UNCERTAINTY IN ANALYTICAL STRUCTURAL RESPONSE ASSOCIATED WITH HIGH LEVEL MODELING DECISIONS

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ABSTRACT :

Current performance based design approach requires an improved understanding of the behavior of structural components which can be realized through nonlinear analysis of structures and structural components. A lot of research exists in characterizing the epistemic uncertainty associated with demand and capacity of a structure or a structural component. However very few researches exist on characterizing the epistemic uncertainty associated with high level analytical modeling decisions for simulation of a structure. In this research, epistemic uncertainty in structural response of reinforced concrete structural system has been investigated through a nonlinear pushover analysis of a reinforced concrete column. While performing nonlinear analysis of the structural component with a particular analytical model, the analyst is required to make certain high level decisions such as with respect to element type, integration rules and material models, to obtain the response of a structural component subjected to a certain kind of loading. The research highlights the loss of objectivity in response associated with high level modeling decisions by an analyst.

KEYWORDS: Nonlinear finite element, component level modeling, uncertainty analysis, fiber beam column element, RC columns.

1. INTRODUCTION

Performance-based-design guidelines of a structure by federal emergency management agency (FEMA) require structures to be analyzed by using nonlinear static push-over analyses and/or nonlinear dynamic analyses to control global and local demands of the structure. Performance based design and analysis requires robust, computationally efficient structural component models for performing analyses in a reasonable amount of time. Thereby, typically for performance based design and analysis, component based macroscopic finite element models are utilized which depends upon formulation of super-elements for structural components.

A lot of research has been performed to determine and characterize the uncertainty associated with demand and capacity of a structure and/or structural component through use of component based finite-element models. In this kind of epistemic uncertainty estimation, a suitable analytical model is chosen based on some high level modeling decisions and then variations are imposed with respect to the demand and capacity of the structure. However, it should be realized that an inherent uncertainty lies in choosing the analytical model and its associated high level modeling decisions for the analysis. There exists very few, if any, research on this type of uncertainty estimation and characterization. This epistemic uncertainty is associated with high level modeling decisions by an analyst and has been explored in details in this research. These high level modeling decisions typically depends on the element and material type that are being used for the investigation. The objectives of the paper will be to make a practicing engineer, using a component based analytical model, aware of uncertainties in structural response associated with high level modeling decisions such as with respect to element type, integration rules and material models.

2. MODELING OF REINFORCED CONCRETE COLUMN COMPONENT

Reinforced concrete columns are integral part of any building or bridge structure which is subjected to axial and bending forces. Typically frame elements are utilized for modeling of reinforced concrete columns. The frame elements typically utilize Euler-Bernoulli beam-column type of element formulation which is able to consider the effect of bending as well as axial loading. Both displacement-based (Hughes 1987) as well as force-based formulation (Spacone et al. 1996) can be utilized to develop elements for the beam-column region. The advantages of force-based element formulation over displacement-based element formulation have been discussed in detail in Neuenhofer and Filippou (1997). The primary advantage can be briefed as follows: In the displacement-based formulation, several elements are required per member to obtain good approximation of response since the displacement fields along the element are expressed as functions of nodal displacements to approximate the actual displacement in the member. In force-based formulation internal force fields are expressed as functions of nodal forces to approximate the actual force distribution in the member, which is a known quantity. Thereby, only one element per member is required for force-based formulation leading to considerable savings in total number of degrees of freedom in the structural model. Thereby, in this research force based fiber beam-column elements (Spacone et al. 1996, Neuenhofer and Filippou 1997) have been utilized for modeling of these structures and/or structural components. Modeling of the structure is accomplished using OpenSees (McKenna et al. 2005), an object oriented, open-source software framework for finite element analysis platform which is used as a simulation component of network for earthquake engineering simulation (NEES) and is constantly under development by researchers at pacific earthquake engineering research center (PEER).

