FLOW ANALYSIS OF WATER SUPPLY NETWORKS POST-EARTHQUAKE

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ABSTRACT :

In this paper, the flow analysis of water supply networks post-earthquake is performed based on the reliability analysis of pipelines. The limit state function of a buried pipeline (bell and spigot joint) is given according to the wave propagation theory. The key variables of the limit state function are assumed to be normally distributed and uncorrelated or non-normally distributed and correlated. For these two cases, The Ditlevsen or the H-L formula may be used to compute the reliability index of pipelines. It is very convenient that the Ditlevsen formula is adopted to calculate reliability index of pipelines by the optimization tool of the spreadsheet software (Excel 2000). If the H-L formula is used the arithmetic of partial derivatives, equivalent normally and linear transformation will involved on the user' part. It usually is very difficult to do it. But in this paper the MATLAB program library (Symbolic Math Toolbox) is used to find partial derivatives of the limit state function of buried pipelines and other functions so that the computation of the reliability index of a pipeline can become very simple. The flow analysis of damaged water supply networks is made accounting for the leakage in water systems. The point leakage model is adopted to simulate the leakage, and the anti-sin function of losing probability of pipelines is proposed to express the relation of the leakage area and the losing probability of pipelines.

KEYWORDS: Limit state function, Reliability, Flow, H-L method, Spreadsheet

1. INTRODUCTION

Seismic events in the past show that the failure of water supply networks not only influence on the fire fighting capability, but also may disrupt daily life of inhabitants and other social activities, resulting tremendous economic losses. Therefore, it is very important issue to evaluate the seismic performance of water supply networks in an urban area. It is usually necessary that its connectivity and flow analysis are performed based on estimating the likely earthquake damage to the buried pipelines. In this paper, the limit state function of a buried pipeline (bell and spigot joint) is given according to the wave propagation theory. Two kinds of method for computing reliability index of buried pipelines are presented. The first method is that the Ditlevsen formula is used to calculate reliability index of pipelines by the optimization tool of the spreadsheet software (Fu 2004), for second one the H-L formula is used ,but the reliability index of pipelines is carried out by the MATLAB -Symbolic Math Toolbox (Fu 2007a, b) . Here the former is called as the spreadsheet method, the latter is the H-L method.

For the spreadsheet method, the uncertainty and correlation of various variables in the limit state function may be accounted in the original coordinate. Usually the variables are assumed as normally distributed. If variables are non-normally distributed it needs to do equivalent normally transformation. It is very convenient to calculate reliability index of pipelines in a small network using the spreadsheet method, but the procedure is not appropriate for evaluating the seismic performance of a huge water delivery system.

Therefore, in this paper the H-L formula for calculating the reliability index in structural system is Therefore, in this paper the H-L formula for calculating the reliability index in structural system is proposed to compute reliability index of buried pipelines of a huge water delivery system.

In this procedure, the iterative formula for calculating reliability index is obtained from the Lagrange multiplier, the arithmetic of partial derivatives of the limit state function , equivalent normally and linear transformation will involved on the user' part. It usually is very difficult to do it. But in this paper the MATLAB program

library (Symbolic Math Toolbox) is used to find partial derivatives of the limit state function of buried pipelines and other functions so that the computation of the reliability index of a pipeline can become very easy.
The flow analysis of water supply systems can be carried out after the reliability index and losing probability of each pipeline in the water supply system have been given. To carry out flow analysis of damaged water supply networks, in this paper, the leakage of pipelines is simulated by the point leakage model, and the anti-sin function of losing probability of pipelines is proposed to express the relation of the leakage area and the losing probability of pipelines (Fu 2007a, b).
The special software program which is used to estimate the losing probability of buried pipelines and the flow of water supply networks was developed. And this program was coupled to the GIS system for evaluating the seismic performance of water supply networks in an urban area.

2. RELIABILITY ANALYSIS OF BURIED PIPELINES

In this chapter, two kinds of method for computing reliability index of buried pipelines during an earthquake are described.

