THE ROBUST COLLOCATION METHODS OF INTEGRATION INVERSION OF GPS AND EARTHQUAKE MONITORING DATA

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ABSTRACT:
This paper focuses on the mathematical model and computational methods for earthquake monitoring and forecasting by Robust Collocation Methods (RCM) of integration inversion utilizing GPS data, gravity data, geomagnetic data, and seismic data. The method of the paper is based on the various data combined collocation, according to the mathematical principles of signal processes, spectral analysis, and time series analysis. The mathematical modeling for earthquake monitoring and forecasting by combining GPS monitoring data and various common seismic monitoring data (e.g. gravity data, geomagnetic data, seismic data, etc.) is completely set up. The various seismic monitoring data mentioned above is transformed to the same basic framework and optimally combined in the framework. The mathematical modeling of robust covariance functions and robust collocation by the combined data mentioned above is independently proposed and are derived from the corresponding formula.
Some important conclusions are attained form the application of GPS monitoring data and common seismic monitoring data is studied in the earthquake monitoring and forecasting. The robust covariance functions and Robust Collocation Method (RCM) by combined GPS monitoring data and common seismic monitoring data. Overcomes the short comings of conventional methods and the Least Squares Collocation (LSC) in the earthquake monitoring and sets up the foundation for increasing the accuracy of earthquake monitoring and forecasting by optimally combining GPS monitoring data and common seismic monitoring data. The new methods will be more and more useful thanks to the growing precision and the generalization of GPS monitoring.
KEYWORDS: Robust Collocation Methods (RCM), GPS monitoring, Earthquake monitoring and forecasting, Integration inversion, Spectral analysis.

1. DEMAND FOR INTEGRATED MULTIPLE DATA FOR EARTHQUAKE FORECASTING

1.1 Introduction

The purpose of earthquake study is to monitor the interactions between tectonic plates, the field of forces applied to the plates along tectonic edges, and the corresponding stress fields. The goal of earthquake forecasting is to provide the possible occurring time of primary shocks, the location of epicenters, and their magnitudes. At this moment, it is still very difficult to forecast the earthquake due to its complexity.
As some geosciences, such as geology, geophysics, and geospatial information technology, have been advancing very rapidly, people have made great progress in understanding the internal structure of the earth, tectonic movements, earthquake mechanism, and earthquake forecasting. The accuracy, intensity, and coverage of gravity surveying, geomagnetism surveying, earthquake wave recording, and the image technology for earthquake signs are far better than ever before. Ocean surveying by satellites and global positioning systems can provide abundant data with high accuracy as a result of recent advances in geospatial information technology. For example, 70% surface of the earth is covered by sea water, and gravity data in ocean regions had long been scare before satellite altimetry was invented. This technology enables the gravity data in ocean regions to reach even higher accuracy and resolution than the data in land areas (Mao 1995).Global positioning systems provide us with displacement change data with high accuracy observed globally. There is no doubt that displacement changing monitored by GPS is one of the most important data resources for earthquake monitoring and forecasting. In the last two decades, many countries in the world have established large-scale tectonic movement monitoring systems with GPS. In China, Chinese Tectonic Movement Observation Networks was also founded, and a great amount of earthquake monitoring

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data with GPS has been collected. In other words, China has obtained more data with high accuracy for predicting earthquakes. In regards to earthquake monitoring data, a few decades ago, there were few kinds of the data with low resolution and less coverage. Theories in earthquake forecasting were far more advanced than earthquake monitoring technology. At this moment, the overall trend on earthquake forecasting is completely reversed from that. Earthquake monitoring technology is far ahead of computational methods for earthquake forecasting. We should integrate various kinds of monitoring data relevant to earthquakes. Special attention should be given to the earthquake monitoring data from GPS since it is abundant and of much higher resolution.In regard to earthquake monitoring and forecasting, gravity anomaly in ocean can be calculated from satellite altimetry. Data on gravity anomaly in land as well as in ocean with high accuracy (as high as mg) can help us understand the internal structures of the earth. Underground rocks can be recognizable horizontally if the accuracy of gravity surveying data is at the level of tenths of nT. For the earth, the internal structures, mass distribution, and dynamic changing processes can be monitored based on the data on gravity and geomagnetism. The earthquake monitoring data, including earthquake waves and images, can help us understand the changes in geological structures near where earthquakes occurred. GPS can provide earthquake monitoring data covering a large region and with high accuracy. Spatially, data from GPS covers ranging from local to global; temporally, from few seconds/minutes to several decades/centuries. GPS can provide data on geometric displacements, geological deformation, and tectonic movements at 10-9 relative accuracy, observe the dynamic processes of the earth, and discover the quantitative evidence of crust movements. At present, one of the most important tasks facing us is how to fully utilize the earthquake monitoring data mentioned above. The computational methods in earthquake forecasting would have some new development as we have a new way to integrate these kinds of earthquake monitoring data.