The following research demonstrates the effect of uncertainties associated with high level modeling decisions in simulation of a 10 feet column, fixed at one end and free on the other end subjected to axial and lateral loading, with typical cross-section as shown in figure 1. The material properties used in the simulation are 4 ksi for unconfined compressive strength of concrete, 60 ksi for yield strength of longitudinal reinforcing steel, 40 ksi as yield strength of hoop steel; a strain hardening ratio of 1% and an ultimate strength of 1.5 times the yield strength of steel. The axial load applied to the system is 15% of $f_c A_g$ (product of concrete compressive strength and gross cross sectional column area). For the prototype model *Concrete04* material without tensile response was utilized for concrete, *Reinforcing Steel* material was utilized for the reinforcing steel with a minimum and maximum cut-off strain. Euler-Bernoulli beam theory was considered for the beam-column elements. The uncertainty in response obtained from using different type of integration rules along the element length, different types of material models to represent the uniaxial compressive response of concrete, residual strength in compression and tensile strength are presented in this research.

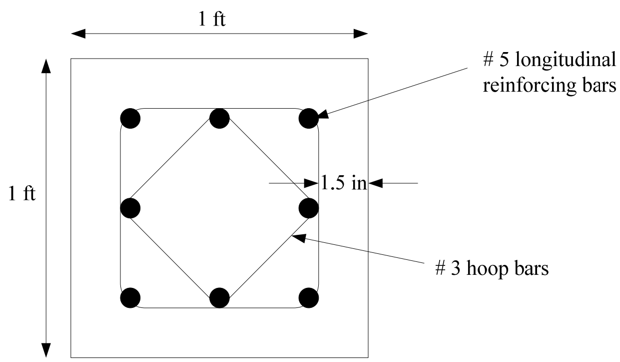


Figure 1. Reinforced concrete column cross section

3. UNCERTAINTY OF RESPONSE WITH INTEGRATION RULES ALONG THE ELEMENT LENGTH

Nonlinearity in material response for a beam-column element can either be introduced throughout the length of the element or as a localized zone of nonlinear response referred to as the plastic hinge region within the element. For nonlinearity throughout the length of the element, a weighted summation of response at a select number of integration points throughout the length of the element results in the element response. This type of formulation would be hereby referred to as NonLinearBeamColumn or distributed plasticity element. For localized region of nonlinearity, a weighted summation of response of a select number of integration points within the nonlinear plastic hinge region and the linear elastic region defines the element response. The localized nonlinear region elements are hereby referred to as BeamWithHinges or localized plasticity elements. Both the two different methodologies are evaluated and compared in this research.

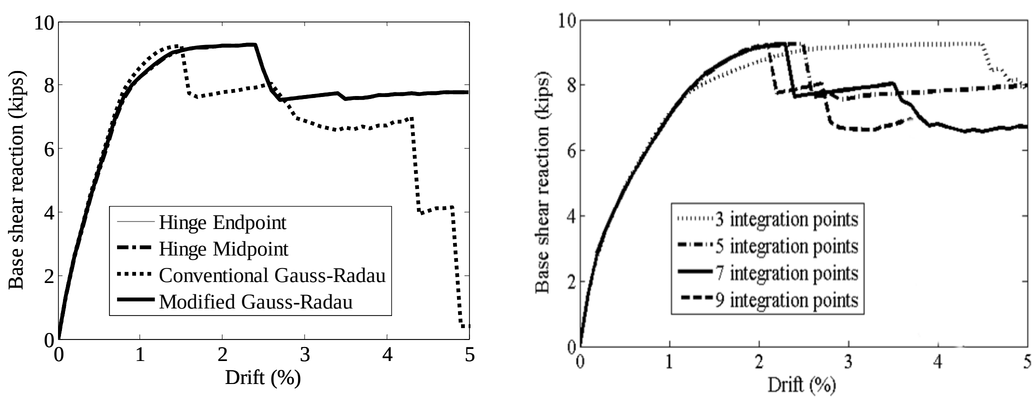
The plastic hinge length considered for the localized plasticity elements is equal to the length specified by Paulay and Priestley (1992) for reinforced concrete members. For the current application, the plastic hinge length amounts to 8% of the total length of the column. It should also be noted that plastic hinge length was applied at the fixed end of the column and no plastic hinge was provided at the free end. Four different integration rules, available in OpenSees, were considered for the analysis (for more detailed explanation on the different types of integration rules, the readers are referred to Scott and Fenves 2006):

