# DISTRIBUTED DETECTION OF LOCAL DAMAGE IN LARGE-SIZE STRUCTURES UNDER EARTHQUAKE EXCITATION

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**ABSTRACT:** Detection of local damage in large-size structures under earthquakes is an important but challenging task. In this paper, based on substructural approach, a technique for distributed detection of local damage in large-size linear structures subject to earthquake excitation is proposed. Interaction effect between adjacent substructures is considered by the interaction forces at substructural interfaces. The 'unknown external inputs' to the substructures concerned are estimated by an algorithm of recursive least squares estimation. The proposed technique not only simplifies the difficulties in the identification of complex structures by decreasing the numbers of unknown parameters in the identification process but also allows for local structural damage detection at critical parts where damage may occur. Moreover, it provides an efficient distributed computing strategy for structural damage detection which can be implemented by wireless smart sensor network based on the distributed computing capacity of smart sensors. A numerical example of detecting structural damage at element level in multi-story building is used to illustrate the proposed technique. Simulation results demonstrate that the proposed technique is capable of detecting local damage with satisfactory accuracy and it is efficient for the detection of local damage in large-scale structures.

**KEYWORDS:** Damage detection, system identification, unknown inputs, recursive least-squares estimation, finite element model

## 1. INTRODUCTION

Strong earthquake usually cause damage in structures. Accurate detection of damage in structures subject to earthquake excitation is an important but challenging task. Various damage detection techniques have been developed. System identification (SI) of structural system, or structural identification, has been used for the detection of structural damage. When an element of a structure is damaged, its dynamic parameters will vary. It is straightforward to identify structural local damage based on the variations of the identified values of dynamic properties at element level, e.g., the degrading of stiffness parameters. SI based structural damage detection techniques have received great attention in recent years (Chang 2003, 2005, Ou *et al.*, 2005). However, as an inverse problem, damage detection by the conventional structural identification is challenging, especially when the system involves a larges number of unknown parameters due to the facts: i) The computational efforts, including the computer storage and computer time, increase rapidly; ii) it is difficult to obtain reasonably accurate results of damage detection, and iii) it require a dense array of sensors to be deployed in structures. This is the inherent challenge and limitation of the conventional SI based damage detection approaches.

In practical application, the modeling of large-size structures often involves a large number of degrees of freedom (DOFs). Moreover, damage in structures is an intrinsically local phenomenon. For a complex large-size structure, there may be only limited number of critical parts where damage may likely to occur, and hence the detection can be restricted to such critical parts of the structures. Consequently, a large-size structure can be decomposed into smaller substructures for the purpose of local damage detection. Substructure identification approaches have been proposed (e.g., Koh *et al.*, 2003, Tee *et al* 2005)

In this paper, a technique for the distributed detection of local damage in large-size linear structures under earthquake excitation is proposed. Based on its finite element model, a large-scale linear structure is divided into a set of smaller substructures. Interaction effect between adjacent substructures is taken into account by considering the interaction forces at substructural interfaces as the 'unknown external inputs' to the substructures concerned. The unknown structural dynamic parameters and 'unknown external inputs' in the substructures are identified by an algorithm based on recursive least squares estimation with unknown excitations (Lei *et a*l. 2007). The proposed technique not only allows local structural damage detection at critical parts of a complex structure but also just requires a limited number of measured responses (sensors) at some DOFs of the substructures. A numerical example of detecting structural damage at element level in a multi-story shear-type building is studied to illustrate the efficiency of the proposed technique.

## 2. THE DAMAGE DETECTION APPROACH BASED ON SUBSTRUCTURES

Consider a large-scale linear structure subject to earthquake excitation. Based on the finite-element model of the structure, the equation of motion of the structure can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\{\mathbf{I}\}\ddot{x}_g(t) \tag{2.1}$$

where $\mathbf{x}$, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are vectors of displacements, velocity and acceleration response of the structure, respectively; **M, C** and **K** are the mass, damping, and stiffness matrices, respectively; $\{\mathbf{I}\}$ is an unit vector; $\ddot{x}_g(t)$ is the earthquake excitation. A large-size structure involves a large number of DOFs. To reduce computational burdens and the difficulty in obtaining reasonably accurate results of damage detection, it is reasonable to apply substructural approach for large-size structures.

