NUMERICAL RESEARCH ON THE EQUIVALENT TRANSFORMATION BETWEEN STRUCTURAL DYNAMIC ANALYSIS IN TIME-DOMAIN AND FREQUENCY DOMAIN

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ABSTRACT :

Numerical analysis methods in time-domain and frequency-domain are commonly considered as two important ways for seismic evaluation of structure responses. In terms of the seismic wave excitation and the structural output response, the expressions of time histories are usually applied in the time-domain method, while the complex harmonic waves or their summation generally used in the frequency-domain method, it is a focus in the field of structure engineering to construct equivalent expressions of seismic wave excitation and structural responses between time-domain and frequency-domain solution methods. In this paper, as far as general dynamic analysis of structure is concerned, a formula of trigonometric coefficients in time domain is deduced to compute frequency spectrum values at arbitrary frequency points for time history data, which avoids the disadvantage of the conventional discrete Fourier transform (DFT) method that merely suiting for the discrete frequency points. Hence, according to the seismic analysis of structural responses, a quantified assessment of equivalent expressions for various wave signals are given in detail, which builds a transformation bridge between time-domain and frequency-domain solution methods. Finally, the validity and feasibility of the transformation algorithm in time-domain and frequency-domain are numerical verified by the dynamic response analysis of multiple particles damp system and long span structure with seismic wave excitations. It's also shown from the results, as the trigonometric coefficient method (TCM) is concerned, only inputted the real part or the imaginary part of the seismic wave exaction time history, the equivalence response of structure in frequency domain can be obtained by a simple combination of trigonometric coefficients in time domain.

KEYWORDS: Seismic analysis of structural responses, Seismic wave, Trigonometric series, Fourier transform

1. INTRODUCTION

Numerical analysis methods in time and frequency domain are powerful tools for the dynamic analysis of structure. Since in frequency domain, the amplitudes of the structure dynamic response at different frequencies can be accurately described in simple formulation, and it is convenient to express the frequency content of the ground motion; so it is commonly used in the derivation of structure response analysis, such as soil-structure interaction analysis [1] and ground response [2]. In general, the structure response in frequency domain can be considered as the steady-state of the response in time domain. With the development of computer, more and more dynamic analysis methods in time domain are used in earthquake engineering. Compared with the methods in frequency domain, methods in time domain are able to account for the characteristics of nonlinear behavior of the structure [1, 3, 4, 5], and it can be express the dynamic response of structure at any time. So many methods of structure dynamic analysis are transferred from frequency domain to time domain, for example, structural damage detection [6]. As a result, it is necessary to practice the numerical research on the equivalent transformation between structural dynamic analysis in time-domain and frequency-domain.

As we all known, vibratory motion is a physical quantity varied with time, it is can be described in frequency domain and time domain. So it is possible for vibratory motion processing between frequency and time domain. In frequency domain, researchers usually used the discrete Fourier transform (DFT) to obtain the amplitude of the vibratory motion at the specific frequencies, but DFT method limits the output frequencies because of the frequency interval $\Delta\omega$ .So it is impossible for DFT method to get the spectrum at any frequencies. While in this paper, based on the notations of the vibratory motion: trigonometric form in time domain and complex exponential form in frequency domain, the equivalent transformation condition of vibratory motion formulation between frequency domain and time domain is derived. The equivalent transformation relationship can be expressed by trigonometric coefficients, which is described by analytical formulation. And therefore, the spectrum at any frequencies can be obtained by using the methods in time domain. Furthermore, how to construct the equivalent seismic wave excitation and how to understand the response of the structure in time and frequency domains? It is an interesting and attractive topic in civil engineering. In this paper, with respect to the dynamic response analysis of damped system subjected to harmonic loading, the response solutions in time and frequency domain are derived. An assessment on relationship of response solution between time and frequency domains is carried out. From the assessment, it is obvious that, only input the real part or imaginary part of the excitation seismic loading, the response in frequency domain can be obtained by the combination of trigonometric coefficients, which can be calculated by the response of structure in time domain. Finally, the validity and feasibility of the equivalent transformation condition between time-domain and frequency-domain are numerical verified by the dynamic response analysis of multiple particles damp system and long span structure subjected to seismic wave excitation. The numerical results showed that the trigonometric coefficient method (TCM) and DFT method are identical in calculating the spectrum of frequency. Furthermore, the advantage of the former is the continuity of output frequencies.