1.2 The Earthquake Forecasting in the Sense of Geophysics

Integrated multiple data can help us to monitor and forecast earthquakes. Mathematically and geophysically, finding the solutions of integrated multiple data may belong to either solving the Molodensky boundary value problem or inverting the Molodensky surface determination problem.Molodensky problems with integrated multiple data can be solved based on statistic, model, or associated calculation approaches. In the last two to three decades, some scholars from geophysics, physical geodesy, and earthquake backgrounds have studied how to predict earthquakes based on integrated gravity and geomagnetism wave data with these three approaches. Although the theoretical foundation of combining multiple data with inversion for earthquake forecasting has not been developed, many research results based on this approach have been reported. It can be classified to be an indeterminate boundary value problem solving the Molodensky problem with integrating GPS, gravity, geomagnetism, and earthquake monitoring data. An indeterminate boundary value problem is a boundary value problem may require more rigorous and extra boundary conditions. The solutions obtained from an indeterminate boundary value problem may not be compatible with each other because the boundary values are those monitoring data with errors under the same boundary. Therefore, to obtain an optimal solution it should adopt statistic or random process method. This paper gives the theory and computational method for multiple data integration for earthquake forecasting. These data are GPS, gravity, geomagnetism, and earthquake monitoring data.

2. DATA CHARACTERISTICS AND THE CONCEPT OF INTEGRATED MULTIPLE DATA FOR EARTHQUAKE

For the last two or three decades, surveying technologies for collecting earthquake monitoring data have been advanced rapidly. Among these technologies, the progresses made satellite altimetry and crust monitoring with GPS are considered breakthrough. Data on gravity anomaly derived from the results of satellite altimetry has been covered globally. Data on crust monitored by GPS has high geometric accuracy. These advances have provided us with abundant data for forecasting earthquakes based on integrated multiple data. One of the most crucial tasks facing us in earthquake forecasting based on multiple monitoring data is its computational methods. Three requirements are essential to the computational methods: (1) we should be able to forecast earthquakes computationally with every kind of earthquake monitoring data; (2) we should be able to establish the theory for the concept of integrated multiple data for earthquake forecasting; (3) we should be able to forecast earthquakes computationally with integrated multiple data. There are many studies devoted to forecast earthquakes computationally with a specific kind of earthquake monitoring data. This paper covers very little on it. This paper discusses requirements 2 and 3 as mentioned above. The computational methods for integrated multiple data focus on the inversion of the Robust Collocation Method (RCM).

2.1 Characteristics of Gravity Data, Geomagnetism Data, and GPS Data

The main purpose of earthquake monitoring and forecasting is to study changes in the displacement field and the stress field. Therefore, it is more scientific and convenient to express gravity data in terms of gravity anomaly $(\Delta\vec{g})$, geomagnetism data in terms of geomagnetism anomaly $(\Delta\vec{T})$, GPS data in terms of GPS displacement $(\Delta\vec{P})$.The convention adopted in this paper is the right-hand coordinate system with X and Y axes on the ground surface and Z axis pointing downwards. Any point in the space is expressed in terms of (x, y, z) and the mass point of the earth in terms of $(\varepsilon, \eta, \zeta)$.The integrated multiple data for earthquake forecasting, such as gravity anomaly $(\Delta\vec{g})$, geomagnetism anomaly $(\Delta\vec{T})$, earthquake monitoring data, and GPS displacement data, are shown in Figure 1.

