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PLASTIC HINGE SETTING FOR NONLINEAR PUSHOVER ANALYSIS OF PILE FOUNDATIONS

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ABSTRACT :

For a pile-soil system, the use of the concentrated plastic hinge method in simulating the inelastic flexural behavior of piles usually gives unsatisfactory results because the location of maximum moment in piles may vary due to the effects of soil-structure interaction. To solve this problem, this study suggests adopting a distributed plastic hinge model to trace the propagation of yielding in a pile interacted with nonlinear soils. To construct the rotational property of a plastic hinge, this paper modifies the definition of plastic curvature suggested in ATC-40. This modification is crucial for plastic hinges in simulating correctly the nonlinear deformation of a beam. Besides, to arrange distributed hinges properly, this paper proposes simple formulae to estimate the least range of plastic zone for a pile-soil system, so that the proper intervals of plastic hinges can be determined. To demonstrate the method of modeling proposed, numerical examples of a pile-soil system were analyzed using the computer program SAP2000 with distributed plastic hinge modeling. Results are in good agreement with those from ABAQUS analyses by using nonlinear elements for pile modeling.

KEYWORDS: pile foundations, pushover analysis, material nonlinearity, plastic hinges

1. INTRODUCTION

When the plastic hinge method was applied to a pile-soil system, the results obtained were not satisfactory. The main difficulties arise from:

(1) The moment vs. plastic-rotation relationship of a pile is very difficult to be obtained directly.

Lots of experimental data of beam or column load tests can be used to deduce reasonable and proper M-θ_p relationships for frame components, such as suggested in ATC-40 (1996), FEMA 273 (1997). These suggested relationships have been coded into some commercial software, such as SAP2000, ETABS. However, due to lack of sufficient experimental data, the M-θ_p relation of a pile is usually evaluated from theoretical analysis.

(2) The location of plastic zone for a pile-soil system is not easy to be determined.

For a frame system, the locations of plastic hinges can be determined in advance since the maximum moment usually occurs near the joints of a frame structure. However, for a pile-soil system, the location of maximum moment may vary with the development of soil plasticity around the depth of a pile.

Therefore, the purpose of this study is to propose a proper plastic hinge method for the nonlinear pushover analysis of a pile-soil system, including the determination of the rotational property of plastic hinges and the arrangement of plastic hinges in a pile.

2. PLASTIC HINGE METHOD

In structural analysis, the inelastic flexure of a beam or column may be modeled by using the concentrated or distributed hinge model. The concentrated hinge model is applied for cases where the yielding will most probably occur at the member ends, unlikely along a member. The distributed hinge model is applied for cases where the yielding may occur along a member.

In the concentrated hinge model, all plastic flexural deformations within a plastic zone are represented by a zero-length point hinge. Therefore, the dimension of plastic zone for a plastic hinge (i.e., plastic hinge length) should be given in advance to calculate the plastic rotation. However, instead of using a single hinge for a defined plastic zone, the distributed plastic hinge model inserts many plastic hinges along the expected plastic zone of a member. The plastic hinge length can be determined based on the spacing of plastic hinges. When the moment at the location of a plastic hinge exceeds its yield moment (implying the whole subsection has yielded), the plastic hinge yields and produces a plastic rotation. The range of yielding plastic hinges defines the actual plastic zone. This model thus saves the trouble to define the precise location of the plastic zone.

For a pile-soil system, the concentrated hinge model can not be applied when the location of maximum moment in a pile will vary as the development of soil plasticity around the pile. Instead, the distributed hinge model will be more appropriate by setting plastic hinges over the possible range of plasticity in a pile to trace the development of plastic zone in the pile. Accordingly, as the load increases the plasticity can spread, rather than being concentrated at a single point. Thus, this paper recommends adopting the distributed plastic hinge method for complete nonlinear pushover analysis of pile foundations.

