ADVANCEMENT IN MODELING OF RC SHEAR WALLS

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ABSTRACT :

Reinforced concrete (RC) shear walls are considered as effective lateral force resisting system that has been widely used in the last decades. RC walls can provide the required lateral stiffness and strength for resisting the lateral loads due to wind or earthquakes. Hence, several experimental and analytical studies were conducted to investigate the behaviour of RC shear walls under the lateral loads in order to enable the designers to predict their seismic response in a building when subjected to a severe ground motion. Predicting the behaviour of RC walls under lateral loads requires enhanced numerical tools that are calibrated using controlled experimental tests. These tools should take into account most of the important factors that could affect the response of RC walls. Hence, modeling of RC walls involves several challenges in representing the combined effects of moment, shear and axial forces, in addition to bar slip, buckling, damping, boundary conditions, as well as rehabilitation method, if any. This paper presents different modeling techniques that have been used by researchers in modeling of RC shear walls. These range from macro-models such as lumped plasticity, multi-axial spring models, combined models, up to micro-models such as finite element models and fibre models. The paper discusses the efficiency of each model in representing both the global and local behaviour of RC shear walls. The objective of this paper is to provide a state-of-the-art on the recent advancements and challenges in the area of modeling of RC shear walls.

KEYWORDS: Reinforced concrete, shear walls, micro-modeling, macro-modeling, hysteretic models.

INTRODUCTION

RC walls are classified according to CSA A23.3 (2004) as bearing walls, non-bearing walls, shear walls, flexural shear walls, and squat shear walls. Shear walls are part of the lateral force resisting system that carry vertical loads, bending moments about the wall strong axis, and shear forces parallel to the wall length. Shear wall system is one of the most common and effective lateral load resisting systems that is widely used in medium- to high-rise buildings. It can provide the adequate strength and stiffness needed for the building to resist wind and earthquake loadings, provided that a proper design is considered, that cares for both the wall strength and ductility. During the recent years, an enormous effort has been done to provide analytical models that are able to simulate the actual behaviour of RC elements including shear walls. The rapid increase in the computational efficiency of computers helped the researchers to develop more sophisticated models that can account for several phenomena of RC shear walls that were used to be ignored in the analysis due to their complexity. For these models to be verified, experimental research is continuously conducted on RC shear walls tested under monotonic, cyclic, or dynamic loading. The numerical modeling of RC walls is not involved only in the applications for new construction, but it is also extended to the applications of retrofitting of existing structures. In that case, it is important to construct a representative model that is able to evaluate the expected response of an existing RC shear wall under certain lateral load hazard, and to predict its expected mode of failure in order to be able to choose the most suitable and effective retrofitting technique for that wall that would meet a target performance.

The numerical modeling of RC elements started by Clough et al. (1965) when they proposed the first nonlinear macro-model, and by Ngo and Scordelis (1967) who proposed the first application of the finite element method of analysis in RC elements. Since then several advancements were done in the area of modeling of RC elements including shear walls. The objective of this paper is to present the different numerical models proposed by researchers for the analysis of RC shear walls. These ranged from macro-models such as one- and two-component elements, multi-axial spring model, truss models, combined models, up to micro-models such as the finite element models and fibre models. The paper provides a state-of-the-art on the recent advancements and challenges involved in the area of modeling of RC shear walls.

1. FACTORS THAT AFFECT THE RESPONSE OF RC SHEAR WALLS

The behaviour of shear walls is primarily affected by a combination of flexural, shear and axial deformations. Medium- to high-rise wall behaves mostly in a flexural manner, while low-rise walls are controlled mainly by the shear deformations. The previously conducted tests on RC shear walls showed that their nonlinear response may vary according to several factors. Some of these factors are:

1- The wall dimensions and its aspect ratio.
2- The axial load level applied on the wall (axial-flexure interaction).
3- The amount of wall reinforcement and the bond between the reinforcement and concrete.
4- The wall flexure capacity relative to the wall shear capacity.
5- The rigidity of wall foundation and the interface between the wall and its foundation.
6- Rocking of the wall about its foundation due to slippage of vertical reinforcement from the foundation (rigid-body rotation).
7- The dimensions and reinforcement of the wall boundary columns if applicable.
8- The effect of the structural elements connected to the wall (e.g. coupling beams, moment resisting frame, etc.).