2.1 Seismic Limit State Function

The limit state function of a buried pipeline (bell and spigot joint) during an earthquake is given according to the formula specified in the Chinese National Standard (Seismic 2003):

$$0.64\sum_{i=1}^{J}[u_a]_i - \Delta_{plk} = 0 \tag{2.1}$$

Where $[u_a]$= allowable displacement value at the joint of a buried pipeline; Δ_{plk} = the axial displacement of a buried pipeline in a half wave length due to earthquake shaking,

$$\Delta_{plk} = \sqrt{2}\varsigma U_{0k} \tag{2.2}$$

where

$$U_{ok} = \frac{k_H g T_g^2}{4\pi^2} \qquad \zeta = \frac{1}{1 + \dfrac{EA}{K}\left(\dfrac{2\pi}{L^{'}}\right)^2}$$

$$A = \pi(D+t)t \qquad K = \pi(D+2t)k_1$$

J = joint node numbers in a half wave length

$$J = L^{'}/(2l) = 0.7\sqrt{2}V_s T_g/(2l)$$

where k_H = the acceleration factor of ground shaking; T_g = the period of a site; V_s = the shear wave velocity of the soil layer; E = elastic modulus of a pipe; D = the diameter of a buried pipe; t = the thickness of a buried pipe wall; k_1 = resistance factor of soils; l = the length of a pipe. In the reliability analysis of pipelines, k_H , T_g ,V_s , D and $[u_a]$ are assumed as random variables , their coefficients of variation (in following example) are respectively assumed as 0.2, 0.10, 0.1, 0.05 and 0.10, while others are assumed as deterministic variables.

2.2 Spreadsheet Method

The reliability index β of a system is expressed using Ditlevsen's formula (Low 1996) as flow

$$\beta = \min_{X \in F}\sqrt{(X-m)^T C_0^{-1}(X-m)} \tag{2.3}$$

Where X = the random variable vector which is assumed as normally distributed; m = the median value of variables; C_0=the covariance matrix; F= the failure surface which is expressed by the limit state function above. The reliability index may be computed by the optimization tool of the spreadsheet software (Low 1996, Fu 2004).Then the losing probability P_f of pipelines is expressed by the standard cumulative normal distribution function.

$$P_f = \phi(-\beta) \tag{2.4}$$

2.3 H-L Method
For evaluating the seismic performance of a huge water supply system the H-L method is described in this chapter.
2.3.1 Case I variables are the normally distributed and uncorrelated
The reliability index and the limit state function in n dimensional spaces are expressed by using following equations (Dong C. 2003)

$$\beta = \min \sqrt{\sum_{i=1}^{n} z_i^2} \qquad g(\overline{Z}) = 0 \tag{2.5}$$

$$\bar{z} = (z_1, z_2, \cdots, z_n), \qquad z_i = \frac{x_i - m_i}{\sigma_i}$$

Where x_i, m_i, and σ_i are respectively random variables (corresponding to k_H, Tg, D, $[u_a]$ *and* Vs), median value and standard deviation. The iterative formula for calculating reliability index is obtained from the Lagrange multiplier

$$\bar{z} = -\frac{\beta G}{\sqrt{GG^T}} \tag{2.6}$$

$$G = \left(\frac{\partial g}{\partial z_1}, \frac{\partial g}{\partial z_2}, \cdots, \frac{\partial g}{\partial z_n} \right) \tag{2.7}$$

$$g(\beta) = 0 \tag{2.8}$$

This is a nonlinear equation whose solution may be given by the secant method. The arithmetic of partial derivatives of the limit state function will be first involved. It usually is very difficult to do it. But it could become much simpler if the MATLAB program library (Symbolic Math Toolbox) is used to find partial derivatives of the limit state function and other functions.
2.3.2 Case II variables are non-normally distributed and correlated
When variables are non-normally distributed and correlated it is necessary to do equivalent normally and linear transformation. Then the reliability index of buried pipelines can be calculated by the H-L procedure
It is assumed that the equivalent normally transformation of n dimensional victories have been carried out. Its the covariance matrix is expressed by the following formula

$$C_x = \begin{bmatrix} \sigma_{x_1}^2 & Cov(x_1,x_2) & Cov(x_1,x_3) & \cdots & Cov(x_1,x_n) \\ Cov(x_2,x_1) & \sigma_{x_2}^2 & Cov(x_2,x_3) & \cdots & Cov(x_2,x_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Cov(x_n,x_1) & Cov(x_n,x_2) & Cov(x_n,x_3) & \cdots & \sigma_{x_n}^2 \end{bmatrix} \tag{2.9}$$

For normalizing we now let

$$Y^* = D_x^{-1}(X - E(X)) \tag{2.10}$$

$$D_x = diag(\sigma_{x_1}, \sigma_{x_2}, \cdots, \sigma_{x_n}) \tag{2.11}$$

Then the covariance matrix is

$$C_{y^*} = \begin{bmatrix} 1 & \rho_{x_1,x_2} & \rho_{x_1,x_3} & \cdots & \rho_{x_1,x_n} \\ \rho_{x_2,x_1} & 1 & \rho_{x_2,x_3} & \cdots & \rho_{x_2,x_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho_{x_n,x_1} & \rho_{x_n,x_2} & \rho_{x_n,x_3} & \cdots & 1 \end{bmatrix} = \rho_x \tag{2.12}$$

