

SEISMIC RESPONSES OF A CABLE-STAYED BRIDGE TO SPATIALLY VARYING GROUND MOTIONS

Y. B. Wang¹, Y. H. Zhang² and Y. Zhao³

¹ Student, Dept. of Engineering Mechanics, Dalian University of Technology, Dalian, China

² Associate Professor, Dept. of Engineering Mechanics, State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian, China

³ Lecturer, Dept. of Engineering Mechanics, Dalian University of Technology, Dalian, China
Email: zhenzhen32@sina.com, zhangyh@dlut.edu.cn, yzhao@dlut.edu.cn

ABSTRACT :

A random vibration methodology is formulated for the seismic analysis of cable-stayed bridges subjected to spatially varying ground motions. The ground motion spatial variability consists of the wave passage, incoherence and site-response effects. It is shown that all these effects have significant influences on the seismic response of the structure. The bridge investigated in this paper is a 7-span composite concrete-steel cable-stayed bridge. The overall length of the bridge is 1150 m, with a central span of 720 m between its two towers. The towers are Y-shaped reinforced concrete structures, with a height of 233.8 m. The bridge deck consists of three independent box girders. The central one is a closed steel box girder with streamline shape, and the two side spans are concrete box girders. The width and height of the deck are 30 m and 3.5 m, respectively. The cables are arranged in a double-plane fan type. Each cable plane has 23 pairs of cables. These cables are anchored to the deck at a 15 m interval on the main span and an 8 m interval on the side spans. For the large uncertainties in the specification of earthquake ground motions, it is inevitable to conduct parametric studies to determine the range of variability of the structures responses. The method used in this paper is quite efficient and convenient for such analyses.

KEYWORDS: bridge, earthquake, wave passage effect, incoherence effect, site-response effect

1. INTRODUCTION

Long-span bridges are being increasingly constructed in China and in the world. The conventional seismic analysis methods have appeared some severe restrictions as they can hardly account for the inhomogeneous characteristics of earthquake excitations along the bridge spans. In order to deal with such kind of so-called multi-excitation seismic analysis problems, various means based on the response spectrum approach (RSA), random vibration approach as well as time-history method (THM) have been explored. For example, in early 1980's, Lee and Penzien (1983) investigated the safety problem of the pipeline system of a nuclear power station subjected to seismic multi-excitations, by means of a highly simplified structural model, both in the frequency domain and time domain, and concluded that neglecting the cross-correlation terms between the participant modes or the inhomogeneous property of the field will both lead to considerable errors in the seismic analysis; and that the random vibration approach is more accurate than RSA, more efficient than THM, and so should be recommended in the seismic designs of nuclear power stations. Lin et al. (1990) applied strict random vibration approach to the solution of an oil pipeline system subjected to multiple stationary seismic random excitations by regarding the pipeline system as a continuous beam with several supports. The random vibration approach used can deal with only very simple structural models. Yamamura and Tanaka (1990) developed response spectrum and time-history methods to evaluate the response of MDF systems subjected to multiple-support seismic excitations, with the support motions grouped into independent subgroups with perfect correlation between the members of each subgroup. Zerva (1990) analyzed two- and three-span beams of various lengths (short, moderate and long) subjected to input motions that exhibit loss of coherence only, with various degrees of correlation, and compared the response to the one induced by fully correlated motions. The results indicates that fully correlated motions may produce higher or lower response than partially correlated

