STUDY ON CRITICAL ANGLE TO THE SEISMIC RESPONSE OF CURVED BRIDGES BASED ON PUSHOVER METHOD

B. Song[1], J.S. Pan[1], Q. Liu[1]

[1] *Civil and Environmental Engineering School, University of Science and Technology, Beijing, China*
Email: orc_hard@163.com

ABSTRACT :

Because of the planar irregularity, the curved bridge response during the earthquake has its own characteristic, for the maxim seismic response of curved bridges is correlative with the input angle of earthquake. This paper presents a relationship between the seismic response and the curvature at the bottom of the pier, based on the principle of Pushover method. The formula to calculate the critical angle is gained in the paper with non-liner Pushover method. Through a case of curved bridge, the critical angle in the seismic response of curved bridges is calculated with the non-linear statical analytical method. The formula is accurate as it is compared to the results of 0-180 degree dynamical time history analysis. According to the formula, it makes sense in the engineering project to use Pushover method to find the critical angle to the seismic response of curved bridges.

KEYWORDS: curved bridge, Pushover method, direction of input seismic wave, curvature, critical angle

1. INTRODUCTION

With the development of bridge structure and complication of urban traffic, the structure of curved bridge keeps increasing. Due to relative particularity of curved bridge forms, its dynamic response on physical parameters presents apparently different compared with common straight bridge. Hence, extensive concern and researches are conducted to seismic response of curved bridge. In view of the collapse of curved bridges happened in American San Fernando earthquake in 1971，David Williams (1979) carried out shaking table test study and calculation on curved bridge, with the emphasis on the effect of the expansion joint to earthquake resistance. Seismic response of curved bridge differs from common bridge because curved bridge presents planar irregularity. Structural vibration responses of curved bridge usually should be considered in longitudinal and transverse direction while the direction of input seismic wave has great effects on maxim seismic response. Planar irregularity and two-axis related bending of curved bridge play great role on it. Thus, pier session design should depend on analysis results of the critical angle in the seismic response of curved bridges.
The research (J. Penzien & M. Watabe, 1975) showed that structural seismic action in horizontal plane could be decomposed as a primary seismic action and another perpendicular one. E.L.Wilison (1982) had considered this in his research on multi-direction seismic action to irregularity structure, though his conclusions were just suitable for single mode. A method based on the energy standard of input angle was discussed by Y.T. Fen(1991) and seismic principal axis was put forward in his paper. Maxim stress at any point could be calculated with given seismic input in two different directions. The methods of multi-component seismic response analysis for curved bridges were systemically analyzed and compared by X.A. Gao (2005).A computional example and time history analysis comparison were also presented in his research. In the study (D.S. Zhu *et al*., 2000) on the effect of pier type and support form to seismic behavior, earthquake response spectrum was utilized to calculate the critical angle. The determining standard to critical angle was discussed by L.C. Fan (2003) and L.Y. Nie (2003) *et al*. Superposition principle and CQC method were adopted to get the critical angle in their researches.
Considering complicated structures，elastic-plastic dynamical time history analysis is obviously a reliable method, for it can judge yield mechanism, weakness points and potential destruction type. As for urban curved bridge, attention should be paid to multi-direction seismic action which needs a heavy workload and results a complex analysis. Hence, wide attentions are attracted to a relatively simple method, pushover seismic analysis

Foundation project: NSFC MAJOR RESEARCH PLAN (No. 90715007)
Author: B. Song (1962-), Male, Professor, Ph.D.

methods (Y. Zhou *et al*., 2001; J.R. Qian *et al*., 2001). Pushover method is not used for bridge analysis as popular as for building construction analysis (C.Y. Liu & G. Lin, 2005). Recently pushover method has been used in research extensively such as on bridge double column pier (X.G. Xu, 2005), on *P*-△ effect (G.H. Cui, 2003) and on both two-dimensional and three-dimensional simulation of multi-span simply-supported bridge (M. Ala Saadeghvaziri, 2007).
Dynamic analysis to complicated structures could be carried out with displacement and curvature which are generalized displacement dynamic response. The maxim reaction of displacement appears at pier top and curvature at pier bottom. In view of the pier with elastic status and the same displacement at pier top, curvature at pier bottom changes according to the pier height. This paper presents a formula to calculate the critical angle of curved bridge seismic response, based on the principle of Pushover method. And it makes great sense to reduce calculating workload in practical project.