The linear transformation is performed accounting the following formula.

$$Y = B \cdot Y^* \tag{2.13}$$

Where B = the orthogonal matrix whose column vectors are obtained from the eigenvectors of the matrix ρ_x. The diagonal matrix C_y is expressed by following formula

$$C_y = B \cdot \rho_x \cdot (B)^T = diag(\sigma_{Y_1}^2, \sigma_{Y_2}^2, \cdots, \sigma_{Y_n}^2) \tag{2.14}$$

Median values of Y

$$E(Y) = B \cdot E(Y^*) \tag{2.15}$$

For normalizing

$$Z = C_y^{-1/2} \cdot (Y - E(Y)) \tag{2.16}$$

The following equations are obtained from the above formulas

$$Z = [B \cdot \rho_x \cdot (B)^T]^{-1/2} \cdot B \cdot D_x^{-1} \cdot (X - E(X)) \tag{2.17}$$

$$X = E(X) + D_x \cdot (B)^T \cdot [B \cdot \rho_x \cdot (B)^T]^{1/2} Z \tag{2.18}$$

Then the limit state function may be written in the form

$$g(Z) = 0 \tag{2.19}$$

So far the reliability index can be calculated using H-L method (Dong C. 2003). But the MATLAB program library (Symbolic Math Toolbox) is first used to carry out the mathematical analysis above.

2.4 Example

For the non-normally distributed and correlated variable case an example is given as follow.

The buried pipelines are made of cast iron pipes with diameters 60 cm. The ground acceleration is assumed as the distribution of extreme value, variables T_g, Vs, D and $[u_a]$ are respectively normal distribution, the median values of random variables are respectively k_H=0.2, T_g=0.45s, Vs=100m/s, D=60cm and $[u_a]$ = 0.058cm, their coefficients of variation are respectively assumed as 0.2, 0.10, 0.1, 0.05 and 0.10, while other variables are deterministic, their median values are E=11×10^{5}N/mm^2, k_l=0.06N/mm^3, t=1.54cm, l=500cm. The correlation coefficient of between D and $[u_a]$ is 0.4, one between Tg and Vs is 0.6. The fragility curves of the pipeline obtained from both the spreadsheet and the H-L methods are compared in Figure 1. It can be seen from the figure that both results are fairly consistent.

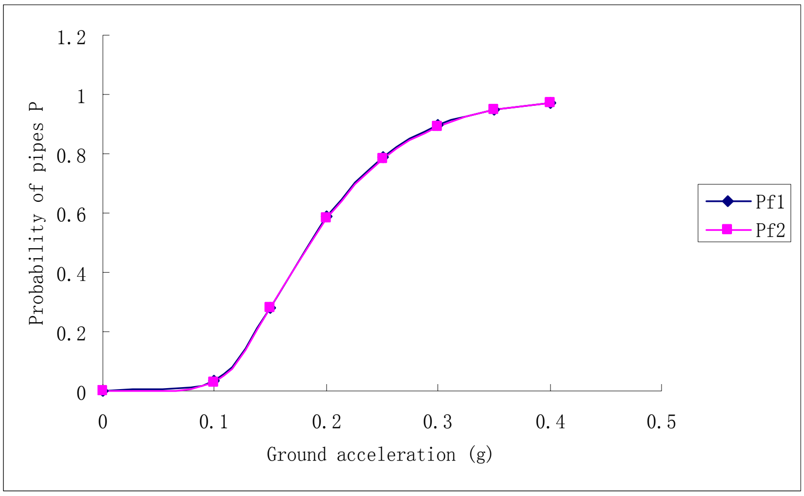

Figure 1 Comparison of fragility curves from both methods P_{f1}- spreadsheet method, P_{f2}- H-L method

3. FLOW ANALYSIS OF WATER SUPPLY NETWORKS

3.1 Flow analysis

Since the input flows at the given demand node equal to the output flows at the same node the following equation is obtained (Chen 2004)

$$Aq + Q + Q_L = 0 \tag{3.1}$$

Where **q** = the flow vector in a pipeline; **Q** = the flow vector at a node; $\mathbf{Q_L}$ = the leakage flow vector; **A**=[a_{ij}] is the link matrix which is expressed as

$$a_{ij} = \begin{cases} 0 & \text{j pipe does not link to i node} \\ 1 & \text{i node is the begin node of a pipeline} \\ -1 & \text{i node is the end node of a pipeline} \end{cases}$$

The flows of the pipe i-j are expressed by Hazen-Williams formula

$$q = 0.278 \cdot C \cdot D^{2.63} \cdot (H_j - H_i)^{0.54} / L^{0.54} \tag{3.2}$$

Where q = the flows of a pipeline (m^3/s); D = the diameter of a pipe (m); Hi = water heads at i node (m); L = the length of a pipelines (m); C = Hazen-Williams coefficient.