motions, depending on the dynamic characteristics of the structure. Berrah and Kausel (1993) proposed a modified spectrum method for the design of extended structures which considers the spatial variability effect arising from the incoherence. Nazmy and Abdel-Ghaffar (1992) studied the seismic responses of cable-stayed bridges considering the seismic-wave traveling effect and accounted for time delay and phase difference by using the time history method. Der Kiureghian et al. (1992) developed a response spectrum method considering the effects of wave passage, incoherence and site-response. Heredia-Zavoni and Vanmarcke (1994) developed a random vibration method for the seismic-response analysis of linear multi-support structural systems, which reduces the response evaluation to that of a series of linear one-degree systems in a way that fully accounts for the multiple-support input and the space-time correlation structure of the ground motion. Harichandran et al. (1988, 1996) proposed a random vibration algorithm to reduce the cost of large-scale stationary and transient random vibration analysis of structures excited by multiple partially correlated nodal and/or base excitations, presented stationary and transient response analyses of the Golden Gate suspension bridge, and the New River Gorge and Cold Spring Canyon deck arch bridges. Allam and Datta (1999, 2000) used frequency domain spectral method and response spectrum method to estimate the seismic responses of cable-stayed bridges subjected to partially correlated stationary random ground motion. Dumanoglu and Soyuk (2003) investigated the relative importance of ground motion variability effects on the dynamic behavior of plane models of cable-stayed and suspension bridges. Lin et al (1992, 1997, 2005) proposed an efficient pseudo excitation method to compute the stationary and non-stationary seismic responses of multi-support structures, for which the wave passage and incoherence effects are included. For complex structures, it can raise computational efficiency by thousands or even more times, therefore for structures with thousands of degrees of freedom, tens of supports, using 200~300 modes for mode superposition, only a few minutes are required for accurate CQC (complete quadratic combination) seismic random vibration analysis (including displacement and internal force responses) on an ordinary personal computer. It is even faster than using RSA. It is sufficient to meet general engineering requirements.

Based on PEM, the seismic random vibration analysis of a bay-bridge with a length of 1150 m was computed. It will be shown in this paper that for the case of uniform ground motion, the numerical results by using the three methods are in general quite close to one another. It is also shown that the wave passage, incoherence and site-response effects all significantly affect the structural responses. The Specifications for Seismic Design of Highway Engineering, JTJ004-89 (published in 1989) currently used in China applies only to bridges with the main-span shorter than 150 m. Therefore, no multiple excitation problems are concerned. In the Specifications for Seismic Design of Highway Bridges (published in September, 2008), corresponding revisions have been made, i.e. for bridges with longer main spans, the spatial effects due to multiple excitations are suggested to be taken into account in the design stage. Meanwhile, PEM is also involved as an optional tool for random vibration analysis. However, for average engineers and some research workers, they still want to know more about the use of PEM, as well as the differences among PEM, RSA and THM. This is the purpose of this paper. It is hoped that the principle, notes, and numerical comparisons given in this paper will help them to get a deep insight into the new specification.

2. SPATIALLY VARYING GROUND MOTION MODEL

Consider a structure with N ground supports. Assume that the seismic ground motion is a stationary random process, then the ground accelerations \ddot{u}_i ($i = 1, 2, \dots, N$) at these supports can be represented by a vector:

$$\ddot{\mathbf{u}}_b(t) = \{\ddot{u}_1(t) \quad \ddot{u}_2(t) \quad \cdots \quad \ddot{u}_N(t)\}^T \quad (2.1)$$

The spatial effect of the field can be characterized in terms of a cross-PSD function $S_{kl}(\omega)$ between the ground accelerations $\ddot{\mathbf{u}}_k(t)$ and $\ddot{\mathbf{u}}_l(t)$ at the k th and l th supports

$$S_{kl}(\omega) = \gamma_{kl}(\omega) \sqrt{S_{kk}(\omega) S_{ll}(\omega)} \quad (2.2)$$

where ω is the angular frequency; $S_{kk}(\omega)$, $S_{ll}(\omega)$ are the auto- PSD functions of the ground accelerations $\ddot{\mathbf{u}}_k(t)$ and $\ddot{\mathbf{u}}_l(t)$; while $\gamma_{kl}(\omega)$ is the coherency factor between $\ddot{\mathbf{u}}_k(t)$ and $\ddot{\mathbf{u}}_l(t)$ (Der Kiureghian 1996).