### *2.1 Substructure approach*

Based on the finite-element model, the large-scale structure can be divided into a set of substructures. Then, the equation of motion of a substructure concerned can be extracted from the equation of motion of the whole structure, Eq.(2.1), to yield

$$\begin{bmatrix}\mathbf{M}_{rr} & \mathbf{M}_{rs}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{x}}_r(t)\\ \ddot{\mathbf{x}}_s(t)\end{bmatrix} + \begin{bmatrix}\mathbf{C}_{rr} & \mathbf{C}_{rs}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{x}}_r(t)\\ \dot{\mathbf{x}}_s(t)\end{bmatrix} + \begin{bmatrix}\mathbf{K}_{rr} & \mathbf{K}_{rs}\end{bmatrix}\begin{bmatrix}\mathbf{x}_r(t)\\ \mathbf{x}_s(t)\end{bmatrix} = -\begin{bmatrix}\mathbf{M}_{rr} & \mathbf{M}_{rs}\end{bmatrix}\begin{Bmatrix}\mathbf{I}_r\\ \mathbf{I}_s\end{Bmatrix}\ddot{x}_g(t) \tag{2.2}$$

where subscript 'r' denotes internal DOFs of the substructure concerned, subscript 's' denotes interface DOFs. The above equation can be re-arranged as

$$\mathbf{M}_{rr}\ddot{\mathbf{x}}_r(t) + \mathbf{C}_{rr}\dot{\mathbf{x}}_r(t) + \mathbf{K}_{rr}\mathbf{x}_r(t) = -\mathbf{M}_{rr}\{\mathbf{I}_r\}\ddot{x}_g(t) - \mathbf{M}_{rs}\ddot{\mathbf{x}}_s(t) - \mathbf{C}_{rs}\dot{\mathbf{x}}_s(t) - \mathbf{K}_{rs}\mathbf{x}_s(t) \tag{2.3}$$

Treating the interaction effects as 'unknown external inputs' to the substructure concerned, the above equation can be expressed as

$$\mathbf{M}_{rr}\ddot{\mathbf{x}}_r(t) + \mathbf{C}_{rr}\dot{\mathbf{x}}_r(t) + \mathbf{K}_{rr}\mathbf{x}_r(t) = -\mathbf{M}_{rr}\{\mathbf{I}_r\}\ddot{x}_g(t) + \mathbf{B}_r^*\mathbf{f}_r^*(t) \tag{2.4}$$

where $\mathbf{f}_r^*(t)$ are the 'unknown external inputs' to the substructure, $\mathbf{B}_r^*$ is the location matrix of 'unknown external inputs', and

$$\mathbf{B}_r^*\mathbf{f}_r^*(t) = -\mathbf{M}_{rs}\ddot{\mathbf{x}}_s(t) - \mathbf{C}_{rs}\dot{\mathbf{x}}_s(t) - \mathbf{K}_{rs}\mathbf{x}_s(t) \tag{2.5}$$

Therefore, the substructure is excited by both the ground earthquake and the 'unknown external inputs' at the substructural interfaces. It is required to explore an algorithm to identify the dynamic parameters of the substructure under the unknown external inputs.

*2.2 Recursive least squares estimation with unknown excitations*

Structural identification without excitation information has also been attempted in the past (Kathuda, *et al*. 2005, Yang *et al.* 2006). However, current approaches require that information about structural displacement and velocity responses are available or they are obtained through integration of measured acceleration responses. In this paper, the algorithm based on recursive least squares estimation with unknown excitations, which has been proposed by the authors (Lei *et al*. 2007), is used to identify the unknown structural parameters and the "unknown external inputs" in the substructure concerned.

Usually, mass of a structure can be estimated based on its geometry and material information. Therefore, Eq.(2.4) can be represented as

$$\mathbf{M}_{rr}\ddot{\mathbf{x}}_{\mathbf{r}}(t)+\mathbf{F}_r(\mathbf{x}_r(t),\dot{\mathbf{x}}_r(t),\theta_r)=-\mathbf{M}_{rr}\{\mathbf{I}_r\}\ddot{\mathbf{x}}_g(t)+\mathbf{B}_r^*\mathbf{f}_r^*(t) \quad (2.6)$$

where $\mathbf{F}_r(\mathbf{x}_r(t),\dot{\mathbf{x}}_r(t),\theta_r)$ is the restoring force in which $\theta_r$ is a vector consisting of the unknown time-invariant structural parameters such as stiffness and damping parameters. Eq.(2.6) can be transferred into state equation as