2. FORMULA TO CALCULATE CRITICAL ANGLE BASED ON PUSHOVER METHOD

2.1. Relation between single pier curvature and displacement

Given that the plastic hinge appears at the pier bottom in a single pier, and that horizontal load of earthquake appears at the center of mass，horizontal deformation is mainly induce by pier moment without considering torsion effect. Thus, in the range of elastic status, the displacement △ at the pier top could be expressed by:

$$\Delta = \int_0^L \int_0^y \varphi(y)dydy = \frac{ML^2}{3EI} \tag{2.1}$$

The relationship of curvature at the pier bottom φ and moment M is given by:

$$\varphi = \frac{M}{EI} \tag{2.2}$$

That is to say,

$$\Delta = \frac{\varphi L^2}{3} \tag{2.3}$$

If the structure comes into plastic status, it is amused that the plastic hinge centralized at the pier bottom with the length of L_P and plastic hinge curvature φ (Figure 1). So, horizontal displacement at the top could be gained as follow:

$$\Delta = \Delta_0 + (\varphi_u - \varphi_0)L_p(L - 0.5L_p) \tag{2.4}$$

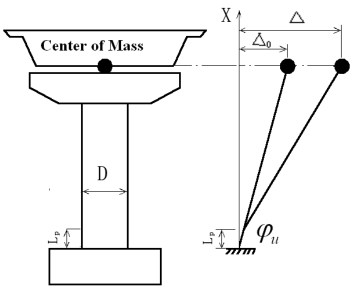

Figure 1 Relation between bottom curvature and top displacement of a single pier

While the elastic status and plastic status are uniformed in one form, it presents as：

$$\Delta_{top} = \Delta_0 + b(\varphi_b - \varphi_0) \tag{2.5}$$

Where $\Delta_0 = \begin{cases} \dfrac{\varphi_b L^2}{3} & (\text{For } \varphi_b < \varphi_0) \\ \dfrac{\varphi_0 L^2}{3} & (\text{For } \varphi_b \ge \varphi_0) \end{cases}$，$b = \begin{cases} L_p(L - 0.5L_p) & (\text{For } \varphi_b < \varphi_0) \\ 0 & (\text{For } \varphi_b \ge \varphi_0) \end{cases}$

$$L_p = 0.2H - 0.1D \text{ and } 0.1D \le L_p \le 0.5D.$$

Where φ_0 is yield curvature at the bottom, φ_b curvature at the bottom, H pier height and D pier diameter.

2.2 Two-dimensional bidirectional seismic wave input

It is showed the maxim deformation response in Figure 2 when seismic wave input in unidirectional arbitrary angle α, then the deformation response could be decomposed along X-axis and Y-axis as follows:

$$u = u_x \cos\alpha + u_y \sin\alpha \tag{2.6}$$

Where u_x and u_y is respectively the X-axis and Y-axis component of deformation response.

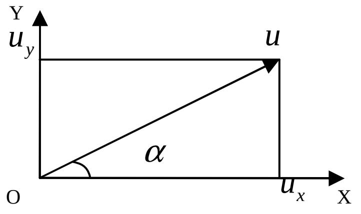

Figure 2 Displacement decomposition of one-direction input seismic wave

Considering spatial structure of curved bridges, the Pushover method is adopted in the calculation of bottom curvature in global coordinate system. Thus, it could be decomposed as φ_x in XZ plane and φ_y in YZ plane, which are less than the global curvature. That is to say, the curvature components are in the elastic range even if the global curvature comes into plastic range. The curvature φ_x and φ_y has respectively relationship with displacement component u_x and u_y as follows:

$$u_x = u_0 + b_x(\varphi_x - \varphi_0) \tag{2.7}$$

$$u_y = u_0 + b_y(\varphi_y - \varphi_0) \tag{2.8}$$

While two-dimensional bidirectional seismic wave input is regarded as Figure 3, u_1 and u_2 could be used to respectively express the displacement in direction 1 and direction 2, which are the two seismic wave input directions.

$$u_1 = u_{1,x} \cos\alpha + u_{1,y} \sin\alpha \tag{2.9}$$

$$u_2 = -u_{2,x} \sin\alpha + u_{2,y} \cos\alpha \tag{2.10}$$

Where, u_{1x} , u_{1y} , u_{2x} and u_{2y} could be gained from Eqn. 2.7 and Eqn. 2.8 as follows:
For $i=1,2$ and $j=x,y$

$$u_{i,j} = u_{i,j0} + b_{i,j}(\varphi_{i,j} - \varphi_0) \tag{2.11}$$

It is assumed seismic response is independent in the perpendicular direction, so the global displacement could be obtained from the displacement components.