3. ROTATIONAL PROPERTY OF A PLASTIC HINGE

In order to use the plastic hinge method for modeling the nonlinear flexure of a beam, the relationship of moment vs. plastic-rotation of each plastic hinge adopted has to be defined in advance. Since every plastic hinge is used to reflect the post-yielding rotation of its tributary length, the term "plastic hinge length" is introduced. For distributed plastic hinge method, the plastic hinge length will be decided on the spacing of plastic hinges introduced. Once the plastic hinge length has been chosen by users, the moment vs. plastic-rotation (M-θ_p) curve can be calculated based on the procedure suggested in ATC-40 (1996). First, the moment-curvature relation of the beam section is calculated using realistic estimates of material stress-strain relations to deduce the yield and ultimate curvatures, ϕ_y and ϕ_u, respectively. From this relation, when the curvature (induced by M) is larger than M_y, the plastic curvature defined by ATC-40 is shown in Fig. 1 and can be calculated as

$$\phi_p = \phi - \phi_y \tag{3.1}$$

Next, according to the plastic hinge length l_p set, the M-θ_p relation can be computed by

$$\theta_p = \phi_p \cdot l_p \tag{3.2}$$

where θ_p is the plastic rotation; ϕ_p is the plastic curvature, and l_p is the plastic hinge length.
The above mentioned ATC method has been widely used in practice, such as the computer code SAP2000. However, applying this method may yield errors in estimating the rotation and deformation of a member as compared to a real nonlinear member.

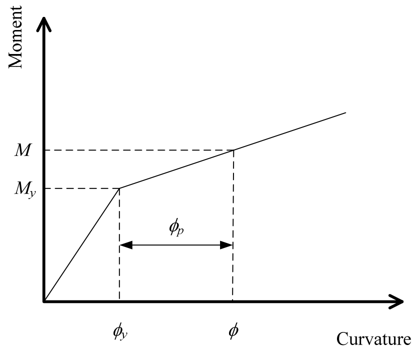


Figure 1 Plastic curvature defined in ATC-40 (1996)

To give correct computations of nonlinear deformations, the plastic curvature has to be modified as

$$\phi_{pm} = \phi - \frac{M}{EI_e} \tag{3.3}$$

where EI_e is the elastic flexural rigidity.

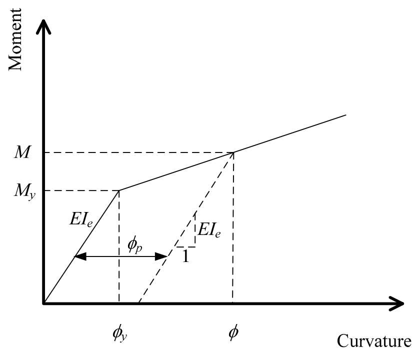


Figure 2 Modification for definition of plastic curvature

Equation (3.3) shows the plastic curvature should be obtained by subtracting the curvature of elastic rebound from the curvature at the post-yield state as shown in Fig. 2. According to the modified plastic curvature, the plastic rotation should be calculated by

$$\theta_p = (\phi - \frac{M}{EI_e}) \cdot l_p \tag{3.4}$$

4. PLASTIC HINGE ARRANGEMENT FOR A PILE-SOIL SYSTEM

A proper arrangement of distributed plastic hinges for a pile is important to identify the location of the plastic zone developed in the pile. Although the density of distributed hinges should be as dense as possible, a reasonable interval is necessary for the sake of computational cost. Generally, a convergence test can be conducted to decide an appropriate arrangement by varying the density of plastic hinges. It is still time consuming. Therefore, having a criterion about plastic hinge arrangement for piles is useful for reference. A basic requirement for a proper arrangement of plastic hinges is that the plastic hinge length of each hinge should be chosen much less than the actual range of the plastic zone that can be developed. Therefore, if the possible range of plastic zone L_p can be estimated in advance, the plastic hinge length of distributed hinges l_p can be chosen to be a fraction of this range, for example, l_p may be set as $L_p/10$ to grasp the gradual spreading of the plastic zone. This way can eliminate the need to perform the convergence check. To this end, this section attempts to deduce a guideline for estimating the possible range of plastic zone for a pile-soil system.