$$\gamma_{kl}(\omega) = \gamma_{kl}^{(i)}(\omega)\gamma_{kl}^{(w)}(\omega)\gamma_{kl}^{(s)}(\omega) \quad (2.3)$$

in which: $\gamma_{kl}^{(i)}(\omega)$ characterizes the real valued incoherence effect, $\gamma_{kl}^{(w)}(\omega)$ indicates the complex valued wave passage effect and $\gamma_{kl}^{(s)}(\omega)$ defines the complex valued site response effect. Several models have been proposed for the incoherence effect due to reflections and refractions of waves through the soil during their propagation (Harichandran and Vanmarcke 1986, Loh and Yeh 1988). The wave passage effect resulting from the difference in the arrival times of waves at support points is defined as

$$\gamma_{kl}^{(w)} = \exp[i(\theta_{kl}^{(w)}(\omega))] = \exp\left[-\frac{i\omega d_{kl}^L}{v_{app}}\right] \quad (2.4)$$

here: d_{kl}^L is the projection of d_{kl} in the earthquake propagation direction and; v_{app} is the apparent velocity of the seismic waves. Suppose that the wave front reaches the origin of the coordinate system, i.e. the reference point, at $T = 0$, and then reaches the N supports of the structure at times T_1, T_2, \dots, T_N , respectively. Without losing generality, we can assume $T_l \geq T_k$ and hence

$$d_{kl}^L/v_{app} = T_l - T_k, \quad \gamma_{kl}^{(w)} = \exp[i\omega(T_k - T_l)] \quad (2.5)$$

The site response effect due to the differences in the local soil conditions is obtained as

$$\gamma_{kl}^{(s)}(\omega) = \exp[i\theta_{kl}^{(s)}(\omega)], \quad \theta_{kl}^{(s)}(\omega) = \theta_k^{(s)}(\omega) - \theta_l^{(s)}(\omega) \quad (2.6)$$

in which

$$\theta_k^{(s)}(\omega) = \tan^{-1} \frac{\text{Im}(H_k(\omega))}{\text{Re}(H_k(\omega))} = \tan^{-1} \frac{-2\xi_k \omega_k \omega^3}{\omega_k^2 (\omega_k^2 - \omega^2) + 4\xi_k^2 \omega_k^2 \omega^2} \quad (2.7)$$

ω_k and ξ_k are the resonant frequency and damping ratio of the soil layer.

3. PSEUDO EXCITATION METHOD WITH SPATIALLY VARYING GROUND MOTIONS

For a linear MDOF system with N ground supports, its equations of motion with spatial coherency considered can be written as

$$\begin{bmatrix} \mathbf{M} & \mathbf{M}_C \\ \mathbf{M}_C^T & \mathbf{M}_G \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{u}}_G \end{Bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{C}_C \\ \mathbf{C}_C^T & \mathbf{C}_G \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{u}}_G \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{K}_C \\ \mathbf{K}_C^T & \mathbf{K}_G \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{u}_G \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{F}_G \end{Bmatrix} \quad (3.1)$$

in which: \mathbf{u}_G is the m -dimensional vector of enforced support displacement components; \mathbf{u} is an n -dimensional vector containing all nodal displacements except those at the supports; \mathbf{F}_G represents the enforced forces at all supports; the $n \times n$ matrices \mathbf{M} , \mathbf{C} and \mathbf{K} are respectively the mass, damping and

stiffness matrices associated with \mathbf{u} ; the $m \times m$ matrices \mathbf{M}_G , \mathbf{C}_G and \mathbf{K}_G are the mass, damping and stiffness matrices associated with \mathbf{u}_G and; \mathbf{M}_C , \mathbf{C}_C and \mathbf{K}_C are the $n \times m$ coupling matrices shown. In order to solve Eqn. 3.1, \mathbf{u} is usually decomposed into the two parts