$$u^2 = u_1^{\ 2} + u_2^{\ 2} \tag{2.12}$$

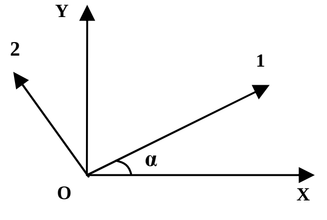

Figure 3 Displacement decomposition of bidirectional input seismic wave

Then, Eqn. 2.12 presents as follow by substitution of Eqn. 2.9 to Eqn. 2.11.

$$\{u_0+b[\varphi_u(\alpha)-\varphi_0]\}^2$$
$$=[u_{1,x0}+b_{1,x}(\varphi_{1,x}-\varphi_0)]^2+[u_{2,y0}+b_{1,y}(\varphi_{1,y}-\varphi_0)]^2\}\cos^2\alpha+\{[u_{2,x0}+b_{2,x}(\varphi_{2,x}-\varphi_0)]^2+$$
$$[u_{1,y0}+b_{1,y}(\varphi_{1,y}-\varphi_0)]^2\}\sin^2\alpha+\{[u_{1,x0}+b_{1,x}(\varphi_{1,x}-\varphi_0)][u_{1,y0}+$$
$$b_{1,y}(\varphi_{1,y}-\varphi_0)]-[u_{2,x0}+b_{2,x}(\varphi_{2,x}-\varphi_0)][u_{2,y0}+b_{1,y}(\varphi_{1,y}-\varphi_0)]\}\sin 2\alpha \tag{2.13}$$

Because Eqn. 2.13 is a function about parameter α, the maxim of curvature response and critical angle of seismic wave input could be calculated by means of derivation to parameter α.

$$\tan 2\alpha=2\{[u_{1,x0}+b_{1,x}(\varphi_{1,x}-\varphi_0)][u_{1,y0}+b_{1,y}(\varphi_{1,y}-\varphi_0)]-$$
$$[u_{2,x0}+b_{2,x}(\varphi_{2,x}-\varphi_0)][u_{2,y0}+b_{1,y}(\varphi_{1,y}-\varphi_0)]\}/\{[u_{1,x0}+b_{1,x}(\varphi_{1,x}-\varphi_0)]^2+$$
$$[u_{2,y0}+b_{1,y}(\varphi_{1,y}-\varphi_0)]^2-[u_{2,x0}+b_{2,x}(\varphi_{2,x}-\varphi_0)]^2-[u_{1,y0}+b_{1,y}(\varphi_{1,y}-\varphi_0)]^2\} \tag{2.14}$$

In view of results form, the two angles have angle difference about ninety degrees. So substitution checking calculation is necessary to determine the right angle from the results. Four curvatures appear in the formula such as φ_{1x} , φ_{1y} , φ_{2x} and φ_{2y}, any one of which may larger than yield curvatures or not. Thus, classified discussion is necessary.

(1)For $\varphi_{ij}<\varphi_0$ ($i=1,2$; $j=x,y$)

$$\tan 2\alpha=2\frac{\varphi_{1,x}\varphi_{1,y}-\varphi_{2,x}\varphi_{1,y}}{(\varphi_{1,x}{}^2+\varphi_{1,y}{}^2)-(\varphi_{2,x}{}^2+\varphi_{1,y}{}^2)} \tag{2.15}$$

(2)For $\varphi_{ij}<\varphi_0$ ($i=1,2$; $j=x,y$)

$$\tan 2\alpha=2(u_0-b\varphi_0)(\varphi_{1,x}+\varphi_{1,y}-\varphi_{2,x}-\varphi_{2,y})+b(\varphi_{1,x}\varphi_{1,y}-\varphi_{2,x}\varphi_{1,y})/$$
$$\{(u_0-b\varphi_0)[(\varphi_{1,x}+\varphi_{1,y})-(\varphi_{2,x}+\varphi_{2,y})]+b[(\varphi_{1,x}{}^2+\varphi_{1,y}{}^2)-(\varphi_{2,x}{}^2+\varphi_{1,y}{}^2)]\} \tag{2.16}$$

And Eqn. 2.14 is a better choice as for the other situation.