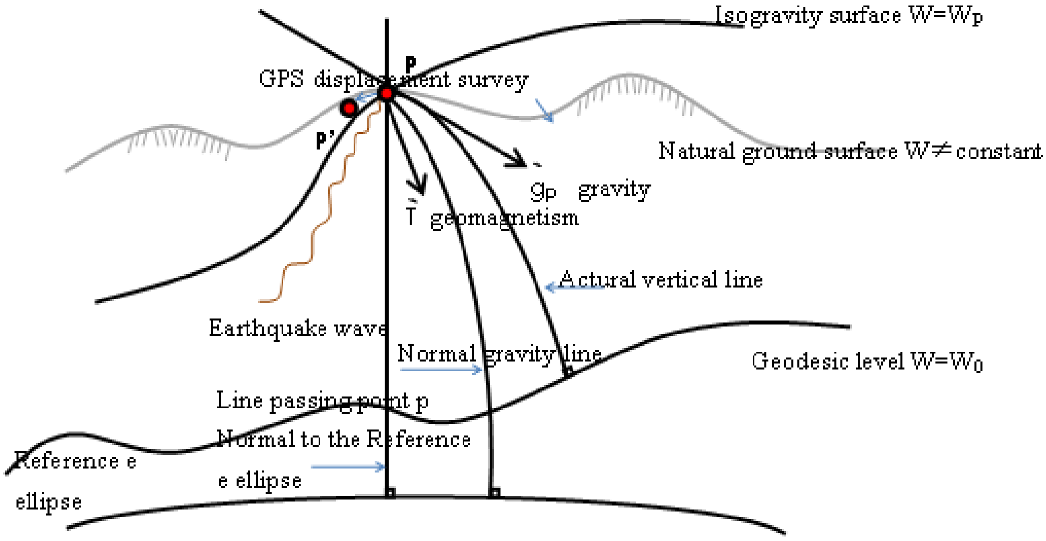

Fig 1 The Robust Collocation Methods of Integration Inversion of Multiple Data

（1）gravity anomaly $\Delta\vec{g}$

$$\Delta\vec{g}(x, y, z) = \frac{\partial}{\partial Z} W_G(x, y, z) \qquad (2—1)$$

$W_G(x, y, z)$ is gravity potential：

$$W_G(x, y, z) = W \iiint_V \sigma_G(\varepsilon, \eta, \varsigma) \frac{1}{r} d\varepsilon d\eta d\varsigma \qquad (2—2)$$

In the formula： $r = \sqrt{(x-\varepsilon)^2 + (y-\eta)^2 + (z-\varsigma)^2}$ （2—3）

Gravity potential should satisfy the following Poisson equation：

$$\frac{\partial^2 W_G}{\partial x^2} + \frac{\partial^2 W_G}{\partial y^2} + \frac{\partial^2 W_G}{\partial z^2} = -4\pi G \sigma_G \qquad (2—4)$$

（2）geomegnetism anomaly $\Delta\vec{T}$

$$\Delta\vec{T}(x,y,z)=\frac{\partial W_T}{\partial V}(x,y,z) \quad (2—5)$$

W_T is the geomagnetism potential：

$$W_T(x,y,z)=\iiint_V J(\varepsilon,\eta,\varsigma)grad\frac{1}{r}d\varepsilon d\eta d\varsigma \quad (2—6)$$

The r in above equation is same as the r in equation （2—3）
Geomagnetism potential should satisfy the following Poisson equation：

$$\frac{\partial^2 W_T}{\partial x^2}+\frac{\partial^2 W_T}{\partial y^2}+\frac{\partial^2 W_T}{\partial z^2}=-4\pi J \quad (2—7)$$

（3）earthquake power spectrum

$$P(k)=\left|X(k)\right|^2 \quad (2—8)$$

In the equation $X(k)=\sum_{n=1}^{N}x(n)\exp(\frac{-j2\pi kn}{N})$ （2—9）

In the equation, $x(n)$ is the earthquake monitoring data， $(n=1,2,\cdots,M)$

（4）GPSdisplacement $\Delta\vec{P}$

$$\Delta\vec{P}(x,y,z)=f_p(\Delta x,\Delta y,\Delta z) \quad (2—10)$$

2.2 Concept of Inversion Based on Integrated Multiple Data

Calculating various physical parameters of the earth based on geophysical monitoring data is called inversion. It is called Strokes boundary value (or inversion) problems to obtain the geophysical solution by treating the geodetic level surface as the boundary surface. It is called the Molodensky boundary value (or inversion) problems to solve the geophysical parameters by treating the regional surface as the boundary surface. It is a Molodensky boundary value (inversion) problem with integrated multiple data if the earthquake monitoring parameters (Φ_q)can be determined based on various data observed at ground surface, such as those data on gravity anomaly, geomagnetism anomaly, earthquake monitoring, GPS displacement monitored. The inversion that is based on one kind of earthquake monitoring data has been well studied by many researchers. The focus of this paper is on the theory and computing procedures of inversion earthquake forecasting based on integrated multiple data.