Assume a pile has a nonlinear moment-curvature relation defined by the yield moment M_y and the ultimate moment M_u. When the moment of pile section exceeds the yield moment M_y, the pile starts to yield. As the load gradually increases, the plastic zone expands over an appreciable length of the pile until the moment of a pile section reaches to its ultimate moment M_u. The distance between the locations where M_y and M_u occur is the range of plastic zone.

To decide the range of plastic zone for a pile-soil system, two possible locations that a plastic zone may develop should be taken into account: (1) the plastic zone of a pile occurs at a certain depth below the ground surface and (2) the plastic zone of a pile occurs at the pile head under a restrained-head condition (fixed-head condition).

(1) the plastic zone of a pile occurs at a certain depth below the ground surface

In this case, the ultimate moment M_u will occur at a certain depth of the pile and the plastic zone will extend upwards and downwards to the locations where the moment is equal to the yield moment M_y. Take the upper plastic zone of a pile as a free body as shown in Fig. 3(a). Suppose that the moments at the upper and lower edges of the free body reach to the yield moment M_y and the ultimate moment M_u, respectively. Let the length be L_p. Assume that the lateral soil pressures p on the pile is uniform. According to the beam theory, assuming the moment distribution of the pile is continuous and differentiable along the pile, the location of maximum moment is the location of zero shear. Thus, the lower end of the plastic zone where the ultimate moment M_u is reached, the shear force is zero. From equilibrium, the upper end where the yield moment M_y is reached, the shear force is $p{\cdot}L_p$.

The equilibrium of moments about point A yields

$$M_u - \frac{pL_p^{\,2}}{2} - M_y = 0 \tag{4.1}$$

Solving Eqn. (4.1) for L_p as

$$L_p = \sqrt{\frac{2(M_u - M_y)}{p}} \tag{4.2}$$

From Eqn. (4.2), the range of plastic zone increases with the difference between M_u and M_y, but decreases with the increasing of soil pressure. When the difference between M_u and M_y is zero, the range of plastic zone becomes zero. This means the plastic zone forms only when M_u is larger than M_y.

If the soil pressure p is set to be its ultimate value, p_{ult}, one gets

$$L_p \geq \sqrt{\frac{2(M_u - M_y)}{p_{ult}}} \tag{4.3}$$

Equation (4.3) gives the least estimate of the range of plastic zone (approximately upper half) for this case.

(2) the plastic zone of a pile occurs at the pile head under a restrained-head condition

For this case, the free-body diagram of the plastic zone at the pile head is shown in Fig. 3(b), where the upper end is the restrained head. For a fixed-head pile, the maximum moment and shear force occur at the pile head. Thus, the upper end of the plastic zone (pile head) is subjected to the ultimate moment M_u and the corresponding ultimate lateral force H, and the lower end of the plastic zone is subjected to the yield moment

M_y. The plastic zone is assumed to be subjected to a uniform soil pressure p. From equilibrium of horizontal forces, the shear force at the lower end is H- $p{\cdot}L_p$.
The equilibrium of moments about point B gives