The results mentioned above are obtained based on the single pier, which is independent with curved bridge plane shape. But the curvatures are obtained based on the Pushover method which considering the effect of bridge plane shape. Thus, influence coefficient β is brought in to make up for the effect of bridge plane shape to the single pier. The modified formula is as follows:

$$\tan 2\alpha_0=\beta\tan 2\alpha \tag{2.17}$$

3. EXAMPLE AND ANALYSIS

Take a six-span curved bridge for example (Figure 4). The bridge has a full length of 222.5 meter, with a span combination of 34.5+4×38.7+33.2 meters. All the sections of piers are circular sections with equal radius which is 4.0 m. Rubber Bearing is adopted so that the structure is movable in tangential and normal direction. The bridge plane sketch is showed in Figure 5.

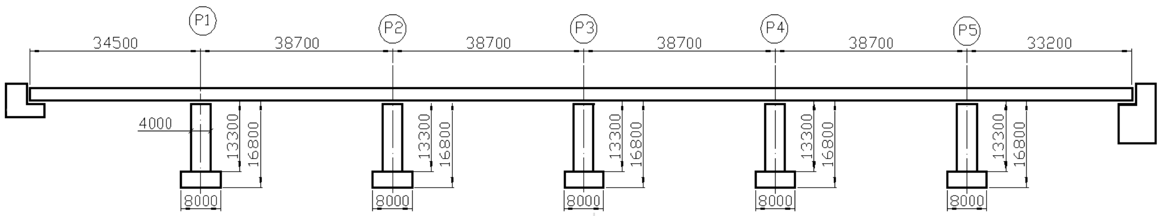

Figure 4 Six-span curved bridge elevation

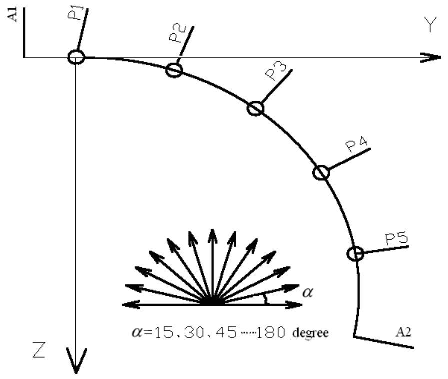

Figure 5 Plane sketch of six-span curved bridge

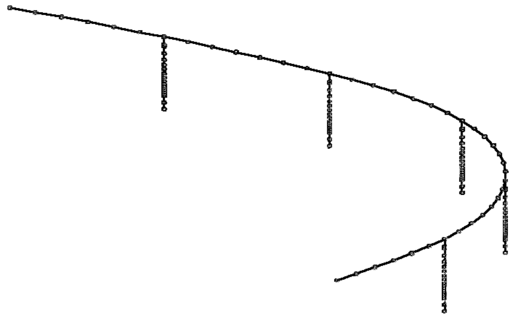

Figure 6 Simplified model to curved bridge calculation

As the simplified calculation model presented in Figure 6, upper structures are simulated with elastic beam element, while piers are simulated non-liner element. The earthquake wave recorded in the Kobe earthquake in 1995 is adopted as time history analysis seismic wave input.

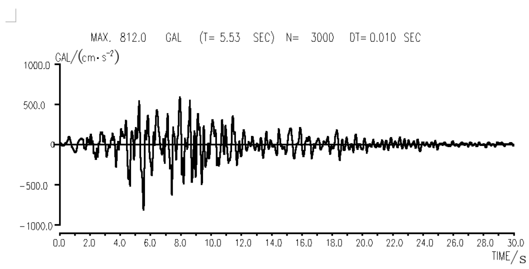

Figure 7 seismic waves recorded in Kobe earthquake in 1995

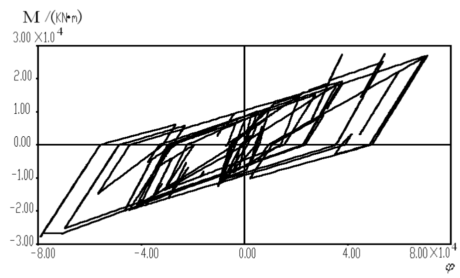

Figure 8 Relation between curvature and bending time–history-analysis results at Pier P2 bottom

With Newmark-β integration method(β=1/4), integration time interval ΔT is 0.002 second, the direction of input angle of earthquake wave ranges from 0 to 180 degree with increment of 15 degree in the course of calculation. Considering the two-axis related bending influence, take pier P2 for example with input angle of 135 degree, the time history analysis results (Figure 8) shows the relation between curvature and bending at Pier P2 bottom. Based on *M*-*φ* relation of each pier in the time history analysis, the critical angles of each pier are obtained in Figure 9, which is the corresponding angle to the peak point.