3. INVERSION OF THE ROBUST COLLOCATION THEORY WITH INTEGRATED MULTIPLE DATA

Inversion of integrated multiple data can be mathematically viewed as multi-stage stable random process on a globe. Actually, spectral analysis is another way to express the random process by means of Fourier transformation. The robust collocation is an application of most probable spectral analysis or greatest entropy spectral estimating.
In order to implement the robust collocation of integrated multiple data; all kinds of data employed should be converted to a single coordination system.

3.1 Mathematic Characteristics of Random Process

The distribution function $F(t,x)$ of a random process $\{x(t),t\in T\}$; if the random variance for a fixed time $t\in T$ is $x(t)$, and then its mathematical expectation (average function) and square difference (covariance function) are： $U_x(t)=E[x(t)]=\int_{-\infty}^{\infty}xdF(t,x)$ （3—1）

$$x^2(t)=D[x(t)]=E[x(t)-u_x(t)]^2=\int_{-\infty}^{\infty}[x-u_x(t)]^2 dF(t,x) \quad (3—2)$$

And then from the auto-covariance function (correlated function) is：

$$R_x(t_1,t_2)=E[X(t_1)X(t_2)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x_1,x_2dF(t_1,t_2,x_1,x_2) \qquad (3—3)$$

From auto-covariance function (covariance function) is：

$$C_x(t_1,t_2)=E[x(t_1)-u_x(t_1)]\ [x(t_2)-u_x(t_2)] \qquad (3—4)$$

Similarly it can extend to many random processes. Associated problems with multiple data are many stable random process problems. Therefore, these problems have various characteristics of multiple stable random processes.

3.2 Spectral Analysis for Stable Random Process

Stable random processes can be described in the spectral field by means of Fourier transformation. When angular frequency ω is introduced into the Fourier transformation can be written as：

$$X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-i\omega t}dt \qquad (3—5)$$

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}x(\omega)e^{i\omega t}dw \qquad (3—6)$$

And it can obtain $\int_{-\infty}^{\infty}x^2(t)dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2d\omega$ (3—7)

Where $|X(\omega)|^2$ is the density of energy spectrum, $\lim_{T\to\infty}\frac{|X(\omega)|^2}{2T}$ is the density of power spectrum, and $G_x(\omega)d\omega$ is the density of power spectrum of survey elements $\{X_n,n=0,\pm1,\pm2,\cdots\}$.

When $X(E)$ and $Y(E)$ are two stable random processes, if its correlated function $R_{xy}(t)$ can satisfy $\int_{-\infty}^{\infty}|R_{xy}(t)|dt<\infty$， it can be called as the Fourier transformation of $R_{xy}(t)$, and then $G_{xy}(t)$ the density of the co-power spectrum of $X(t)$ and $Y(t)$ is： $G_{xy}(\omega)=\int_{-\infty}^{\infty}R_{xy}(t)e^{i\omega t}dt$

The relation of the density of the co-spectrum and co-covariance function is：

$$G(\omega)=\sum_{r=-\infty}^{\infty}R(r)e^{-i\omega r} \qquad (3—8)$$

3.3 Earthquake Observation Function for Single Observation Data

The object function for earthquake forecasting with single monitoring data has been studied by several researchers. It is not the focus of this article. Following are the assumptions：

$$\Delta\bar{g}(x,y,z)=f_g(\Delta g)(t,x,y,z,M) \qquad (3—9)$$

$$\Delta\bar{T}(x,y,z)=f_t(\Delta T)(t,x,y,z,M) \qquad (3—10)$$

$$X(k)(x,y,z)=f_k(X)(t,x,y,M) \qquad (3—11)$$

$$\Delta\bar{p}(x,y,z)=f_p(\Delta P)(t,x,y,M) \qquad (3—12)$$

Where t is the time of an earthquake forecasted, (x, y, z) is the location where an earthquake forecasted, M is the magnitude of an earthquake forecasted. We can use the methods mentioned above to express various kinds of monitoring data in spectrum formats so that values of spectral field and power spectrum for the data can be obtained.