$$\frac{d\mathbf{X}_r(t)}{dt}=\mathbf{g}_r\left(\mathbf{X}_r(t),\boldsymbol{\theta}_\mathbf{r},\mathbf{f}_r^*(t),\ddot{\mathbf{x}}_g(t)\right)+\mathbf{w}_r(t) \quad (2.7)$$

in which $\mathbf{X}_r(t)=[\mathbf{x}_r(t)\ \dot{\mathbf{x}}_r(t)]^\mathbf{T}$ is the state vector; $\mathbf{w}_r(t)$ is the model noise vector; $g_r(\bullet)$ is a function of state vector $\mathbf{X}_r(t)$, the unknown parameters $\theta_r$, earthquake excitation $\ddot{\mathbf{x}}_g(t)$ and the unknown inputs $\mathbf{f}_r^*(t)$.

With a limited number of measured acceleration responses of the structure, discrete form of the observation equation at time t= (k+1)*Δt (Δt is thesampling time step) can be represented as

$$\mathbf{y}_r[k+1]=\mathbf{D}_r\ddot{\mathbf{x}}_r[k+1]+\mathbf{v}_r[k+1] \quad (2.8)$$

where $\mathbf{y}_r$ is a measured acceleration vector, $\mathbf{D}_r$ is the matrix associated with the locations of accelerometers, and $\mathbf{v}_r$ is the measurement noise which is assumed to be a Gaussian white vector with zero mean and a covariance matrix $E[\mathbf{v}_{ri}\mathbf{v}_{rj}^T]=\mathbf{R}_i\delta_{ij}$ where $\delta_{ij}$ is the Kroneker delta.

From Eq.(2.7), it is noted that state vector $\mathbf{X}_r(t)$ is a function of the unknown parameters $\theta_r$. Therefore, observation equation (2.8) can be rewritten as

$$\mathbf{y}_\mathbf{r}[k+1]=\mathbf{h}_r\left(\mathbf{X}_r(\boldsymbol{\theta}_r),\ \boldsymbol{\theta}_r,\ \mathbf{f}_r^*[k+1],\ \ddot{\mathbf{x}}_g[k+1]\right)+\mathbf{v}_r[k+1] \quad (2.9)$$

where $\mathbf{h}_r(\bullet)$ is a nonlinear function. Let $\hat{\theta}[k|k]$ and $\hat{\mathbf{f}}_r^*[k|k]$ be the estimated values of the unknown parameters $\theta_r$ and the excitations $\mathbf{f}_r^*$ at the previous time t=k*Δt, respectively, given the observations $\mathbf{y}_r[1],\mathbf{y}_r[2],...,\mathbf{y}_r[k]$. The nonlinear function $\mathbf{h}_r(\bullet)$ in Eq.(4) can be liberalized around $\hat{\theta}[k|k]$ and $\hat{\mathbf{f}}_r^*[k|k]$

based on Taylor's expansion, leading to the representation of Eq.(2.9) as follows,

$$\mathbf{y_r}[k+1] = \hat{\mathbf{h}}_r\left(\hat{\mathbf{X}}_r\left(\hat{\boldsymbol{\theta}}_r[k|k], \hat{\mathbf{f}}_r^*[k|k]\right),\ \hat{\boldsymbol{\theta}}_r[k|k],\ \hat{\mathbf{f}}_r^*[k|k],\ \ddot{\mathbf{x}}_g[k+1]\right) + \mathbf{H}_r[k+1|k]\left(\boldsymbol{\theta}_r - \hat{\boldsymbol{\theta}}_r[k|k]\right) + \mathbf{D}_r^*\left(\mathbf{f}_r^*[k+1] - \hat{\mathbf{f}}_r^*[k|k]\right) \tag{2.10}$$

where $\mathbf{H}_r[k+1|k]$ and $\mathbf{D}_r^*$ are two matrices.

$$\mathbf{D_r^*} = \left[\frac{\partial \mathbf{h}_r\left(\mathbf{X}_r[k+1], \boldsymbol{\theta}_r,\ \mathbf{f}_r^*[k+1],\ \ddot{\mathbf{x}}_g[k+1]\right)}{\partial \mathbf{f}_r^*[k+1]}\right] \tag{2.11}$$