2. EQUIVLENT CONDITION OF VIBRATORY MOTION FORMULATION BETWEEN FREQUENCY DOMAIN AND TIME DOMAIN

Vibratory motion can be described in terms of displacement $x(t)$ by three ways, which will be presented as follows:

(A) The motion $x(t)$ can be expressed by trigonometric notations:

$$x(t) = \sum_{n=0}^{\infty} A_n \cos(\omega_n t - \varphi_n) \tag{2.1}$$

Where A_n represents the displacement amplitude, ω_n is the circular frequency, and φ_n is the phase angle.

(B)The motion $x(t)$ can be considered as a summation of simple harmonic functions; it can be expressed by using trigonometric notations:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos\omega_n t + b_n \sin\omega_n t) \tag{2.2}$$

Where the trigonometric coefficients are

$$a_0 = \frac{1}{T_a}\int_0^{T_a} x(t)dt \tag{2.3a}$$

$$a_n = \frac{2}{T_a}\int_0^{T_a} x(t)\cos\omega_n tdt \tag{2.3b}$$

$$b_n = \frac{2}{T_a}\int_0^{T_a} x(t)\sin\omega_n tdt \tag{2.3c}$$

And $\omega_n = 2\pi n / T_a$, T_a is the duration of the motion, a_0 represents the average value of $x(t)$ over the range $t = 0$ to $t = T_a$.

(C) The motion $x(t)$ can also be expressed in exponential form.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} c_n \cdot e^{i\omega_n t} \cdot d\omega_n \tag{2.4}$$

Where c_n is the spectrum of circular frequency ω_n, $|c_n|$ is the spectrum amplitude of the circular frequency ω_n, it can be presented as

$$c_n = \int_{-T_a/2}^{T_a/2} x(\tau)e^{-i\omega_n \tau} d\tau \tag{2.5}$$

Equation (2.2), and equation (2.4) is different formulation of $x(t)$, they are equivalent in the framework of mathematics [7].

The relationship between c_n and a_n, b_n can be derived directly from the exponential form of the motion by using Euler's law.

$$c_n = \frac{T_a}{2} a_n - i\frac{T_a}{2} b_n \tag{2.6}$$

Form the equation (2.6), we can conclude that the trigonometric coefficients a_n and b_n reflect the real part and imaginary part of c_n, respectively. Hence, except for DFT method, we introduce a new method to obtain the spectrum; it is referred as the trigonometric coefficient method (TCM). In fact, at the specific frequencies, we may be not got its spectrum by DFT method because of the discrete frequencies interval $\Delta\omega$. In order to remedy this point, it is usual to add a long segment of zero value at the end of motion duration, and then capture the spectrum approximatively. In doing so, the information of the time history of the motion will be destroyed to a certain extent. Alternatively, there are analytical formulations for a_n and b_n, so the spectrum of any circular frequency ω_n can be obtained accurately by TCM**.**

3. FORMULATION OF DYNAMIC RESPONSE ANALYSIS OF STRUCTURE IN FREQUENCY DOMAIN AND TIME DOMAIN

In the process of dynamic response analysis of structure, the time history of the motion mainly consists of seismic wave excitation and output structure response. In the frequency domain, the seismic wave excitation can be described as complex exponential form or their summation; accordingly, the output response of the structure can also be described as complex exponential form.

In the time domain, the seismic wave excitation is expressed in the form of time history, based on that, how to construct the equivalent seismic wave excitation between time domain and frequency domain and how to understand the output response of the structure, it is one of most important and interesting topics in earthquake engineering. In general, this problem can be summarized as two points. One point is: only the real part or imaginary part of the seismic wave excitation is inputted, the complete response of structure in the frequency domain can be obtained. The other point is: the real part and imaginary part of the input motion need to input, respectively, and then the complete response of structure in the frequency domain can be obtained by combining the response of the structure in time domain. In the next section, we will take damped system for example and discuss this problem in detail.