Therefore, modeling of RC shear walls should take into account the previous factors, especially for the axial-flexure interaction and the representation of the wall boundary conditions, in order to simulate the wall behaviour efficiently. The analytical model should be able to estimate the monotonic capacity of the wall, as well as its behaviour under reversed cyclic loading. The ideal numerical model should also be able to represent other phenomena like concrete cracking, stiffening in tension, opening and closing of cracks with recovery of stiffness, strength degradation with cyclic loading, confinement effects in compression, etc. In most cases, one or more of these factors are neglected in the analytical model for simplicity, provided that this approximation would not lead to a significant impact on the model accuracy in simulating different behaviours of RC walls.

2. MICRO-MODELING VERSUS MACRO-MODELING

The two main approaches for modeling of RC members are micro-modeling and macro modeling. Micro-modeling such as the finite element analysis or fiber analysis is based on representing the behaviour of different materials that compose the RC element and the interaction between them. The member is discretized into small elements and principles of equilibrium are applied. This approach is complex and needs high numerical processing efforts, and hence it might not be practical for large structures and it is limited to model individual structural components such as a column, a beam or a wall. On the other hand, macro-modeling is based on representing the overall behaviour of the RC element, such as the wall deformations, strength, and energy dissipation capacity. The global behaviour of the RC element using a macro-model should be calibrated using an experimental verification to adjust the parameters needed for the model. This approach is simple and does not require high numerical efforts, which makes it suitable to simulate the response of large structures.

3. HYSTERETIC MODELS

The cyclic behaviour of RC shear walls should be defined using a hysteretic model that is able to simulate different inelastic phenomena of reinforced concrete materials. The modeling of the hysteretic behaviour of the RC element can affect the element response significantly (Anderson and Townsend 1977). These models can be used to represent the axial, flexure and shear behaviour of the element. The hysteretic model consists of a primary curve (backbone curve) that control the monotonic loading and some hysteresis rules that control the loading and unloading element behaviour under cyclic loading. The control parameters of the hysteresis rules are adjusted to simulate the actual cyclic behaviour of the tested wall. In case of wall modeling using concrete and steel springs, the behaviour of the element is controlled by the constitutive laws predefined for the model springs. Appropriate definition of the hysteretic behaviour of the RC wall is an important issue, and a special attention should be directed to the interaction of the hysteresis models used for different element springs (e.g. flexure and shear springs). Several improvements in the hysteretic models were accompanied by the advancements in the wall modeling, and that will be discussed in the following sections.

4. MICRO-MODELING OF RC SHEAR WALLS

4.1 Finite element method of analysis

The most general technique that is used for simulating the behaviour of RC elements using micro-modeling is the finite element method (FEM) of analysis. In this method, the reinforced concrete member is discretized into a finite number of small elements (concrete and steel elements) interconnected at a finite number of nodal points. The number of the finite elements is chosen according to the level of accuracy required and the available analysis tool. The FEM of analysis is capable of tracking the member's global behaviour (e.g. member forces and displacements) in addition to its local behaviour (e.g. crack pattern, material stresses and strains). The first FE model used for RC element was proposed by Ngo and Scordelis (1967). The proposed two-dimensional linear model used constant strain triangular (CST) finite elements to model the concrete and steel elements, linkage elements were used to represent the bond between steel and concrete elements, and the effect of cracking was included in the model. Since then, the FEM has become a powerful tool for the analysis of RC structures, including three-dimensional and nonlinear analysis. For this purpose, many finite element analysis softwares were developed and used by researcher, such as; ANSYS (Desalvo and Swanson 1983), ABAQUS (Hibbitt 1984), VecTor 2 and 3 (Vecchio 1989), ADINA (1992).

4.2 Fibre (layer) model

In this model, the member is divided longitudinally into several segments, and each segment consists of parallel layers. Some layers would represent the concrete material and other layers would represent the steel material. In other type of models, each single layer was divided into a finite number of fibres as shown in Figure 1 (a). The constitutive laws for concrete and steel materials are defined, and hence the moment-curvature relationship of the member can be calculated at each load level. This model accounts for the distribution of flexibility along the member length and the axial-flexure interaction. Park et al. (1972) used the fibre model to represent a RC member under cyclic loading. Emori and Schnobrich (1981) used this model for RC column members and they found that it showed the detailed behaviour of the inelastic zone of the column. Monti and Spacone (2000) accounted for the bond-slip of the reinforcement bars in the fibre-section model (Figure 1[a]). Recently, this model was used by Kotronis et al. (2005) to simulate the behaviour of RC shear walls under dynamic excitations. The assumption of linear shear deformations, the complexity of simulating the boundary conditions, and neglecting the effect of bond slip were the limitations of this model. Belmouden and Lestuzzi (2007) used the layer model to predict the nonlinear behaviour of the RC shear walls under reversed cyclic loading based on the tests conducted at the Swiss Federal Institute of Technology (ETH) in Zürich, Switzerland (Dazio et al. 1999). Nonlinear shear behaviour and effect of bar slip were considered in the wall analysis. Figure 1(b) shows the layer model used by Belmouden and Lestuzzi (2007).