Let $$q = r \cdot h \tag{3.3}$$

Where $$h = H_j - H_i$$

$$r = \frac{0.278 \cdot C \cdot D^{2.63}}{L^{0.54} \cdot (H_j - H_i)^{0.46}} \tag{3.4}$$

Because $$\mathbf{h} = \mathbf{A}^{\mathbf{T}} \cdot \mathbf{H} \tag{3.5}$$

If (3.5) is substituted into (3.3), and then the result from (3.1) and (3.3) is

$$\mathbf{ARA^TH} + + \mathbf{Q} + \mathbf{Q_L} = \mathbf{0} \qquad R = \begin{bmatrix} r_1 & & 0 \\ & \ddots & \\ 0 & & r_m \end{bmatrix}$$

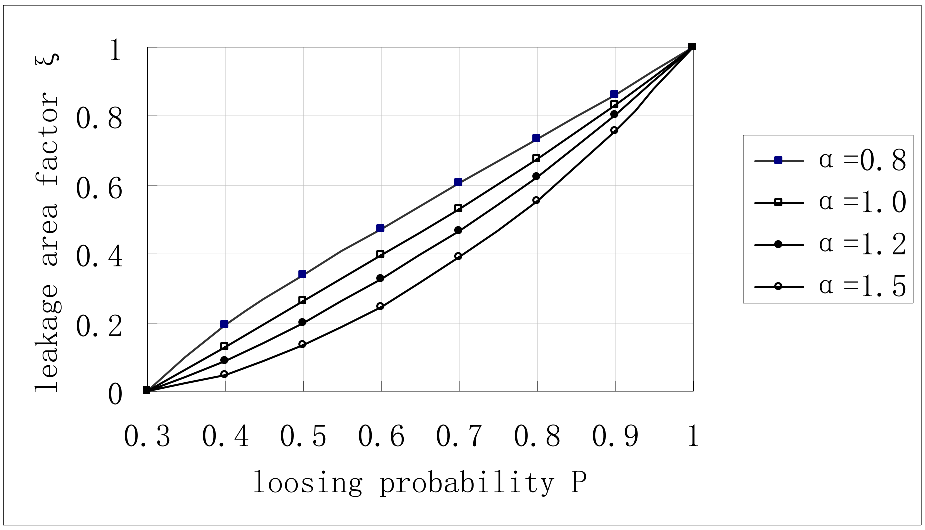

Figure 2 Relation between leakage area factor and loosing probability

To calculate leakage flows of a pipeline the formula specified by Chinese City and Town Water Supply Association (Chen 2004) is the following

$$Q_L = 0.421 A_L \sqrt{H_L} \tag{3.6}$$

Where H_L = the head at leakage point (m); Q_L = the leakage flows (m^3/s); A_L = the leakage area (m^2). In this paper, the authors propose the following expressions to calculate the leakage area A_L

$$A_L = \xi A_0, \qquad \xi = \frac{\beta}{\pi}[\arcsin(P - P_0)]^{\alpha} \tag{3.7}$$

Where ξ=leakage area factor; A_0 = the sectional area of a pipeline; P = the losing probability of a pipeline; P_0 =the initial probability. Let $P_0=0.3$. The values of α and β should satisfy the following equations

$$P \le 0.3, \ \xi = 0$$

$$P = 1.0, \ \xi = 1.0$$

When α=0.8, 1.0, 1.2 and 1.5 the curves of ξ are shown in Figure 2. In this paper, the leakage points are assumed to local at the middle of a pipeline, and the flow analysis is performed based on the method above (Chen 2004,).

3.2 Example

The seismic performance of the water supply network whose database comes from the reference (Zhao 2003) was analyzed using the software program (based on above procedures). The node numbers, pipeline numbers, flows at demand nodes, input flows and the water heads at input nodes are illustrated in Figure 3. The lengths and diameters of various pipelines are shown in Table 3.1. For the example, it is assumed that various pipes are joined by the bell and spigot joint, and the random variables of the limit state function are assumed as normally distributed and uncorrelated. The some analytic results are given as follow

The flows in a pipeline and water heads at various nodes may be obtained from the follow analysis. The water heads at various nodes in the water supply network are reduced with increases in peak ground accelerations as shown Figure 4.