3.1. Response of damped system subject to periodic loading in frequency domain

In order to evaluate the dynamic response of a damped system, the differential equation of motion must be solved. First, the equation of motion can be written as

$$\ddot{u} + 2\zeta\omega\dot{u} + \omega^2 u = P_0 e^{i\omega_n t} \tag{3.1}$$

Where ω represents the natural circular frequency of the system, the response of the system can be related to the loading by

$$u(t) = U_0 e^{i\omega_n t} \tag{3.2}$$

Where U_0 is the amplitude of response in frequency domain, substituting equation (3.2) into the equation of motion gives

$$u(t) = \frac{P_0}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}}(\sin\theta - i\cos\theta)e^{i\omega_n t} \tag{3.3}$$

The amplitude of response in frequency domain is

$$U_0 = \frac{P_0}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}}\sin\theta - i\frac{P_0}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}}\cos\theta \tag{3.4}$$

Where

$$\tilde{\omega} = \omega^2 - \omega_n^2, \hat{\omega} = 2\zeta\omega\omega_n \qquad \sin\theta = \frac{\tilde{\omega}}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}}, \cos\theta = \frac{\hat{\omega}}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}} \tag{3.5}$$

3.2. The relationship between Response of damped system in time domain and in frequency domain

There are two types of external loading in time domain under which the dynamic response of the structure is equivalent to the response in frequency domain.

First, the equation of motion of the damped system subject to the real part of the external loading $P_0 e^{i\omega_n t}$ is written as

$$\ddot{u} + 2\zeta\omega\dot{u} + \omega^2 u = P_0 \cos\omega_n t \tag{3.6}$$

The general solution to the equation of motion for damped forced vibration can be obtained by combining the complementary and particular solutions. Note that the complementary solution, which describes a transient response caused by the requirement of satisfying the initial conditions, decays with time. After the transient dies out, only the steady-state response described by the particular solution remains, so only steady-state response solution is produced as follows

$$u(t) = \frac{P_0}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}}(\sin\theta\cos\omega_n t + \cos\theta\sin\omega_n t) \tag{3.7}$$

The trigonometric coefficients of equation (3.7) a_n and b_n are obtained by the TCM

$$a_n = \frac{P_0}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}} \sin\theta , b_n = \frac{P_0}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}} \cos\theta \qquad (3.8)$$

Compared with the amplitude of response solution in frequency domain, which is written as equation (3.4), it is obvious that the formulation of $a_n - i \cdot b_n$ is consisted with U_0, a_n is the real part of the U_0, $-b_n$ is the imaginary part of the U_0.

Second, the equation of motion of the damped system subject to the imaginary part of the external loading $P_0 e^{i\omega_n t}$ is written as

$$\ddot{u} + 2\zeta\omega\dot{u} + \omega^2 u = P_0 \sin\omega_n t \qquad (3.9)$$

The steady-state response solution of equation (3.9) is written as

$$u(t) = \frac{-P_0}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}} (\cos\theta \cos\omega_n t - \sin\theta \sin\omega_n t) \qquad (3.10)$$

In the same manner, the trigonometric coefficients of equation (3.10) a_n and b_n are expressed as

$$a_n = \frac{P_0}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}} \cos\theta , b_n = \frac{P_0}{\sqrt{\tilde{\omega}^2 + \hat{\omega}^2}} \sin\theta \qquad (3.11)$$

Compared a_n and b_n with equation (3.4), the formulation of $i(a_n - i \cdot b_n)$ is the same as the amplitude of response solution in frequency domain, where a_n is the imaginary part of the U_0, b_n is the real part of the U_0.

From the comparison mentioned as above, if both the real part and imaginary part of the seismic wave excitation are inputted, it is not correct to obtain the amplitude of response solution in frequency domain by combining the response of the structure in time domain. In reality, either the real part or the imaginary part of the seismic wave excitation is inputted in time domain, it is convenient to obtain the amplitude of response solution in frequency domain from the time history of response of structure in time domain. As a result, it also supplies a tool to verify advanced time domain algorithms in frequency domain. In section 4.1, the validity of TCM is proved through dynamic response analysis of multiple particles damp system.

