses of the structure obtained both by two previously described approaches are satisfactory. For other cases, such as Kozani earthquake, the result is not so satisfactory. It proves that the role of the incident signal is not negligible, and the adequate set of spring properties values are strongly dependent on the frequency content of the signal. With the purpose to compare the difference between the results of the different earthquakes, we can introduce a normalized error as follows:

$$Normalized\ \ error = \sqrt{\sum \left( \frac{ID_{SSI} - ID_{optim.}}{ID_{SSI}} \right)^2} \tag{5.4}$$

where, $ID_{SSI}$ is the damage index obtained by the direct approach, $ID_{optim.}$ is the damage index obtained by simplified method, and the error is calculated by sum over the different amplitudes of each motion.




(a) Aegion earthquake (b) Kozani earthquake (c) Lefkas earthquake

(d) San Fernando earthquake (e) Superstition Hill earthquake (f) Umbria earthquake

Figure 6 : Computed responses using the optimum set of spring parameters obtained for the Friuli earthquake.

To characterize the frequency content of the earthquakes, we use the mean period, $T_m$ (Rathje et al., 1998). Figure 7 presents the error in terms of the mean period. The minimum error is obtained for the Friuli earthquake.

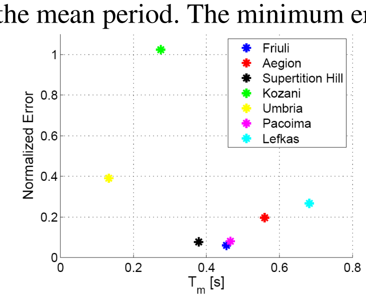



Figure 7 : Normalized errors for the seven used motions using spring parameters optimized for Friuli earthquake

As the mean period approaches to the one of the Friuli earthquake, the normalized error diminishes, thus we can define a range in terms of the mean period where the simplified approach gives satisfactory results. The minimum error, (for Friuli earthquake) is approximately 6%. To reduce this error, we should improve the approximate model, for instance changing the position of springs, placing more springs at the ends of the foundation or as a matter of fact, by using different property values depending on the position of the spring in the foundation. The normalized error for Kozani earthquake does not follow the general tendency of the curve. If we compare the fundamental period of the structure (0.24s) with the mean period of the Kozani earthquake (0.28s), we can explain this abnormal difference by the resonance phenomenon between the structure and the input motion.

## 6. CONCLUSION

The influence of the inelastic behaviour of soil deposit on the amplification of ground seismic accelerations and on the soil-structure interaction effects has been studied in this work. A simplified model is presented by means of the bi-linear elastoplastic behaviour of Winkler springs to simulate the non-linear behaviour of the soil. Compared to the direct Finite Elements method, the proposed simplified model provides satisfactory results and introduces a fast numerical modelling of the soil-structure interaction problem.

The parameters of the model are obtained by matching the dynamic non-linear structural response of the proposed simplified approach and a complete 2D finite element analysis using a realistic constitutive model for the soil. For this purpose, an optimization procedure based on the value of the induced structural damage was introduced. It was shown that for a given optimal set of springs' parameters, the accuracy of the simplified method is strongly correlated with the frequency content of the input. An applicability domain could be established for a set of parameters in terms of the soil classification and frequency contents of the motion.

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