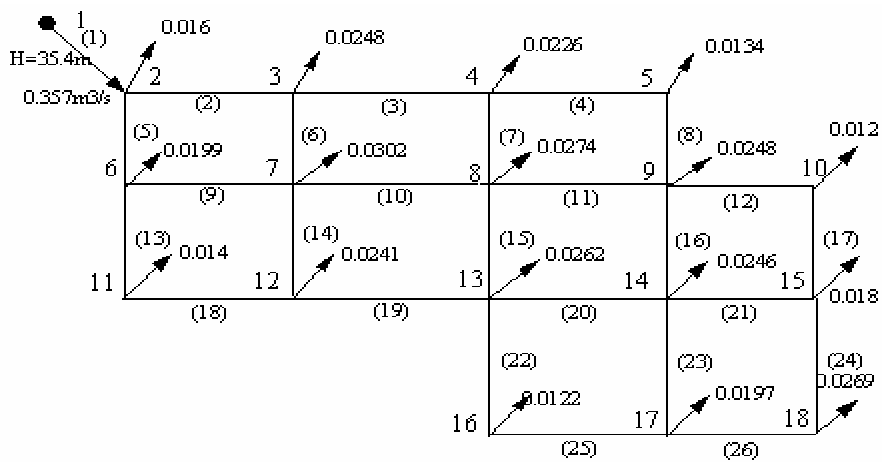

Figure 3 A water supply network

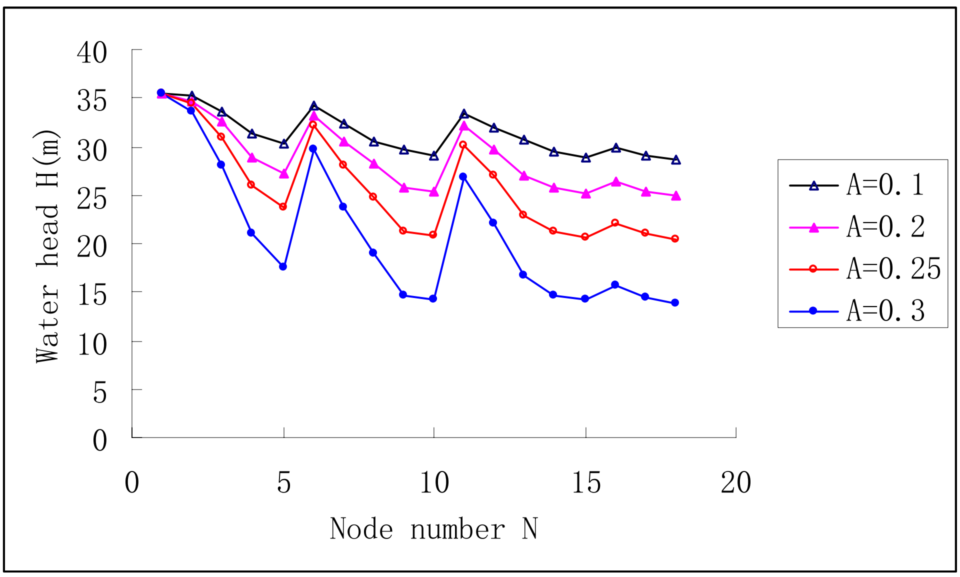

Figure 4 Water heads change with the peak ground accelerations

Table 3.1 Lengths and diameters of various pipelines

No	Length of pipe(m)	Diameter of pipe(m)
1	1000	1.00
2	900	0.50
3	800	0.35
4	700	0.25
5	600	0.50
6	600	0.30
7	600	0.35
8	550	0.20
9	900	0.35
10	800	0.35
11	650	0.30
12	600	0.25
13	350	0.35
14	500	0.30
15	500	0.30
16	500	0.30
17	520	0.20
18	950	0.35
19	800	0.35
20	650	0.3
21	650	0.25
22	480	0.30
23	480	0.20
24	500	0.25
25	650	0.25
26	700	0.25

4. CONCLUSIONS

The flow analysis of water supply networks post-earthquake is performed based on the reliability analysis of buried pipelines. The spreadsheet method and the H-L' method for computing the reliability index of pipelines are presented. The flow analysis of damaged water supply networks is made accounting for the leakage in water systems. The anti-sin function of losing probability of pipelines is proposed to express the relation of the leakage area and the losing probability of pipelines. According to above discussions it can be concluded that this methodology is relatively rational. Since the evaluation of the seismic performance of a water supply network in an urban is a complicated issue, the further improvement of the methodology is needed by means of its application to practical water supply networks.

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