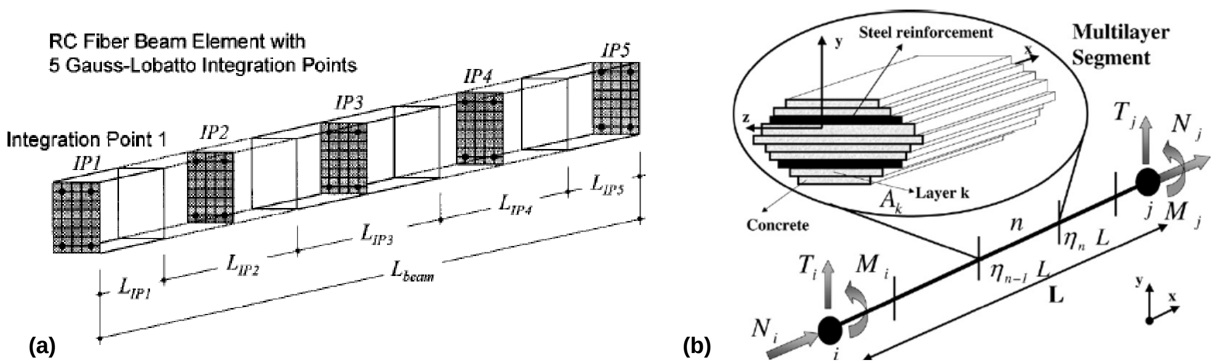


Figure 1 (a) Fibre beam element proposed by Monti and Spacone (2000), (b) Multi-layer finite element model (Belmouden and Lestuzzui 2007)

5. MACRO-MODELING OF RC SHEAR WALLS

5.1 Two-component beam-column element

The beam-column element was the first element to be used for modeling of RC shear walls and the wall members of coupled shear walls. The two-component element was the first nonlinear beam-column model that was used for structural analysis of a reinforced concrete element. The model developed by Clough et al. (1965) consisted mainly of two parallel components; one was fully elastic and the other was perfectly elasto-plastic as shown in Figure 2. The two components were able to represent the material yielding (elasto-plastic behaviour) and the strain hardening (elastic behaviour). The nonlinearity of this model was represented uniformly along the entire member length. The main problem of this model was its inability to represent the element stiffness or strength degradation with cyclic loading. This model was improved by Takizawa (1976) to be able to simulate different hysteretic behaviour of RC elements by using appropriate hysteresis models (general two-component model).

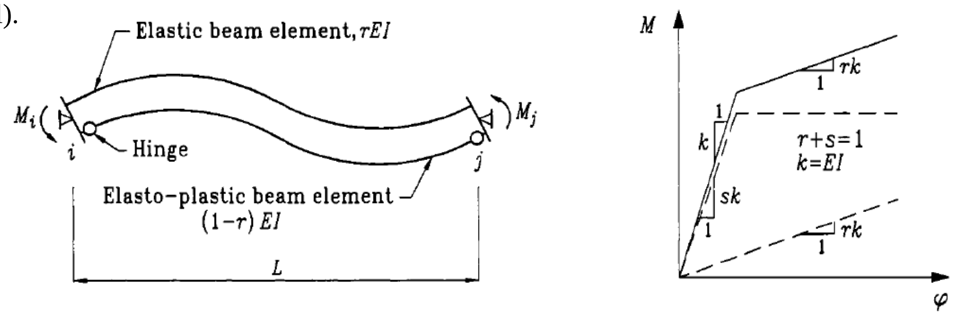


Figure 2 Two-component element model.

5.2 One-component beam-column element

This element which was developed by Giberson (1967) consisted of one linear elastic member with two nonlinear rotational springs at the two member ends as shown in Figure 3. The member's nonlinear deformations were assumed to be lumped at the zero-length end springs (lumped plasticity). For this model, the deformed shape was assumed to have a double curvature with a fixed point of contraflexure at the middle of the member, and the plain sections were assumed to remain plain. The one-component model and the general two-component model need an appropriate hysteretic load-deformation (or moment-curvature) models to be defined. This requires definition of different properties of the member's plastic hinges such as stiffness, strength, ductility, cyclic behaviour, etc., which may be difficult to be defined unless some assumptions were made.