- *Hinge End-point* – The integration points are located at the element ends while the integration weights are taken as the plastic hinge length.
- *Hinge Mid-point* – The integration points are considered at the midpoints of the plastic hinge region with integration weights as the plastic hinge length of the member.
- *Conventional Gauss-Radau* (Hildebrand 1974) – This method involves two-point integration within each of the plastic hinge regions. The integration points are at the ends and 2/3rd of the plastic hinge lengths from the end with integration weights ¼ and ¾ of the plastic hinge length respectively.
- *Modified Gauss Radau* – In this method, the Conventional Gauss-Radau integration is applied over lengths of 4 times the plastic hinge lengths at each of the element ends. The integration points are thereby at the ends of the plastic hinge length and 8/3rd of the plastic hinge length from the end, which happens to be in the elastic region, with integration weights as 1 and 3 times the plastic hinge length respectively.

Figure 2a) shows the variation of system response for a reinforced concrete column subjected to an axial and pushover loading. It can be observed that the global response as obtained for the Conventional Gauss Radau exhibits very different response in comparison to the other methods.

It should be pointed out that the section force, such as the bending moment, is largest at the element ends in

absence of member loads. Thereby, based on this criterion the Hinge Midpoint scheme should not be used for analysis of typical frame structures since the scheme might result in larger flexural capacity than expected, which in fact would be a function of the ratio of the plastic hinge length to the total member length. In this particular case study of the RC column, Hinge Midpoint doesn't exhibit different response since at the middle point of the hinge (which is small and equal to only 8% of the length of column) the curvature response is not significantly different from the end. The reader should be reminded that for both the Hinge Endpoint and the midpoint schemes, a one-point integration is carried out for the plastic hinge region and thereby an error is possible in the representation of linear curvature distribution. Since the integration points for the Hinge End Point and the Modified Gauss Radau are located at the element ends and the integration weights are considered equal to the hinge length, the response obtained from these two methods would be similar if the slope of the linear curvature distribution in the element is very small. Both the Gauss Radau schemes utilizes two integration points and represents well the linear curvature distribution; but for the conventional Gauss-Radau scheme, the integration weights for the end point is quarter of the plastic hinge length and this reduction would cause the element to unload at a faster rate than expected to maintain equilibrium, which is observed in figure 2a. It should be mentioned that a combination of the integration points and weights associated with these points determines the variations in the response.
A detailed discussion of these integration schemes can be obtained in Scott and Fenves (2006). However, it should be noted that the primary objective of the Scott and Fenves (2006) was to develop an objective plastic hinge length and corresponding integration rule for the lumped plasticity element for use in a reinforced concrete column. The objective in this paper is different, since it highlights the differences in response obtained by using different integration rules with a plastic hinge length prescribed by Paulay and Priestley (1992) for lumped plasticity elements for the case of a reinforced concrete column. There are also other prescribed hinge lengths in literature e.g. Coleman and Spacone (2001), which is based on fracture energy concepts. However, the trends in the behavior would remain the same irrespective of which hinge length being used and modified Gauss-Radau integration scheme would result in the most accurate representation of linear curvature distribution within the element. In this paper, only the Paulay and Priestley (1992) hinge length is shown since this hinge length is one of the most commonly used hinge lengths utilized by structural engineers for modeling. The results obtained from figure 2a support Scott and Fenves (2006) conclusion that modified Gauss-Radau scheme provides better approximation compared to the other integration schemes.



a) Integration rules in lumped plasticity element formulation

b) Integration points in distributed plasticity element formulation

Figure 2. Variation in pushover response with integration rules along element length

Nonlinear beam-column elements or distributed-plasticity elements represent a generalization of the BeamWithHinges or localized plasticity element since the nonlinearity in these elements is spread out over the entire element rather than being confined within the plastic hinge region. However, differences in global system response are observed with respect to the peak strength and the post-peak response when using different number of integration points within the element (figure 2b). The location and weights for the integration points are based on the Gauss-lobatto rule. The series of investigation reinforces the conclusions of Coleman and