$$M_u - M_y - H \cdot L_p + \frac{1}{2} p \cdot L_p{}^2 = 0 \tag{4.4}$$

Rearranging Eqn. (4.4), the range of plastic zone L_p can be represented as

$$L_p = \frac{M_u - M_y}{H - \frac{1}{2} p \cdot L_p} \tag{4.5}$$

If p is neglected in Eqn. (4.5), we have

$$L_p > \frac{M_u - M_y}{H} \tag{4.6}$$

The shear force H in Eqn. (4.6) can be further estimated by

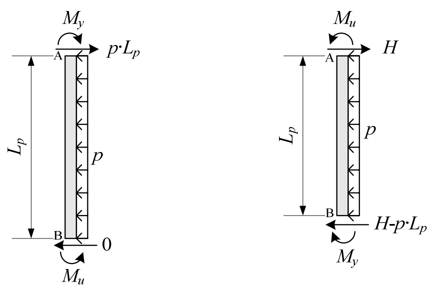
$$H = M_u \cdot (2\beta) \tag{4.7}$$

where $\beta = \sqrt[4]{\frac{E_s}{4EI}}$ is the characteristic coefficient of the pile-soil system, in which EI is the flexural rigidity of pile and E_s is the subgrade reaction modulus.
Equation (4.7) is based on the relation between the pile-head moment and the pile-head shear force for a fixed-head semi-infinite pile embedded in uniform soils (Chang, 1937).

Substituting (4.7) into (4.6) yields

$$L_p > \frac{M_u - M_y}{2\beta \cdot M_u} \tag{4.8}$$

Equation (4.8) gives the least estimate of the range of plastic zone for a fixed-head pile. The range of plastic zone increases with the difference between M_u and M_y, but decreases as β increases. When the soils enter nonlinear response, β tends to decrease due to smaller subgrade reaction modulus. Thus, choosing β corresponding to the linear state of the pile-soil system and substituting into Eqn. (4.8), we can get a smaller estimate of the range of plastic zone for this case.



(a) Plastic zone occurs at a certain depth below the ground surface

(b) Plastic zone occurs at the pile head

Figure 3 Free-body diagrams of plastic zones

Once the range of L_p is estimated, the plastic hinge length l_p can then be set to be a fraction of L_p for distributed plastic hinges. A conservative (smaller) plastic hinge length l_p can be determined based on the least range of L_p.

5. NONLINEAR ANALYSIS FOR A PILE-SOIL SYSTEM

This section employs a numerical model of a pile-soil system to investigate the effectiveness of the proposed modified plastic curvature and arrangement of distributed hinges for the pushover analysis of a pile-soil system.

A pile of length 30 m, fully embedded in ground is selected as shown in Fig. 4.

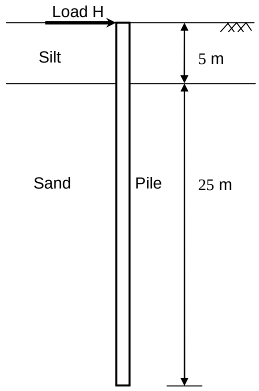


Figure 4 Example problem of a laterally loaded pile

The moment-curvature relation of the pile section is assumed to be bilinear (Fig. 5) and the ground is assumed to have two uniform layers, in which a 5 m-thick silt layer is underlain by a sand layer. The *p-y* curves of these two soil layers are elastic perfectly-plastic, as shown in Fig. 6.

Two programs, ABAQUS (1995) and SAP2000 (2002) are used to analyze this problem, respectively. The ABAQUS analysis will use the nonlinear beam elements to model the nonlinear flexure of the pile, but the SAP analysis will use the plastic hinge model.