$\mathbf{H}_r[k+1|k]$ can be obtained by the chain rule of partial derivative as,

$$\mathbf{H}_r[k+1|k] = \mathbf{H}_{r,\theta}[k+1|k] + \mathbf{H}_{r,\mathbf{X}}[k+1|k]\mathbf{X}_{r,\theta}[k+1|k] \tag{2.12}$$

in which $\mathbf{H}_{r,\theta}[k+1|k]$, $\mathbf{H}_{r,\mathbf{X}}[k+1|k]$ and $\mathbf{X}_{r,\theta}[k+1|k]$ are matrices, respectively, defined as.

$$\mathbf{H}_{r,\theta}[k+1|k] = \left[\frac{\partial \mathbf{h}_r\left(\mathbf{X}_r[k+1], \boldsymbol{\theta}_r,\ \mathbf{f}_r^*[k+1],\ \ddot{\mathbf{x}}_g[k+1]\right)}{\partial \boldsymbol{\theta}_r}\right]_{\substack{\mathbf{X}_r[k+1]=\hat{\mathbf{X}}_r(\hat{\boldsymbol{\theta}}_r[k|k], \hat{\mathbf{f}}_r^*[k|k]) \\ \boldsymbol{\theta}_r[k+1]=\hat{\boldsymbol{\theta}}_r[k|k] \\ \mathbf{f}_r^*[k+1]=\hat{\mathbf{f}}_r^*[k|k]}} \tag{2.13a}$$

$$\mathbf{H}_{r,\mathbf{X}}[k+1|k] = \left[\frac{\partial \mathbf{h}_r\left(\mathbf{X}_r[k+1], \boldsymbol{\theta}_r, \hat{\mathbf{f}}_r^*[k+1],\ \ddot{\mathbf{x}}_g[k+1]\right)}{\partial \mathbf{X}_r[k+1]}\right]_{\substack{\mathbf{X}_r[k+1]=\hat{\mathbf{X}}_r(\hat{\boldsymbol{\theta}}_r[k|k], \hat{\mathbf{f}}_r^*[k|k]) \\ \boldsymbol{\theta}_r[k+1]=\hat{\boldsymbol{\theta}}_r[k|k] \\ \mathbf{f}_r^*[k+1]=\hat{\mathbf{f}}_r^*[k|k]}} \tag{2.13b}$$

$$\mathbf{X}_{r,\boldsymbol{\theta}}[k+1|k] = \left[\frac{\partial \mathbf{X}_r}{\partial \boldsymbol{\theta}_r}\right]_{\boldsymbol{\theta}_r=\hat{\boldsymbol{\theta}}_r[k|k];\ \mathbf{f}_r^*=\hat{\mathbf{f}}_r^*[k|k]} \tag{2.13c}$$

$\mathbf{X}_r[k+1] = \hat{\mathbf{X}}_r(\hat{\boldsymbol{\theta}}_r[k|k], \hat{\mathbf{f}}_r^*[k|k])$ is the estimation of state vector $\mathbf{X}_r(t)$ at time t=(k+1)*Δt given $\hat{\boldsymbol{\theta}}_r[k|k]$ and $\hat{\mathbf{f}}_r^*[k|k]$. Based on Eq.(2.7), it is derived that

$$\hat{\mathbf{X}}_r(\hat{\boldsymbol{\theta}}_r[k|k], \hat{\mathbf{f}}_r^*[k|k]) = \hat{\mathbf{X}}_r(\hat{\boldsymbol{\theta}}_r[k-1|k-1], \hat{\mathbf{f}}_r^*[k-1|k-1]) + \int_{K\Delta t}^{(k+1)\Delta} \mathbf{g}_r\left(\mathbf{X}_r(t), \hat{\boldsymbol{\theta}}_r[k|k], \hat{\mathbf{f}}_r^*[k|k], \ddot{\mathbf{x}}_g(t)\right) dt \tag{2.14}$$

Differentiating both sides of Eq.(2.7) with respect $\boldsymbol{\theta}_r$, one obtains,

$$\frac{d\mathbf{X}_{r,\boldsymbol{\theta}}(t)}{dt} = \left[\frac{\partial \mathbf{g}_r\left(\mathbf{X}_r(t), \boldsymbol{\theta}_r, \mathbf{f}_r^*(t), \ddot{\mathbf{x}}_g(t)\right)}{\partial \boldsymbol{\theta}_r}\right] + \left[\frac{\partial g\left(\mathbf{X}_r(t), \boldsymbol{\theta}, \mathbf{f}^u(t), \mathbf{f}(t)\right)}{\partial \mathbf{X}_r}\right]\mathbf{X}_{r,\theta} = \mathbf{g}_\theta\left(\mathbf{X}_{r,\theta}(t), \boldsymbol{\theta}_r, \mathbf{f}_r^*(t), \ddot{\mathbf{x}}_g(t)\right) \tag{2.15}$$