Spacone (2001) that localization occurs in the integration points for a force-based element formulation and can be eliminated by using more number of integration points. However, it should be pointed out that objective in Coleman and Spacone paper (2001) was to resolve the localization issue by introduction of a plastic hinge length. In this paper, the differences in system response are highlighted with different number of integration points and it is being concluded that more the number of integration points used in an element, the closer the approximation of the actual response can be obtained. It should also be mentioned here that since an objective and robust performance based analysis requires less amount of computational time, 7 integration points are recommended for nonlinear analysis using force based distributed plasticity fiber elements to maintain a balance with accuracy as well as computational time.
Figure 3 identifies the difference in system response observed when the structural components are modeled with lumped plasticity elements (plastic hinge length as specified by Paulay and Priestley 1992) compared to when modeled with distributed plasticity elements with 7 integration points along the member length. It should be noted that there is significant difference in the initial stiffness region of the system response. Figure 3 indicates that the system response obtained from lumped plasticity elements have stiffer initial stiffness compared to distributed plasticity elements. The reason for this is because of 1) the integration weights (being equal to the plastic hinge length) associated with the response as obtained at the end point of the plastic hinge in a localized plasticity element compared to the distributed plasticity element, and 2) the response of the elastic region considered for the localized plasticity element in between the plastic hinge lengths.

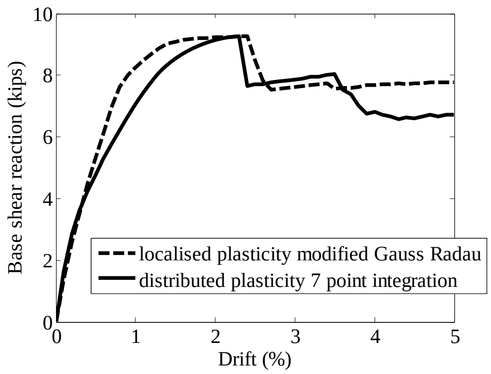


Figure 3. Comparison between RC column pushover response for lumped and distributed plasticity element formulation

Thereby, from these above set of figures it is being demonstrated that the response of a system modeled using a force-based formulation is sensitive to the number, location and weightage of the integration points in determination of the response of the element. The study reiterates that the localization of response prevalent in displacement based formulation (Bazant and Oh 1983, Bazant and Planas 1998, de Borst et al. 1994) is also of concern for force based formulation and occurs at the integration points within the element (Coleman and Spacone 2001, Scott and Fenves 2006). Thereby the analyst should be careful while determination of the plastic hinge length of the structural component while modeling with force based formulation using localized plasticity formulation. It is being recommended that modified gauss radau integration scheme works better for force based fiber beam column localized plasticity element. If the analyst is using distributed plasticity element, then it is recommended that the number of integration points used along the member length should be 7 to obtain an objective post-peak response.

4. UNCERTAINTY OF RESPONSE WITH RESPECT TO MATERIAL MODEL SELECTION

Different types of material model are available to represent the uniaxial compressive behavior of concrete. The choice of different types of material models to represent the material in the structure can have significant effect on the system response. Thereby the modeler or the analyst should be judicious while choosing the material for his/her analysis. In order to justify the statement a pushover analysis of commonly used reinforced concrete structural components such as a column is carried out with the same material properties but with different material models in OpenSees environment. The variations in material models for concrete utilized for analysis are:

- Concrete01 – uniaxial Kent and Park (1971) material model with degraded linear unloading/reloading stiffness according to the work of Karsan and Jirsa (1969) and with no tensile strength.
- Concrete04 – uniaxial Popovics (1973) material model with degraded linear unloading/reloading stiffness according to the work of Karsan and Jirsa (1969) and an optional tensile strength with exponential decay.