In the same way, through TCM, the response of structure, which is subjected to the seismic wave excitation included phase difference and amplitude decay in frequency domain, can also be obtained. It also provides theoretical foundation for dynamic analysis of long span structure subjected to multi-support seismic wave excitation. In order to verify its availability and accuracy, the TCM is practiced on a single span bridge structure in section 4.2.

4. NUMERICAL EXAMPLES

4.1 Dynamic response analysis of multiple particles damp system

The multiple particles damp system shown in Figure 4.1 is at rest when the harmonic loading $F_1 = P_0 e^{i\omega t}$ is applied, where P_0 is 24.5N, ω is 20rad/s. Determine the amplitude of response of particle 2. The Rayleigh damping scale

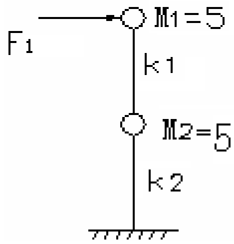

Figure 4.1 calculation model

coefficient is $\zeta = 0.05$, accordingly, the natural circular frequency of first mode shape is ω_1=32.418rad/s，the natural circular frequency of second mode shape is ω_2 =10.05rad/s.

First, the equation of motion is written as

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 9.26176 & -5.42592 \\ -5.42592 & 11.97473 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} 2304 & -2304 \\ -2304 & 3456 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P_0 e^{i\omega t} \\ 0 \end{Bmatrix} \quad (4.1)$$

The solution to this equation can be represented as $u_1 = \bar{u}_1 e^{i\omega t}, u_2 = \bar{u}_2 e^{i\omega t}$, the analytical solution of particle 2 obtained as

$$\bar{u}_2 = -.0115293 - .000171959i \quad (4.2)$$

So the amplitude of the response of particle 2 is $|\bar{u}_2|$=0.01153m.
Second, referring to section 3.2, the equation of motion of the damped system subject to the imaginary part of the external loading $P_0 e^{i\omega_n t}$ can be written as (4.1), where the $P_0 e^{i\omega_n t}$ is replaced by $24.5\sin(20t)$. Furthermore, The solution of equation can be obtained by the Precise Time-Integration Method [8], where $u_2(t)$ is plotted in Figure 4.2.

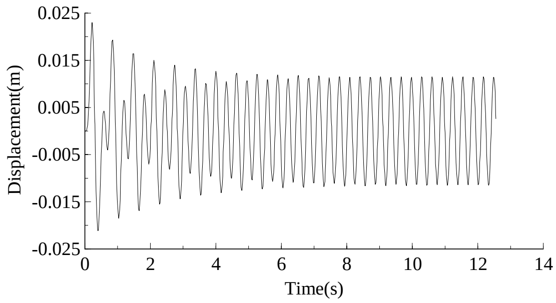

Figure 4.2 Response of particle 2

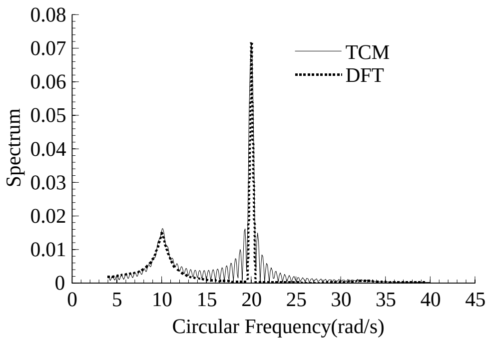

Figure 4.3 Frequency spectrum of particle 2

Using TCM, the trigonometric coefficient a_n and b_n of $u_2(t)$ can be obtained, based on that, the amplitude of response in frequency domain can be got by $i(a_n - i \cdot b_n)$. At the same time, the DFT [9] method is used to get the amplitude of response. The comparison of the results of spectrum is displayed in the Figure 4.3; the value of amplitude of response is compared in the Table 4.1.