Thus, $\mathbf{X}_{r,\theta}[k+1|k]$ is obtained by

$$\mathbf{X}_{r,\theta}[k+1|k] = \mathbf{X}_{r,\theta}[k|k-1] + \int_{k\Delta t}^{(k+1)\Delta t} \mathbf{g}_{\theta}\left(\mathbf{X}_{r,\theta}(t), \hat{\theta}_r[k|k], \hat{\mathbf{f}}_r^*[k|k], \ddot{\mathbf{x}}_g(t)\right) dt \quad (2.16)$$

From Eqs.(2.9-2.10), the error between the observation $\mathbf{y_r}[k+1]$ and the estimated value of $\mathbf{h}_r\left(\mathbf{X}_r(\boldsymbol{\theta}_r),\ \boldsymbol{\theta}_r,\ \mathbf{f}_r^*[k+1],\ \ddot{\mathbf{x}}_g[k+1]\right)$ is

$$\begin{aligned}
\Delta[k+1] &= \mathbf{y}_r[k+1] - \mathbf{h}_r\left(\mathbf{X}_r(\boldsymbol{\theta}_r),\ \boldsymbol{\theta}_r,\ \mathbf{f}_r^*[k+1],\ \ddot{\mathbf{x}}_g[k+1]\right) \\
&= \mathbf{y}_r[k+1] - \hat{\mathbf{h}}_r\left(\hat{\mathbf{X}}_r\left(\hat{\theta}_r[k|k]\right),\ \hat{\theta}_r[k|k],\ \hat{\mathbf{f}}_r^*[k|k],\ \ddot{\mathbf{x}}_g[k+1]\right) - \mathbf{H}_r[k+1|k]\left(\boldsymbol{\theta}_r - \hat{\theta}_r[k|k]\right) \\
&\quad - \mathbf{D}_r^*\left(\mathbf{f}_r^*[k+1] - \hat{\mathbf{f}}_r^*[k|k]\right) = \bar{\mathbf{y}}_r[k+1] - \mathbf{H}_r[k+1|k]\boldsymbol{\theta}_r - \mathbf{D}_r^*\mathbf{f}_r^*[k+1] = \bar{\mathbf{y}}_r[k+1] - \mathbf{H}_r^*[k+1|k]\boldsymbol{\theta}_r^*[k+1]
\end{aligned} \quad (2.17)$$

in which,

$$\bar{\mathbf{y}}_r[k+1] = \mathbf{y}_r[k+1] - \hat{\mathbf{h}}_r\left(\hat{\mathbf{X}}_r\left(\hat{\theta}_r[k|k]\right),\ \hat{\theta}_r[k|k],\ \hat{\mathbf{f}}_r^*[k|k],\ \ddot{\mathbf{x}}_g[k+1]\right) + \mathbf{H}_r[k+1|k]\hat{\theta}_r[k|k] + \mathbf{D}_r^*\hat{\mathbf{f}}_r^*[k|k] \quad (2.18a)$$

$$\mathbf{H}_r^*[k+1|k] = [\mathbf{H}[k+1|k]\ \ \mathbf{D}_r^*] \quad ; \quad \boldsymbol{\theta}_r^*[k+1|k] = [\boldsymbol{\theta}_r\ \ \mathbf{f}_r^*[k+1]]^T \quad (2.18b)$$

With (j+1) measurement time instants in the observation data and parameters $\boldsymbol{\theta}_r$ being time-invariant, the sum square error is

$$\begin{aligned}
J_{j+1} &= \sum_{k=1}^{j+1} \Delta_r[k]^T R_k^{-1} \Delta_r[k] = \sum_{k=1}^{j+1} \left[\bar{\mathbf{y}}_r[k] - \mathbf{H}_r^*[k|k-1]\boldsymbol{\theta}_r^*[k]\right]^T R_j^{-1} \left[\bar{\mathbf{y}}_r[k] - \mathbf{H}_r^*[k|k-1]\boldsymbol{\theta}_r^*[k]\right] \\
&= \left[\bar{\mathbf{Y}}_r[j+1] - \boldsymbol{\Phi}_r[j+1]\boldsymbol{\Psi}_r[j+1]\right]^T W_r[k+1]\left[\bar{\mathbf{Y}}_r[j+1] - \boldsymbol{\Phi}_r[j+1]\boldsymbol{\Psi}_r[j+1]\right]
\end{aligned} \quad (2.19)$$