3.4 Estimate for Capon Maximum Entropy Spectrum

The entropy of each sample of observation series in temporal-spatial field from stable random process：

$$H = \int_{-\frac{1}{2}}^{\frac{1}{2}} \ln G_x(f) df \qquad (3—13)$$

Object function： $\int_{-\frac{1}{2}}^{\frac{1}{2}} \ln G_x(f) df = \max$ （3—14）

Constraint condition： $R_x = \int_{-\frac{1}{2}}^{\frac{1}{2}} \ln G_x(f) e^{-i2\pi f} df$ （3—15）

When variance is chosen to be the minimum requirement, the power density in frequency is the maximum-likelihood spectral estimate, and then unit impulse function is assumed as

$$A = [a_0, a_1, \ldots, a_{p-1}]^T \qquad (3—16)$$

output Y_t is： $y_t = \sum_{k=0}^{p-1} a_k x_k$ （3— 17）

The variance of the output Y_t is： $\sigma_y^2 = A^T RA$ （3— 18）

Where R is the data of the correlated function mentioned above.

Assume the I/O is $\overline{E_0} A = 1$， this is a function problem. There is a maximum entropy estimate：

Object function： $A^T RA = \min$ （3— 19）

Constraint condition： $\overline{E_0} A = 1$ （3— 20）

Create a function： $\varphi = A^T RA - \lambda(\overline{E_0} A - 1) = \min$ （3— 21）

From extremal condition： $\frac{\partial\varphi}{\partial A} = \frac{\partial[A^T RA - \lambda(\overline{E_0} A - 1)]}{\partial A} = 0$ （3— 22）

Can obtian： $\overline{E_0}^T R_0^{-1} E_0 = \max$ （3— 23）

That is $\Phi^T(f) R^{-1} \Phi(f) = \max$ （3— 24）

Where R is a correlated function in the stable random process as mentioned above， and $\Phi\ (f)$ is a transformation function.

3.5 A Mathematic Model for the Robust Collocation Methods with Integrated Multiple Data

Mathematical model： $l = BS + n$ （3— 25）

Object function： $G_{ll}(f) = B^T RB = \min$ （3— 26）

Constraint condition 1： $\Phi^T(f) B = I$ （3— 27）

Constraint condition 2：estimated robust

The minimization of an object function can be implemented by Legendre's minimized trace, and an extremal function can be created：

$$tr(E) = tr[B^T RB - \lambda(\Phi^T(f) B - I)] = \min \qquad (3— 28)$$

That is $\frac{\partial tr(E)}{\partial B} = \frac{\partial tr[B^T RB - \lambda(\Phi^T(f) B - I)]}{\partial B} = 0$ （3— 29）

Afterwards we can estimate $\hat{S}$ and covariance matrix, and the estimated value and its accuracy for the inversion robust collocation with multiple data can be obtained:

$$\hat{S} = G_{st}(G_{tt} - G_{nn})^{-1} l_{q-b} + \frac{1}{k^2} G_{st}(G_{tt} + G_{nn})^{-1} l_b \qquad (3—30)$$

$$\hat{\sigma}^2 = G_{ss} - G_{st}(G_{tt} + G_{nn})^{-1} G_{ts} \qquad (3—31)$$

4. PROCEDURES FOR SOLVING THE ROBUST COLLOCATION METHODS FOR INTEGRATED MULTIPLE DATA

4.1 Unified Basis for Data

When inverting robust collocation with integrated multiple data, a unified datum should first be established. Data reduction and processing should be conducted under this datum. It is recommended that unified datum system be same as World Geodetic System 1984 (WGS-84).

4.2 Preprocess for Various Kinds of Data

The main purpose of pre-processing various data is to modify or correct various kinds of monitoring data.