Table 4.1 Comparison table of results in different methods

	Real part (m)	Spectrum (m)
Analytical solution	-0.01153	0.01153
trigonometric coefficient method (12)	-0.01105	0.01140
Relative Error	-4.16%	-1.13%
trigonometric coefficient method (24)	-0.01113	0.01143
Relative Error	-3.47%	-0.87%
FFT(19.8946)	-0.00827	0.01078
Relative Error	-28.27%	-6.55%
DFT(19.8946)	-0.00647	0.01078
Relative Error	-43.89%	-6.48%

The real part and spectrum of the response of the structure in frequency domain at the specific circular frequency of 20rad/s are listed in table 4.1, the "12" in the bracket means the duration of $u_2(t)$ in time domain is 12s, in the same manner, the "24" in the bracket means the duration of $u_2(t)$ in time domain is 24s, the "19.8946" in the bracket is approximate to the specific circular frequency of 20rad/s, because the DFT method can not capture the frequency of 20rad/s accurately. The analytical solution is obtained by equation (4.2).

From the results shown in Table 4.1, the TCM agrees with the analytical solution very well, and as the duration of the $u_2(t)$ increase, the relative error is decrease. Compared with DFT, there are analytical formulations for a_n and b_n, so the spectrum of any circular frequency ω_n can be obtained accurately by the TCM. While there is a certain interval of the circular frequency ω_n in the DFT method, sometimes, it is inevitable to approximate the specific circular frequency. So the TCM is better than the DFT method in the output of the frequency information.

4.2. Dynamic response analysis of long span structure

For earthquake engineering problems, dynamic loading often results from vibration of the supports of a system rather than from dynamic external loads. To evaluate the availability of the TCM in such systems, an example of dynamic response analysis of long span structure is presented in this section. In Figure 4.4 is a simple single span structure.

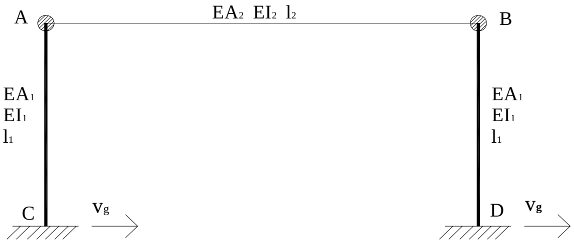

Figure 4.4 calculate model

Where $EI_1 = 1, EI_2 = 2, EA_1 = 1, EA_2 = 2, l_1 = 1, l_2 = 2, m_1 = m_2 = 5$. the natural circular frequency of first mode shape is ω_1=0.447rad/s,the natural circular frequency of second mode shape is ω_2=1.183rad/s. The particle C and D are subjected to a horizontal excitation $e^{i\omega t}$ and $Ae^{i(\omega t-\varphi)}$, respectively, in which A is 0.6, ω is 20rad/s, φ is 1.0. Determine the response of particle B.
First, for this case of base shaking the equation of motion can be expressed as

$$m\ddot{u} + c\dot{u} + ku = -mr\ddot{v}_g \quad (4.3)$$ [10]

Where $r = -k^{-1}k_g$, $\ddot{v}_g = \left\{ e^{i(20t)}, e^{i(20t-1)} \right\}^T$, $u = \left\{ u_1, u_2, u_3, u_4 \right\}^T$.The expression $\ddot{v}_g$ in the right of equation (4.3) represents the free-field input acceleration applied at the base of the structure. In a more general case, where the relative displacements are not all measured parallel to the ground motion, the total displacement may be expressed as the sum of the relative displacement and the quasi-static displacement that would result from a static-support displacement.
According to theory of dynamics of structure, the solution of equation (4.3) is written as

$$u_1 = \bar{u}_1 e^{i\omega t}, u_2 = \bar{u}_2 e^{i\omega t}, u_3 = \bar{u}_3 e^{i\omega t}, u_4 = \bar{u}_4 e^{i\omega t}, \quad (4.4)$$

Substituting equation (4.4) into equation (4.3), the horizontal response of particle B in frequency domain is obtained as $\overline{u}_3 = 0.00227830 + 0.000112366\,i$ .Accordingly, the spectrum value of the particle B is $|\overline{u}_3|$=00022811m

Second, the imaginary part of the complex horizontal excitation is represented at the base of the single span structure, the equation of motion can be written as (4.3), where $\ddot{v}_g = \{\ \sin 20t, 0.6\sin(20t-1)\}^T$, through the coefficient 0.6, the decay of the amplitude of the seismic wave is considered. The time lag between C and D is 1s, which reflects the excitation time is different at different support.