$$\mathbf{u} = \mathbf{u}^s + \mathbf{u}^d \quad (3.2)$$

where \mathbf{u}^s and \mathbf{u}^d are respectively the quasi-static and dynamic displacement vectors, which satisfy the equations:

$$\mathbf{u}^s = -\mathbf{K}^{-1} \mathbf{K}_C \mathbf{u}_G \equiv \mathbf{R} \mathbf{u}_G \quad (3.3)$$

$$\mathbf{M} \ddot{\mathbf{u}}^d + \mathbf{C} \dot{\mathbf{u}}^d + \mathbf{K} \mathbf{u}^d = -(\mathbf{M} \mathbf{R} + \mathbf{M}_C) \ddot{\mathbf{u}}_G \quad (3.4)$$

For the Cartesian system xyz , x and y axes are assumed to constitute the horizontal plane. Provided that the angle between x axis and the traveling direction of the seismic waves is β , Thus, the displacement components along the coordinate axes, \mathbf{u}_G , can be expressed in terms of the components parallel or normal to the wave traveling direction, \mathbf{u}_b , as

$$\mathbf{u}_G = \mathbf{E}_{mN} \mathbf{u}_b \quad (3.5)$$

Using Eqn. 3.5, Eqns. 3.3 and 3.4 can be rewritten as (Lin and Zhang 2005)

$$\mathbf{u}^s = -\mathbf{K}^{-1} \mathbf{K}_C \mathbf{u}_G \equiv \mathbf{R} \mathbf{E}_{mN} \mathbf{u}_b \quad (3.6)$$

$$\mathbf{M} \ddot{\mathbf{u}}^d + \mathbf{C} \dot{\mathbf{u}}^d + \mathbf{K} \mathbf{u}^d = -(\mathbf{M} \mathbf{R} + \mathbf{M}_C) \mathbf{E}_{mN} \ddot{\mathbf{u}}_b \quad (3.7)$$

If $\mathbf{z}(t)$ represents an arbitrary response vector, which can be a nodal displacement vector, an internal force or strain vector, etc. For a linear system, $\mathbf{z}(t)$ can be expressed as

$$\mathbf{z}(t) = \mathbf{T}^T \mathbf{u}(t) + \mathbf{T}_G^T \mathbf{u}_G(t) \quad (3.8)$$

\mathbf{T}^T and \mathbf{T}_G^T are the transformation matrices relying on the variation of structural geometry and stiffness. The dynamic relative displacements can be expressed by the following convolution-integration form:

$$\mathbf{u}^d(t) = -\int_{-\infty}^{\infty} \mathbf{h}(\tau) (\mathbf{M} \mathbf{R} + \mathbf{M}_C) \mathbf{E}_{mN} \ddot{\mathbf{u}}_b(t - \tau) d\tau \quad (3.9)$$

in which, $\mathbf{h}(t)$ is the impulse response function matrix. Hence, using Eqns. 3.2, 3.6 and 3.9, Eqn. 3.8 becomes

$$\mathbf{z}(t) = -\mathbf{T}^T \int_{-\infty}^{\infty} \mathbf{h}(\tau) \mathbf{A} \ddot{\mathbf{u}}_b(t - \tau) d\tau + \mathbf{G} \mathbf{u}_b(t) \quad (3.10)$$

where

$$\mathbf{A} = (\mathbf{M} \mathbf{R} + \mathbf{M}_C) \mathbf{E}_{mN}, \quad \mathbf{G} = \mathbf{T}^T \mathbf{R} \mathbf{E}_{mN} + \mathbf{T}_G^T \mathbf{E}_{mN} \quad (3.11)$$