The envelope of the stress-strain curve proposed by Kent and Park (1971) model is represented in Eq. 4.1 as

$$f_{ci} = f'_c\left(2\frac{\varepsilon_{ci}}{\varepsilon_c} - \left(\frac{\varepsilon_{ci}}{\varepsilon_c}\right)^2\right) \qquad (4.1)$$

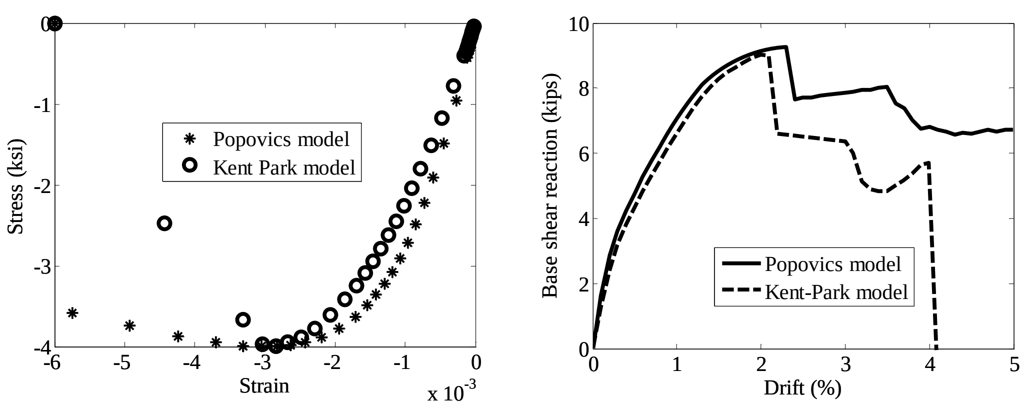
whereas the envelope of the stress-strain curve proposed by Popovics (1973) model is represented in Eq. 4.2 as

$$f_{ci} = f'_c\left(\frac{\varepsilon_{ci}}{\varepsilon_c}\right)\frac{n}{n-1+\left(\frac{\varepsilon_{ci}}{\varepsilon_c}\right)^n}. \qquad (4.2)$$

In both the equations, subscript *i* refers to the current value of stress and/or strain; the stress values being represented with symbol f_{ci} whereas the strains are represented with symbol ε_{ci}; the unconfined 28 day compressive strength of concrete is represented as f'_c; and its corresponding strain is represented as ε_c. The value of *n* is represented in Eq. 4.3 as

$$n = \begin{cases} \dfrac{\left(\dfrac{f'_c}{\varepsilon_c}\right)}{E_c - \left(\dfrac{f'_c}{\varepsilon_c}\right)} & if \left(\dfrac{f'_c}{\varepsilon_c}\right) \geq E_c \\ 400 & otherwise \end{cases} \qquad (4.3)$$

where E_c is the initial stiffness of the concrete compressive envelope, which is specified per the building code as $57000\sqrt{f'_c}$ psi. The difference in the concrete compressive envelope utilizing the above two models is shown in figure 4a.



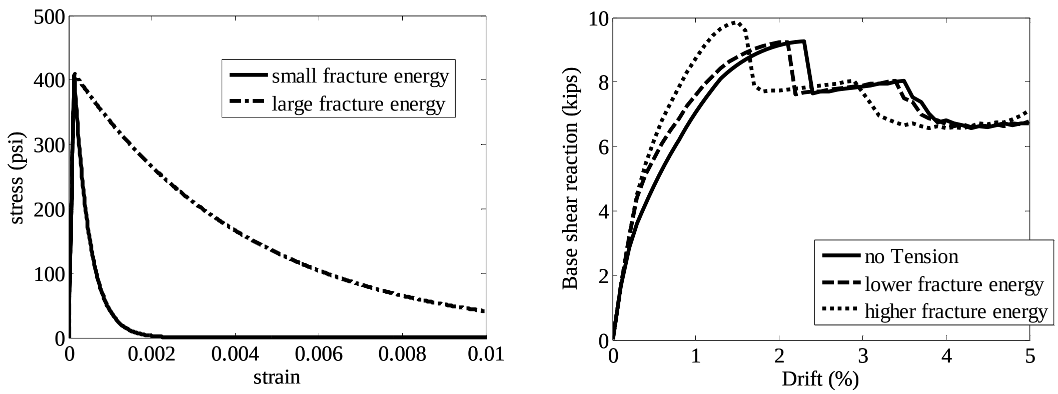
a) Concrete compressive envelope response with concrete empirical models

b) Global response with different concrete empirical models

Figure 4. Differences in response using two different material models for concrete