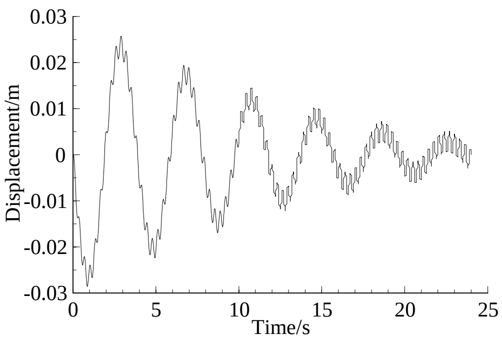

Figure 4.5 Time history of horizontal displacement of particle B

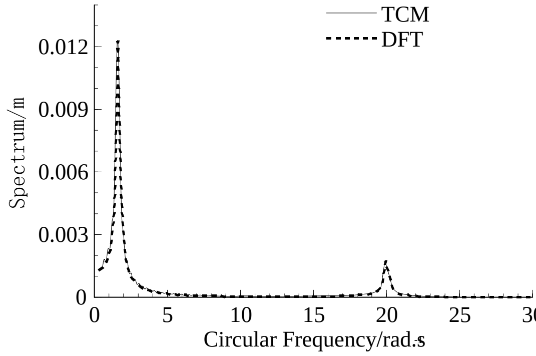

Figure 4.6 The horizontal spectra of particle B

The solution of equation is obtained by the Precise Time-Integration Method [3], where $u_3(t)$ is plotted in Figure 4.5. a_n and b_n of $u_3(t)$ can be obtained by the TCM. And the response solution in frequency domain is expressed as $i(a_n - i \cdot b_n)$. In other sides, the spectrum at the specific frequency ω is also got by the DFT. The comparison between the spectra curve is shown in the Figure4.6. The value of amplitude of response is compared in the Table.4.2

Table 4.2 Comparison between theory solution and the numerical solution on the horizontal spectra of particle B

Duration of time history of response	24s	48s
Theory solution	00022811	00022811
trigonometric coefficient method	00022594	00022608
Relative Error	-0.95%	-0.89%

From the Table 4.2, the horizontal spectra of particle B obtained by the TCM agrees well with the theory solution, the relative error does not exceed 1%. And the accuracy of the calculation will be improved if the duration of time history of response is longer. As shown in Figure 5, the difference between the spectra which calculated by the DFT and the TCM is very small, but in the method of DFT, the duration of the response is divided into N equal intervals Δt , as a result, the spectra can only be obtained at the discrete frequencies. Compared with DFT, the TCM can be output the spectra at the consecutive frequencies.

5 CONCLUSION

In this paper, through derivation and the numerical example of the equivalent condition between the TCM and DFT, the conclusion is summarized as follows:

(1) The spectrum at any frequencies can be obtained by the TCM, accordingly, the relationship between trigonometric coefficients and spectra c_n is written as

$$a_n - ib_n = 2c_n / T_a \tag{5.1}$$

(2) In reality, the TCM is a powerful tool to transfer response of structure from time domain to frequency domain. As far as the dynamic response analysis of structure is concerned, the real part or imaginary part of the external loading can only be applied on the structure, and then calculated the time history of the structure response, the a_n and b_n of the response is obtained by the TCM. Accordingly, the response of the structure in the frequency domain can be expressed as $a_n - i \cdot b_n$ or $i(a_n - i \cdot b_n)$.

(3) We compute the numerical results of the dynamic response analysis of multiple particles damp system and long span structure and compare them with the results obtained by the DFT as well as the theory solution. The comparison verifies the availability and accuracy of the TCM. And it is also shown that the traveling wave effect and amplitude decay of the wave can be considered in this method. In future, this method can be used to do the dynamic response analysis of long span structure subjected to multi-support excitation

(4) In the previous methods of generating artificial ground motions compatible with response spectrum, the target spectrum is usually obtained by the statistical method based on phase-difference spectrum and Fourier amplitude spectrum [11]. The TCM introduced in this paper can be used to get spectrum of arbitrary frequency, so the spectrum of the specific frequency can be set as a target parameter, it is possible to evaluate the constraint equation to produce some artificial ground motions on basis of the parameter optimization.

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