Then the power spectrum density matrix of $\mathbf{z}(t)$ can be obtained as

$$\begin{aligned} \mathbf{S}_{zz}(\omega) = & \mathbf{T}^T \mathbf{H}^*(\omega) \mathbf{A} \mathbf{S}_{\ddot{u}_b \ddot{u}_b}(\omega) \mathbf{A}^T \mathbf{H}^T(\omega) \mathbf{T} + \frac{1}{\omega^4} \mathbf{G} \mathbf{S}_{\ddot{u}_b \ddot{u}_b}(\omega) \mathbf{G}^T \\ & + \frac{1}{\omega^2} \mathbf{T}^T \mathbf{H}^*(\omega) \mathbf{A} \mathbf{S}_{\ddot{u}_b \ddot{u}_b}(\omega) \mathbf{G}^T + \frac{1}{\omega^2} \mathbf{G} \mathbf{S}_{\ddot{u}_b \ddot{u}_b}(\omega) \mathbf{A}^T \mathbf{H}^T(\omega) \mathbf{T} \end{aligned} \quad (3.12)$$

in which $\mathbf{H}(\omega)$ is the frequency response function matrix of the structure and is given by

$$\mathbf{H}(\omega) = \int_{-\infty}^{\infty} \mathbf{h}(t) e^{-i\omega t} dt \quad (3.13)$$

In order to apply PEM, the ground acceleration PSD matrix need be decomposed according to

$$\mathbf{S}_{\ddot{u}_b \ddot{u}_b}(\omega) = \mathbf{B}^* \mathbf{D} \mathbf{\Gamma} \mathbf{D} \mathbf{B} \quad (3.14)$$

in which * denotes complex conjugate, and

$$\mathbf{B} = \text{diag} \left[e^{-i(\omega T_1 + \theta_1^{(s)})} \quad e^{-i(\omega T_2 + \theta_2^{(s)})} \quad \dots \quad e^{-i(\omega T_N + \theta_N^{(s)})} \right] \quad (3.15)$$

$$\mathbf{D} = \text{diag} \left[\sqrt{S_{11}} \quad \sqrt{S_{22}} \quad \dots \quad \sqrt{S_{NN}} \right] \quad (3.16)$$

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{11}^{(i)} & \gamma_{12}^{(i)} & \dots & \gamma_{1N}^{(i)} \\ \gamma_{21}^{(i)} & \gamma_{22}^{(i)} & \dots & \gamma_{2N}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1}^{(i)} & \gamma_{N2}^{(i)} & \dots & \gamma_{NN}^{(i)} \end{bmatrix} \quad (3.17)$$

In general $\mathbf{S}_{\ddot{u}_b \ddot{u}_b}(\omega)$ in Eqn. 3.14 is a positive definite Hermitian matrix, while $\mathbf{\Gamma}$ is a positive definite real symmetric matrix, which can be decomposed into the product of a real lower triangle matrix \mathbf{Q} and its transpose, i.e.

$$\mathbf{\Gamma} = \mathbf{Q} \mathbf{Q}^T \quad (3.18)$$

Thus, Eqn. 3.14 can be written as

$$\mathbf{S}_{\ddot{u}_b \ddot{u}_b}(\omega) = \mathbf{P}^* \mathbf{P}^T \quad (3.19)$$

in which

$$\mathbf{P} = \mathbf{B} \mathbf{D} \mathbf{Q} \quad (3.20)$$

By substituting Eqn. 3.19 into Eqn. 3.12, the PSD matrix of $\mathbf{z}(t)$ can be derived as

$$\mathbf{S}_{zz}(\omega) = \left(\mathbf{T}^T \mathbf{H}(\omega) \mathbf{A} + \mathbf{G} / \omega^2 \right)^* \mathbf{P}^* \mathbf{P}^T \left(\mathbf{T}^T \mathbf{H}(\omega) \mathbf{A} + \mathbf{G} / \omega^2 \right)^T = \mathbf{V}^* \mathbf{V}^T \quad (3.21)$$

here

$$V = (T^T H(\omega)A + G/\omega^2)P \quad (3.22)$$

To use PEM, it is required to constitute the following harmonic excitation vector first