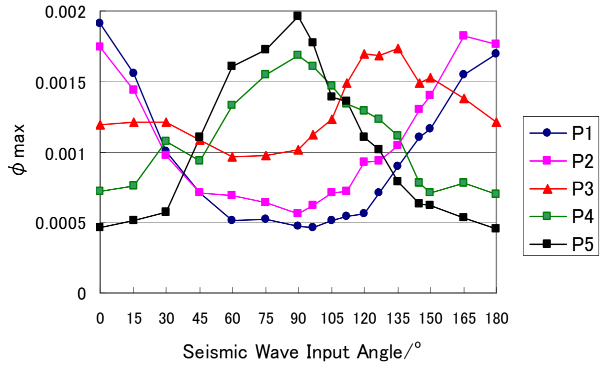

Figure 9 Critical angle results of each pier in time-history analysis

The directions of input angle of earthquake wave adopted in Pushover method analysis are the same as that in time history analysis. The static loads take 100 steps, which are horizontal distribution loads with mass promotion. The results of Pier P1 gained from the Pushover method analysis are listed in Table 1.

Table 1 Bottom curvature of Pier 1 according to Pushover-based method

Pier	P1	
Input angel α	φ_x	φ_y
0	0.00915	0.00039
15	0.00801	0.00089
30	0.00544	0.00149
45	0.00215	0.00111
60	0.00049	0.00044
75	0.00020	0.00041
90	0.00001	0.00032
103	0.00023	0.00046
120	0.00057	0.00049
135	0.00391	0.00203
150	0.00707	0.00225
165	0.00889	0.00150
180	0.00911	0.00038

While utilizing Eqn. 2.14 to Eqn. 2.17 to calculate the critical angle of seismic wave input α_0, it needs the results in two perpendicular direction α_1 and α_2. Take Pier P1 for example, get an input angle α from Table 1, and fetch the corresponding curvature component as φ_{1x} and φ_{1y}. Then, get another input angle perpendicular to α from Table 1, and fetch the corresponding curvature component as φ_{2x} and φ_{2y}. After that, the critical angle of curved bridges with the certain input angle α could be calculated according to Eqn. 2.14 to Eqn. 2.17. Through trial calculation, influence coefficient β to Pier P1-P5 are respectively set as 0.5、1.0、2.0、1.0、0.5.
The yield curvature parameter φ at the bottom of pier should be set according to the upper structure weight load. Two yield curvature parameters are considered due to structure weight load difference. One yield curvature parameters φ_0 equals to 0.0006 and the other 0.0012. And the results of critical angle based on these two yield curvatures are listed in Table 2.

Table 2 Critical angle calculated of Pier P1 with yield curvature 0.0006 and 0.00012(°)

Input Angle (α_1 , α_2.)	critical angle with φ_0 0.0006	critical angle with φ_0 0.00012
(0，90)	2	3
(15，105)	5	6
(30，120)	9	12
(45，135)	20	9
(60，150)	11	9
(75，165)	6	5
(90，180)	2	2

According to Eqn. 2.14 and 2.17, the results of critical angle with the arbitrary two data sets taken out from Table 1 should be same and constant. That is to say, the results are independent from input angle α_1 and α_2. But comparing the results in Table 2, the critical angle to the same pier (one column in Table 2) is not identical no matter with the yield curvature parameters. This is inconsistent to the conclusion mentioned above. This inconsistency is due to the insufficiency of upper structure stiffness while simplifying the model. Thus, the ultimate critical angle is an average to the critical angles calculated under different seismic wave input direction (one column in Table 2).
With different yield curvature parameter φ, results also present different discreteness, because yield curvature is the key parameter for judging whether the structure comes into plastic status or not. And its value is related to the load in vertical plane. If the load in vertical plane could not be neglected, the yield curvature parameter will influence accuracy of results.