4.3 *Unified Process for Data*

$\Delta\vec{g}$ 、 $\Delta\vec{T}$ 、 $X(t)$、 $\Delta\vec{P}$ are different kinds of the geophysical quantity. Their units are different, and the orders of magnitude for these quantities are also very different. It is necessary to unify them since they are independent statistically. Their variance are $\sigma_{\Delta g_i}(i=1,2,\cdots)$、 $\sigma_{\Delta T_i}(i=1,2,\cdots)$、 $\sigma_{X(t)_i}(i=1,2,\cdots)$、 $\sigma_{\Delta p_i}(i=1,2,\cdots)$， after unification $\frac{\Delta g_i}{\sigma_{g_i}}$ 、 $\frac{\Delta T_i}{\sigma_{\Delta T_i}}$ 、 $\frac{X(t)_i}{\sigma_{X(t)_i}}$ 、 $\frac{\Delta P_i}{\sigma_{\Delta p_i}}$ are quantity without units. After unification, gravity anomaly , geomagnetism anomaly , earthquake monitoring data, and GPS displacement data can be expressed $\underline{\Delta\vec{g}}$ 、 $\underline{\Delta\vec{T}}$ 、 $\underline{X(t)}$、 $\underline{\Delta\vec{P}}$ 。

4.4 Calculation for the Coefficients of Associated Square Difference Function

From equation 3.1, integrated multiple data belongs to stable process with multiple random variables, and the relevant functional value matrix associated with their space field is:

$$R(r) = \begin{bmatrix} R_{11}(r) & R_{12}(r) & \cdots & R_{1p}(r) \\ R_{21}(r) & R_{22}(r) & \cdots & R_{2p}(r) \\ \cdots & \cdots & \cdots & \cdots \\ R_{p1}(r) & R_{p2}(r) & \cdots & R_{pp}(r) \end{bmatrix} \qquad (4—1)$$

From equation 3.2, monitoring data can be expressed in the spectral field, and its power spectrum matrix (category P=4) is：

$$G(f) = \begin{bmatrix} G_{11}(f) & G_{12}(f) & G_{13}(f) & G_{14}(f) \\ G_{21}(f) & G_{22}(f) & G_{23}(f) & G_{24}(f) \\ G_{31}(f) & G_{32}(f) & G_{33}(f) & G_{34}(f) \\ G_{41}(f) & G_{42}(f) & G_{43}(f) & G_{44}(f) \end{bmatrix} \qquad (4—2)$$

And the density of the power spectrum is：

$$G_{ll}(f)=\begin{bmatrix} G_{gg}(f) & G_{gT}(f) & G_{gx}(f) & G_{gp}(f) \\ G_{Tg}(f) & G_{TT}(f) & G_{Tx}(f) & G_{Tp}(f) \\ G_{xg}(f) & G_{xT}(f) & G_{xx}(f) & G_{xp}(f) \\ G_{pg}(f) & G_{pT}(f) & G_{px}(f) & G_{pp}(f) \end{bmatrix} \qquad (4—3)$$

$$G_{ts}(f)=\begin{bmatrix} G_{gs}(f) \\ G_{Ts}(f) \\ G_{xs}(f) \\ G_{ps}(f) \end{bmatrix}=\Phi(f)G_{ss}(f) \qquad (4—4)$$

$$G_{nn}(f)=\begin{bmatrix} G_{n_1n_1}(f) \\ G_{n_2n_2}(f) \\ G_{n_3n_3}(f) \\ G_{n_4n_4}(f) \end{bmatrix} \qquad (4—5)$$

4.5 Solving for Robust Collocation

When the coefficients of a covariance function are plugged into equation 3-25, the estimation of integration inversion with multiple data can be obtained. These coefficients can be plugged into equation 3-26, errors of the estimated values can be obtained.

5. CONCLUSIONS

（1） This paper provides a theory and a integrated inversion framework for earthquake forecasting based on multiple earthquake monitoring data.

（2） This paper describes the mathematical model and calculation procedures for the robust collocation inversion of integrated multiple data.

（3） When the mathematical model of earthquake forecasting based on a single data is known, the integrated inversion for multiple data should be implemented by using the method provided in this article. In regard to the mathematical models for earthquake forecasting based on single data, many studies have been conducted.

（4） When the accuracy of the results obtained from using the robust collocation methods of integration inversion is low, we can consider to perform a second robust collocation based on the square difference matrix of the first robust collocation.

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