$$\ddot{U}_b = P \exp(i\omega t) \quad (3.23)$$

Then, by solving the deterministic harmonic equations of motion, the deterministic pseudo dynamic displacements \tilde{u}^d can be thus obtained. And the quasi-static displacements can be computed by solving the linear algebraic equations. Therefore, the pseudo response vector $\tilde{Z}(t)$, which is corresponding to the required random responses $z(t)$, would be

$$\tilde{Z}(t) = -(T^T H(\omega)A + G/\omega^2)P \exp(i\omega t) = -V \exp(i\omega t) \quad (3.24)$$

Hence, the following PEM-based response PSD matrix can be calculated:

$$S_{zz}(\omega) = \tilde{Z}^*(t)\tilde{Z}^T(t) \quad (3.25)$$

With such response PSD matrices obtained, the spectral moments or extreme values of the corresponding response $z(t)$ can be easily obtained. Clearly, while using PEM, no extra assumptions have been introduced. Therefore, this method is not only very convenient, but also accurate. For complex structures, if mode-superposition scheme is used to compute the pseudo response vector $\tilde{Z}(t)$, as $\tilde{Z}(t)$ has involved the contributions of all participant modes, therefore according to Eqn. 3.25, $S_{zz}(\omega)$ involves all product terms between those participant modes. That's why it has been known as a Fast-CQC algorithm.

4. CASE STUDY

A 7-span continuous bay-bridge with span lengths 67+72+76+720+76+72+67 m and total length 1150 m is investigated. It is a double tower cable-stayed bridge constructed by steel and RC concrete materials. The towers are Y-shaped reinforced concrete structures, with a height of 233.8 m. The bridge deck consists of three independent box girders. The central one is a closed steel box girder with streamline shape, and the two side spans are concrete box girders. The width and height of the deck are 30 m and 3.5 m, respectively. The cables are arranged in a double-plane fan type. Each cable plane has 23 pairs of cables. These cables are anchored to the deck at a 15 m interval on the main span and an 8 m interval on the side spans. The foundations of the towers have 42 caisson piles with a diameter of 2.2 m, while those of the abutment piers are 4 caisson piles with a diameter of 2 m. Its finite element model has 7267 nodes, 6928 elements and 18694 degrees of freedom. In the dynamic analysis, the state with gravity applied is taken as the initial state, the designed cable tensions for the constructed bridge are input in the form of initial strains.

The lowest 300 modes of the bridge were computed and used in the mode-superposition analysis of the seismic response analysis. The seismic response spectrum is from Specifications for Seismic Design of Highway Engineering (JTJ004-89). By adopting the seismic ground motion parameters for the level of probability of exceedance 10% for 50 years, as earthquake movement input; and assuming the damping ratio is 0.05. The above design response spectrum curves can be used to produce corresponding design PSD curves and time-history samples (Lin and Zhang 2005). Structural analyses can then be performed based on these input data. The responses of the deck and the south tower are illustrated below for comparisons. These responses include:

the y -direction bending moments M_y and z -direction shear forces Q_z for the deck. When the ground motion is assumed to be uniform, the internal force comparisons for the three methods are shown in Figure 1. When wave passage effect, incoherence effect and site-response effect are taken into account, the comparisons are shown in Figs. 2 and 3, respectively.