where

$$\bar{\mathbf{Y}}_r[j+1] = \begin{Bmatrix} \bar{\mathbf{y}}_r[1] \\ \bar{\mathbf{y}}_r[2] \\ \vdots \\ \bar{\mathbf{y}}_r[k+1] \end{Bmatrix} \quad ; \quad \boldsymbol{\Psi}_r[j+1] = \begin{bmatrix} \boldsymbol{\theta}_r \\ \mathbf{f}_r^*[1] \\ \mathbf{f}_r^*[2] \\ \vdots \\ \mathbf{f}_r^*[j+1] \end{bmatrix} \quad ; \quad \boldsymbol{\Phi}_r[j+1] = \begin{bmatrix} \mathbf{H}_r[1|0] & \mathbf{D}_r^* & 0 & 0 & \cdots & 0 \\ \mathbf{H}_r[2|1] & 0 & \mathbf{D}_r^* & & \cdots & 0 \\ \mathbf{H}_r[3|2] & 0 & 0 & \mathbf{D}_r^* & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_r[j+1|j] & 0 & 0 & 0 & 0 & \mathbf{D}_r^* \end{bmatrix} \quad (2.20)$$

Under the condition that the number of acceleration observations is greater that of ‘unknown external inputs’, the recursive solution for solutions for $\hat{\mathbf{f}}_r^*[k+1|k+1]$ and $\hat{\theta}_r[k+1|k+1]$ can be derived as follows (Lei *et al*, 2007),

$$\hat{\mathbf{f}}_r^*[k+1|k+1] = S_{k+1}\mathbf{D}_{k+1|k}^T\mathbf{R}_{k+1}^{-1}(\mathbf{I} - \mathbf{H}_r[k+1|k]\mathbf{K}[k+1])(\mathbf{y}_r[k+1] - \bar{\mathbf{h}}_r(\mathbf{X}_r(\hat{\theta}_r[k|k]), \hat{\theta}_r[k|k], \ddot{\mathbf{x}}_g[k+1])) \quad (2.21)$$

$$\hat{\theta}_r[k+1|k+1] = \hat{\theta}_r[k|k] + \mathbf{K}[k+1](\mathbf{y}_{k+1} - \bar{\mathbf{h}}_r(\mathbf{X}_r(\hat{\theta}_r[k|k]), \hat{\theta}_r[k|k], \ddot{\mathbf{x}}_g[k+1]) - \mathbf{D}_r^*\mathbf{f}_r^*[k+1]) \quad (2.22)$$

where

$$\overline{\mathbf{h}}_r(\mathbf{X}_r(\hat{\theta}_r[k|k]),\hat{\theta}_r[k|k],\ddot{\mathbf{x}}_g[k+1]) = \hat{\mathbf{h}}_r(\mathbf{X}_r(\hat{\theta}_r[k|k]),\ \hat{\theta}_r[k|k],\ \hat{\mathbf{f}}_r^*[k|k],\ \ddot{\mathbf{x}}_g[k+1]) - \mathbf{D}_r^*\mathbf{f}_r^*[k+1] \quad (2.23)$$

$$\mathbf{K}[k+1] = \mathbf{P}[k+1]\mathbf{H}_r^T[k+1|k](\mathbf{R}[k+1] + \mathbf{H}_r[k+1|k]\mathbf{P}[k]\mathbf{H}_r^T[k+1|k])^{-1} \quad (2.24)$$

$$\mathbf{S}[k+1] = [\mathbf{D}_r^{*T}\mathbf{R}[k+1]^{-1}(\mathbf{I} - \mathbf{H}_r[k+1|k]\mathbf{K}[k+1])\mathbf{D}_r^*]^{-1} \quad (2.25)$$

$$P[k+1] = (\mathbf{I} + \mathbf{K}[k+1]\mathbf{D}_r^*\mathbf{S}[k+1]\mathbf{D}_r^{*T}R[k+1]^{-1}\mathbf{H}_r[k+1|k])\ (\mathbf{I} - \mathbf{K}[k+1]\mathbf{H}_r[k+1|k])P[k] \quad (2.26)$$

From the above recursive solution in Eqs. (2.21)-(2.26), it is noted that in order to obtain the recursive solution for the unknown inputs $\hat{\mathbf{f}}_r^*[k+1|k+1]$, $\mathbf{D}_r^*$ should be a non-zero matrix, which requests that there are response measurements (sensors) at the substructural interfaces.