Table 3 Critical angle results comparison between time history analysis and formula calculation with the two different yield curvature(°)

Pier	P1	P2	P3	P4	P5
Angle based on time history analysis	0	165	135	90	90
Angle calculated with φ_0 0.0006	8	166	163	82	81
Angle calculated with φ_0 0.00012	6	161	148	99	87

Compared with Figure 9, the results from Pushover method and time history analysis presents basically identical with the maxim error of 15 degree. For further observation to the consistency of results from Pushover method and time history analysis, take average value for the critical angle as mentioned above. Then, it is found that the error could be controlled in 13 degree while yield curvature parameter φ is 0.0012.

Through the example, given two perpendicular wave input direction, the critical angle could be gained with non-liner Pushover method results according to the formula mentioned above. And the formula is accurate to the degree, compared with the results of dynamical time history analysis. So it can satisfy the engineering demand.

4. CONCLUSIONS

This paper presents a formula to calculate the critical angle on the base of relation between single pier curvature and displacement under bidirectional seismic waves input. Through a case of curved bridge, the critical angle calculated is compared to the results of dynamical time history analysis.

(1) The critical angle could be found apparently based on the curvature at the bottom of single pier in the seismic response of curved bridges. The input direction of seismic wave has great influence on dynamic response of the curvature as other generalized displacement.

(2) The formula as Eqn. 2.14 and Eqn. 2.17 are gained in this paper to calculate the critical angle to seismic response of curved bridges while rubber Bearing and the same height pier is adopted. The yield curvature has influence on the critical angle. Meanwhile, different piers interplay each other much less on the critical angle.

(3) The method in this paper is based on seismic response of single pier curvature and Pushover method. It can simplify the calculating course of critical angle to of curved bridges. Compared to time history analysis, this method could shorten computing time greatly just with sacrificing a little of results precision, which makes great sense in practical project.

REFERENCE

Williams, D. and Godden, W. (1979). Seismic response of long curved bridge structures: experimental model studies. *EESD* **7:2,** 107-128.

Penzien, J. and Watabe, M. (1975). Characteristics of 3-dimensional earthquake ground motions. *International Journal of Earthquake Engineering & Structural Dynamics* **4:3,** 365-373.

Wilison, E.L. and Button, M.R. (1982). Three-dimension dynamic analysis for multi-component earthquake spectra. *EESD* **10,** 471-476.

Feng, Y.T., Li, M.R. and Lin, C.Z. (1991). Elastic earthquake response analysis for complex structures. *Earthquake Engineering and Engineering Vibration* **11:4,** 77-85.

Gao, X.A. and Zhou, X.Y. (2005). Multi-component seismic response analysis for curved bridges. *Special Structures* **22:1,** 56-59.

Zhu, D.S., Liu S.Z. and YU, L.S. (2002). Research on seismic response of curved girder bridges. *China Journal*

of Highway and Transport **15:3,** 42-48.
Zhu, D.S., Liu, S.Z. and YU, L.S. (2000). The study of earthquake input principal direction for irregular bridges. *Journal of Lanzhou Jiaotong University* **19:6,** 37-40.
Fan, L.C., Nie, L.Y. and Li, J.Z. (2003). Discussion on standard of critical angle of seismic wave in seismic analysis of complicated structures. *Journal of Tongji University* **31:6,** 631-636.
Nie, L.Y., Li, J.Z. and Fan, L.C. (2003). Principle and effect of critical angle of complicated structure in dynamic analysis. *Journal of Earthquake Engineering and Engineering Vibration* **23:3,** 37-41.
Zhou, Y. and An, Y. (2001). Research and development of performance-based seismic design theory and method. *World Earthquake Engineering* **17:2,** 1-7.
Qian, J.R. and Luo, W.B. (2001). Displacement Based Seismic Design Methodology for Building Structures. *Building Structure* **31:40,** 3-6.
Liu, C.G. and Lin, G.. (2005). Research on push-over method for earthquake-resistant performance of bridge structure. *Journal of Dalian University of Technology* **45:3,** 395-400.
Xu, X.G., Tang, G.W. and Zheng, G. (2005). Application of Push-over Analytic Method to Seismic Resistance Performance Evaluation of Double Column Piers. *Technology of Highway and Transport* **6:3,** 88-93.
Cui, G.H. (2003). Studies on Push-over Method of Bridge Structure [D]. Beijing: Institute of Engineering Mechanics China Earthquake Administration.
Ala Saadeghvaziri M. and Yazdani-Motlagh, A.R. (2007). Seismic behavior and capacity/demand analyses of three multi-span simply supported bridges. *Engineering Structures* **17:2,** 106-119.