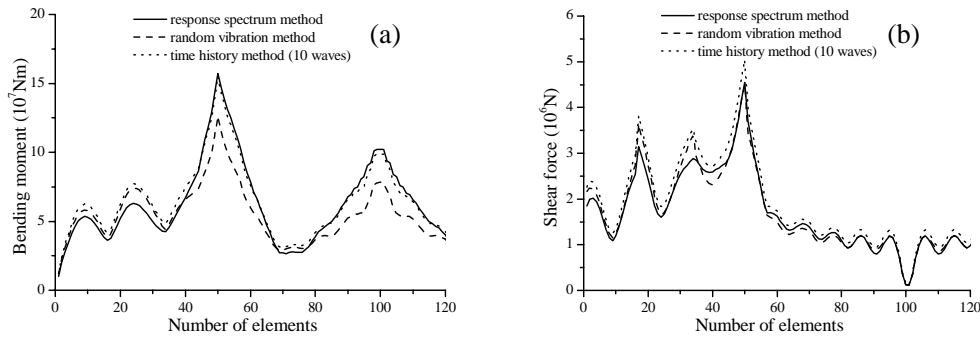


Figure 1 Comparison of shear forces and bending moments of the deck and south tower among three methods with uniform ground motion and level

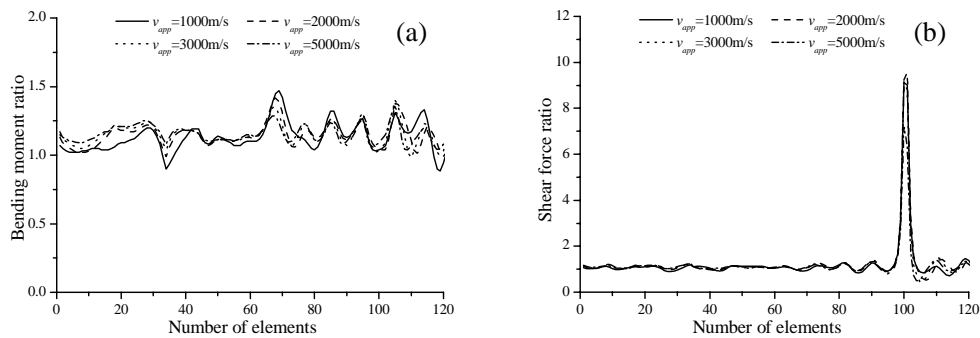


Figure 2 shear forces and bending moments of the deck and south tower with wave passage effect

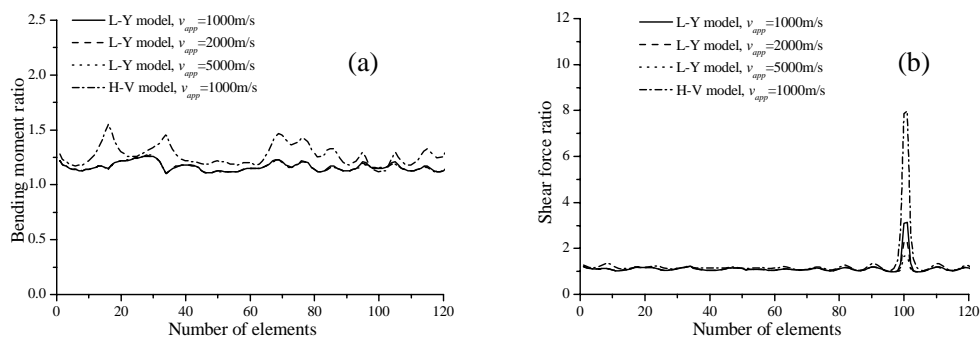


Figure 3 shear forces and bending moments of the deck and south tower with incoherence effect

It can be seen from Figs. 1-3 that under uniform ground motion, RSA, PEM and THM (using 10 artificial seismic waves) all give rather close numerical results for whichever level of exceedance probability. Figs. 2 and 3 show that the wave passage, incoherence and site-response effects all significantly affect the structural responses. In general, complex structures have closely spaced natural frequencies, it is difficult to judge the one that is the most important factor to affect the structural responses. In addition, as the conventionally used RSA can not properly deal with the spatial effects of ground motion that may lead to unreliable design for long-span

bridges. Therefore PEM provides a beneficial alternative. When the visual seismic wave speed can not be accurately determined, a few possible wave speeds can be adopted for structural analyses. The most unsafe value will be used in the practical design.