Consider a four-story shear type frame building subject to both the ground El Centro earthquake excitation and a white-noise external input at the top floor as shown in Fig.1. The mass for each floor is $m_1 = m_2 = m_3 = m_4 = 1000$kg; the values of story stiffness are: $k_1 = k_2 = 120$kN/m, $k_3 = 100$kN/, $k_4 = 60$kN/m, and the damping coefficients are $c_1 = c_2 = c_3 = c_4 = 0.6$kN s/m. In the numerical example, the external white noise input is not measured. Three accelerometers are employed to measure acceleration responses of the 2nd, 3rd and 4th floors, but the acceleration response of the 1st floor is not measured. Based on the above algorithm of recursive least squares estimation with unknown excitations, the unknown external input can be estimated. Fig. 2 compares the identified value of the external input (dotted cure) with that of the theoretical one (solid cure) for the segment from 5.0 to 10 second. It demonstrates that the above algorithm of recursive least squares estimation with unknown excitations lead to quite accurate results in the identification of un-measured external inputs.

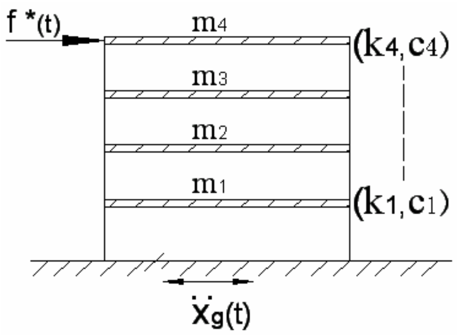



Figure 1 A four-story building under earthquake and an un-measured external input

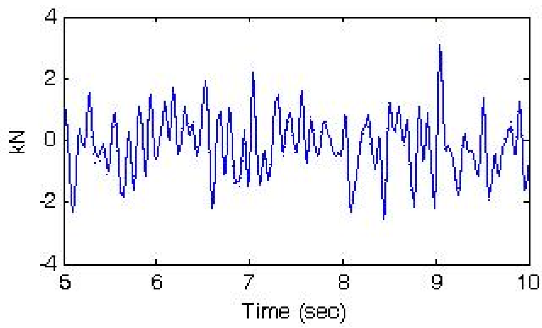
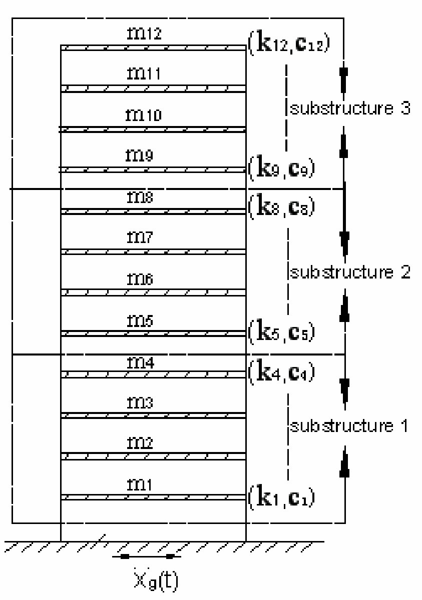



Figure 2 Comparison of the Identified input (dotted cure) with the theoretical one (solid cure)

## 3. NUMERICAL EXAMPLE

Detection of local damage in a tall building under earthquake excitation is taken as an numerical example to illustrate the proposed technique and investigate its damage detection feasibility. The structure of the building is a 12 story shear-type frame with lump masses as shown in Fig.3. The following parametric values are used in the simulation study: mass for each floor $m_1 = m_2 = ..., = m_{12} = 1000$kg ; story stiffness $k_1 = k_2 = ..., = k_{11} = 120$kN/m , $k_{12}$=80kN/m, damping coefficients $c_1 = c_2 = ..., = c_{12} = 0.6$kN s/m . The building is excited by the 1940 El Centro N-S earthquake excitation with a PGA=0.2g. As a numerical example, the building is divided into three substructures. The 1-4 story part of the building is the substructure one, the 5-9 story part of the building is the substructure two, and the 9-12 story part of the building is the substructure three, as shown in Fig. 3. Interaction effect between each two adjacent substructures is considered by interaction forces at the substructural interfaces. The interaction forces are treated as external inputs to the substructures.