Significant difference in system response is observed in figure 4b, if these two different types of models are considered for representing the concrete material response. It is noteworthy to mention here that typically when

simulating the response of reinforced concrete structures subjected to loading, the modeler or analyst selects a model to represent concrete response given the properties of concrete such as the concrete compressive strength. However, what is being demonstrated in here is that this decision of proper selection of the compressive envelope could have significant impact on the system global response. The pushover system response for the above two models shown in figure 4 has been obtained by using 7 integration points distributed over the length of the element utilizing different models for concrete response.
Typically most analysis of concrete structures is performed assuming that the tensile yield strength of concrete is zero. However, in reality concrete does possess finite tensile yield strength even though the value is small in comparison to its compressive strength. Bazant (1996) concluded that no-tension design might not always guarantee safety if the material is brittle as observed with concrete structures, especially if the structure being analyzed is large and/or the concrete structure is under-reinforced. Figure 5b agrees with the conclusions from Bazant (1996) that as fracture energy in increased the initial stiffness of the entire system response increases. It is to be noted that the area under the tensile envelope curve in figure 5a represents the fracture energy divided by the characteristic length.



a) Tensile material model envelope used in Concrete04 model

b) Global response envelope using different Concrete04 tensile models

Figure 5. Effect of tension in pushover analysis of RC columns

The tensile envelope used for the above models analyzed using the Popovics model (Concrete04 model in OpenSees) is shown in Eq. 4.4 as follows:

$$f_i^t = f_t(\beta)^{(\varepsilon_i-\varepsilon_t)/(\varepsilon_{tu}-\varepsilon_t)} \tag{4.4}$$

where $\varepsilon_t = \dfrac{f_t}{E_c}$ and f_t represents tensile strength of concrete considered as 7.5 times the square root of compressive strength of concrete; superscript *i* refers to the current value of stress at any given instant, E_c represents initial elastic stiffness, β is a dimensionless quantity, chosen as 0.1 for small fracture energy in figure 5a, such that at strain ε_{tu} there is a 90% degradation in strength from the tensile strength. The value of variables β as well as ε_{tu} can be modified to obtain smaller or larger fracture energy for the specimen.
Thereby, from the above study it has been demonstrated that the global response of a reinforced concrete structural component is dependent on the material envelopes of its constituents as well as whether the analyst considers tensile response for concrete. It has been observed by the author that in many experimental investigations, the only data provided for concrete material is the concrete compressive strength; which the author believes to be inadequate. In literature on analytical simulations, the concrete compressive and tensile response model also needs to be specified explicitly. Significant differences in global response could result from these modeling decisions by the analyst and thereby these high level modeling decisions should also be considered as sources of epistemic uncertainty. It has also been shown based on the above figures that tensile response of concrete significantly affects the initial stiffness. For seismic design this increase in initial stiffness in the global response could be associated with decrease in the natural time period of vibration of the structure which would in-turn will affect the capacity calculation of the structure.

5. CONCLUSIONS

In performance-based-design of a structure, an analyst makes some high-level modeling decisions in simulating the structure and/or structural component given a set of loading criterions. For epistemic uncertainty estimation, variations are imposed with respect to the demand and capacity of the structure. Through this research study, it has been demonstrated that an inherent uncertainty lies in making the high level modeling decisions for the analysis. A reinforced concrete column subjected to axial and lateral pushover loading has been utilized to demonstrate the variations in global response associated with making high level modeling decisions such as type of element for the column, integration rules and points required for analysis, different types of empirical envelopes chosen to represent compressive response of concrete and the tensile response of concrete. It is recommended that the analyst and/or a practicing engineer, using a component based analytical model, be aware of uncertainties in structural response associated with high level modeling decisions such as with respect to element type, integration rules and material models and thereby take high level modeling decisions judiciously.

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