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REFERENCES

- Allam S. M. and Datta T. K. (1999). Seismic behaviour of cable-stayed bridges under multi-component random ground motion. *Engineering Structures* **21:1**, 62-74.
- Allam S. M. and Datta T. K. (2000). Analysis of cable-stayed bridges under multi-component random ground motion by response spectrum method. *Engineering Structures* **22:10**, 1367-1377.
- Berrah M., Kausel E. (1993). A modal combination rule for spatially varying seismic motions. *Earthquake Engineering and Structural Dynamics* **22:9**, 791-800.
- Davenport A. G. (1961). The application of statistical concepts to the wind loading of structures. *Proceedings of the Institution of Civil Engineers* **19**, 449-472.
- Der Kiureghian A. and Neuenhofer A. (1992). Response spectrum method for multi-support seismic excitations. *Earthquake Engineering and Structural Dynamics* **21:8**, 713-740.
- Der Kiureghian A. (1996). A coherency model for spatially varying ground motions. *Earthquake Engineering and Structural Dynamics* **25:1**, 99-111.
- Dumanoglu A. A. and Soyluk K. (2003). A stochastic analysis of long span structures subjected to spatially varying ground motions including the site-response effect. *Engineering Structures* **25:10**, 1301-1310.
- Harichandran R. S. and Vanmarcke E. H. (1986). Stochastic variation of earthquake ground motion in space and time. *Journal of Engineering Mechanics ASCE* **112:2**, 154-175.
- Harichandran R. S. (1993). An efficient, adaptive algorithm for large scales random vibration analysis. *Earthquake Engineering and Structural Dynamics* **22:2**, 151-165.
- Harichandran R. S. Hawwari A. and Sweidan B. N. (1996). Response of long-span bridges to spatially varying ground motion. *Journal of Structural Engineering, ASCE* **122:5**, 476-484.
- Heredia-Zavoni E. and Vanmarcke E. H. (1994). Seismic random vibration analysis of multi-support structural systems. *Journal of Engineering Mechanics ASCE* **120:5**, 1107-1128.
- Lee M. C. and Penzien J. (1983). Stochastic analysis of structures and piping systems subjected to stationary multiple support excitations. *Earthquake Engineering and Structural Dynamics* **11:1**, 91-110.
- Lin Y. K., Zhang R. and Yong Y. (1990). Multiply supported pipeline under seismic wave excitations. *Journal of Engineering Mechanics ASCE* **116:5**, 1094-1108.
- Lin J. H. (1992). A fast CQC algorithm of PSD matrices for random seismic responses. *Computers & Structures* **44:3**, 683-687.
- Lin J. H., Li J. J., Zhang W. S. and Williams F. W. (1997). Non-stationary random seismic responses of multi-support structures in evolutionary inhomogeneous random fields. *Earthquake Engineering and Structural Dynamics* **26:1**, 135-145.
- Lin J. H. and Zhang Y. H. (2005). Seismic Random Vibration of Long-span Structures, C. de Silva (Ed.), Chapter 30 in *Vibration and Shock Handbook*, CRC Press: Boca Raton, FL.
- Loh C. H., Yeh Y. T. (1988). Spatial variation and stochastic modeling of seismic differential ground movement. *Earthquake Engineering and Structural Dynamics* **16:5**, 583-596.
- Nazmy A. S. and Abdel-Ghaffar A. M. (1992). Effects of ground motion spatial variability on the response of cable-stayed bridges. *Earthquake Engineering and Structural Dynamics* **21:1**, 1-20.
- Yamamura N. and Tanaka H. (1990). Response analysis of flexible MDF systems for multiple-support seismic excitations. *Earthquake Engineering and Structural Dynamics* **19:3**, 345-357.
- Zerva A. (1990). Response of multi-span beams to spatially incoherent seismic ground motions. *Earthquake Engineering and Structural Dynamics* **19:6**, 819-832.