However, the values of the inputs $f_r^*(t)$ are unknown, as shown by Fig. 4. Accelerometers are employed at the 2nd, 3rd, 4th, 6th, 8th, 9th, 10th and 11th floors to measure the floor acceleration responses.



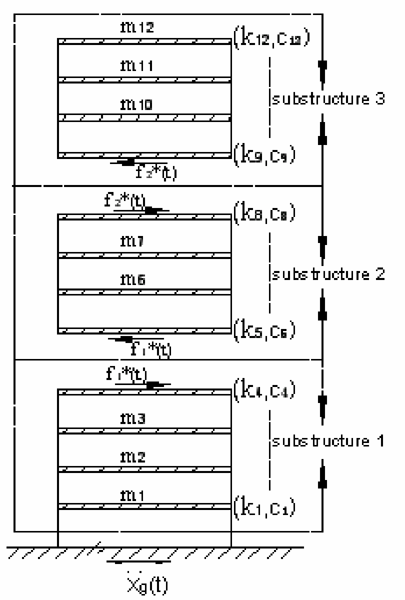




Figure 3 A multi-story shear-type building under earthquake Figure 4 Substructure with "unknown inputs"

Two damage patterns are considered in the numerical example. For damage pattern 1, a damage in the 4th story occurs with $k_4$ being reduced from 120kN/m to 90kN/m. For damage pattern 2, damage occurs in both the 3rd and 12th stories, which leads to $k_4$ being reduced from 120kN/m to 100kN/m and $k_{12}$ being reduced from 80kN/m to 60kN/m. Distributed identification of the damage in substructures are conducted by distributed commutating strategy by the proposed damage detection technique. Based on the proposed method, the identified stiffness parameters of the building in damage pattern 1 and 2 are shown in table 3.1. From the comparison of the identified results with their analytical values, it is shown that the proposed technique can identify structural element stiffness parameters with good accuracy and local structural damage can be detected and located from the degrading of element stiffness parameters.

Table 3.1. Story stiffness parameters of the building

| Story No. | Story Stiffness $k_i$ (kN/m) | | | | |
|---|---|---|---|---|---|
| | Un-damaged | Damage pattern 1 (analytical) | Damage pattern 1 (identification) | Damage pattern 2 (analytical ) | Damage pattern 2 (identification) |
| 3 | 120 | 120 | 120 | 100 | 101 |
| 4 | 120 | 90 | 87 | 120 | 120 |
| 12 | 80 | 80 | 80 | 60 | 62 |

Compared with the previous damage detection techniques, it is observed that proposed technique allows for distributed identification of local damage in large-size structures. It provides an efficient distributed computing strategy for structural damage detection which greatly reduces the amount of data to be transmitted compared with damage detection methodologies. Therefore, the technique for local damage detection presented in this paper is very suitable for the implementation of automated damage detection system based on the wireless structural monitoring sensing network (Lynch *et al*. 2006).

## 4. CONCLUSIONS

In this paper, a technique is proposed for distributed identification of local damage in large-size structures under earthquake excitation. A large-scale structure is divided into a set of smaller substructures. Interaction effect between adjacent substructures is taken into account by considering the interaction forces at substructural interfaces. The interaction forces at substructural interfaces are treated as the "unknown external inputs" to the substructures concerned. Under the conditions: i) the number of response measurement (sensors) is greater than the number of the "unknown external inputs", and ii) there are response measurements (sensors) at the substructural interfaces, identification of the unknown structural parameters and the "unknown external inputs" in the substructures concerned can be implemented by an algorithm based on recursive least squares estimation with unknown excitations. The technique enable distributed identification of local damage in large structures utilizing only a limited number of measured responses (sensors). It not only simplifies the difficulties in the identification of complex structures by decreasing the numbers of unknown parameters in the identification process but also allows for local structural damage detection at critical parts where damage may occur. A numerical example demonstrates that the proposed technique is capable of detecting local damage with satisfactory accuracy and it is suitable for the identification of local damage in large-scale structures.

The proposed technique also provides a distributed computing strategy which is particular suitable for automated damage detection implemented by the wireless smart sensor network based on the distributed computing capacity of smart sensors. Research work is undertaken by the authors.

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