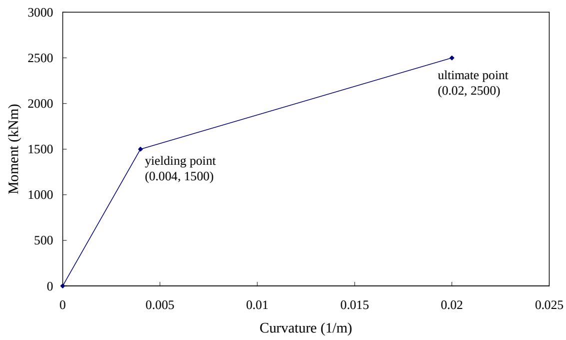


Figure 5 Moment-curvature curves of the pile

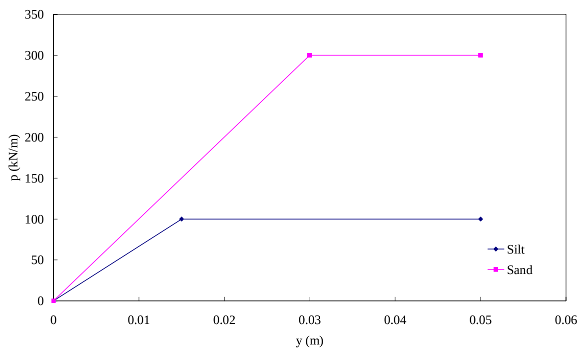
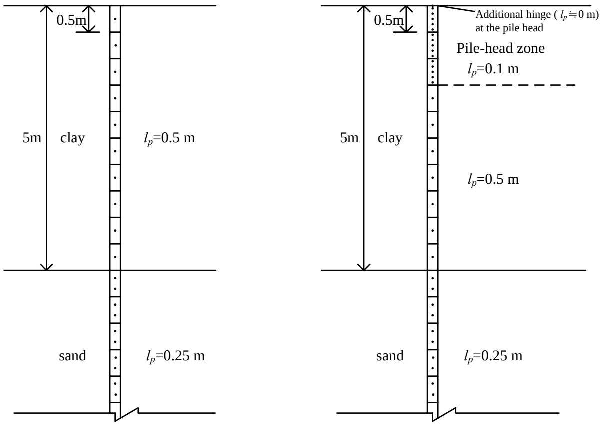


Figure 6 *p-y* curves of soil layers

Two different pile-head conditions will be investigated hereafter: (1) free-head condition and (2) fixed-head

condition.
In the SAP analysis, the distributed plastic hinge model is adopted to model the nonlinear flexure of the pile. Both the ATC method and the modified plastic curvature are used to compute the rotational property of the plastic hinges. The arrangement of plastic hinges for this case is shown in Fig. 7.
Figures 8 and 9 are the pile-head capacity curves for free- and fixed- head conditions, respectively. The capacity curves obtained from the modified plastic curvature and plastic hinge arrangement are in good agreement with those from ABAQUS analysis. On the other hand, the ATC method overestimates the pile-head displacements when the pile yields.



(a) Free-head condition (b) Fixed-head condition

Figure 7 Arrangement of distributed plastic hinges in SAP analysis

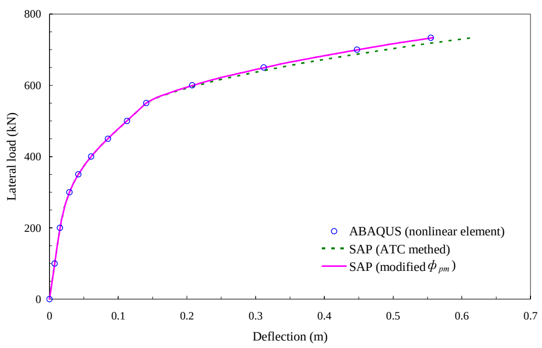


Figure 8 Capacity curves for free-head condition

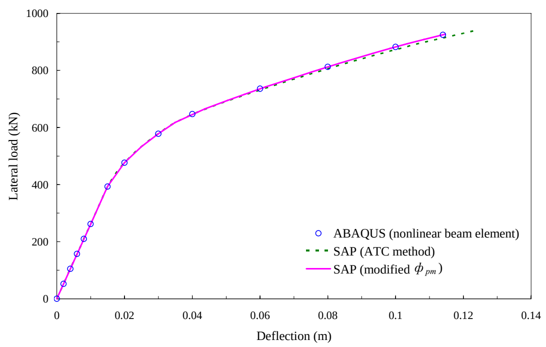


Figure 9 Capacity curves for fixed-head condition

6.CONCLUSIONS

When using the concept of plastic hinge to model the nonlinear flexural deformation of a pile, the model of distributed plastic hinges must be used to trace the development of plastic zone in the pile. The illustrative examples presented herein demonstrate clearly the effectiveness of the proposed modified plastic curvature and the arrangement of distributed plastic hinges in modeling a pile-soil system.

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