The simple bilinear elasto-plastic model was the first nonlinear hysteretic model to be used. The earlier hysteretic models did not directly consider the nonlinear shear deformations, which made these models unsuitable for walls dominated by the shear mode of failure. Hence, several enhancements were done to these models in order to better simulate different characteristics of RC elements. Takeda et al. (1970) proposed a trilinear force-displacement primary curve to account for cracking, yielding and strain hardening of the concrete element. The stiffness degradation was considered in the seven-condition hysteretic model of the element.

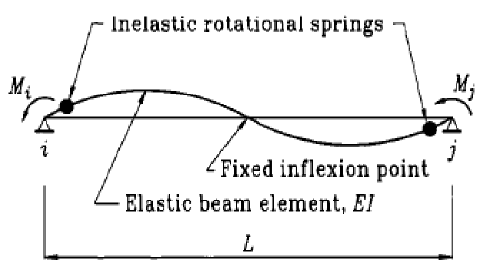


Figure 3 One-component element model.

The model was used to evaluate the dynamic behaviour of a single-degree-of-freedom system. Otani (1974) accounted for the effect of the bond slippage of the reinforcing steel in the Takeda hysteretic model. Anderson and Townsend (1977) proposed a trilinear hysteretic model to analyze multi-degree-of-freedom systems. They compared different models and they found that the most representative model is the degrading trilinear connection model (one-component model with trilinear degrading hysteretic behaviour). Soleimani et al. (1979) enhanced the one-component element model by having a variable length of the inelastic zone at the member's ends instead of the zero dimension inelastic zones that were previously assumed. Saatcioglu and Derecho (1980) included the effect of axial-flexure interaction in the hysteretic behaviour of coupled shear walls by shifting the element's primary curve with the increase of axial load level as shown in Figure 4. Takayanagi et al. (1980) added two nonlinear rotational shear springs at the ends of the one-component element in order to account for the shear deformations. Recently, the one-component element model was used by Tremblay et al. (2001) and Panneton et al. (2006) to investigate the higher mode effects on the behaviour of high-rise RC walls. In both studies, the modified Takeda hysteretic model (Otani 1974) with degrading stiffness was used to model the inelastic behaviour of the rotational springs (Figure 5), while the shear deformations were assumed to be linear.

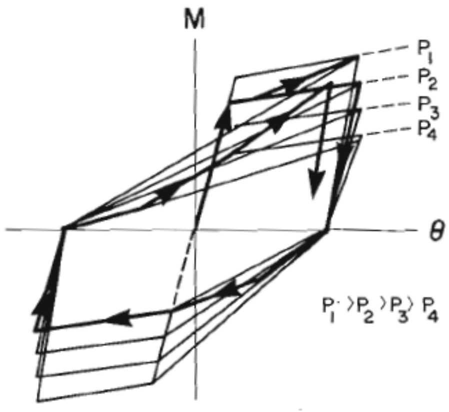


Figure 4 Hystersis loops with effects of changing axial force (Saatcioglu and Derecho 1980).

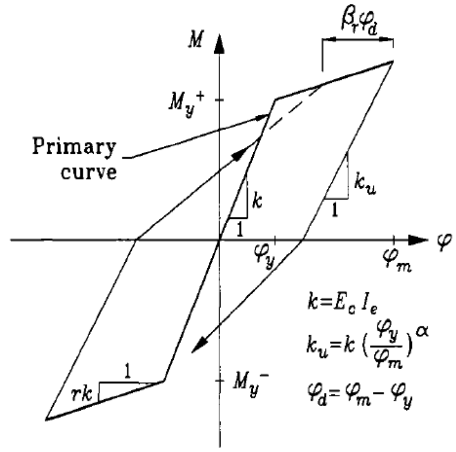


Figure 5 Modified Takeda hysteretic model (Otani 1974).

5.3 Multiple spring model

This model was proposed by Takayanagi and Schnobrich (1976). The multiple spring model consisted of a number of inelastic springs that are connected in series using rigid members as shown in Figure 6. The inelastic properties of each spring were varied according to the segment properties and the level of axial load on that segment, however the segment properties were assumed to be constant along the segment length. The model was used to represent the behaviour of coupled shear walls, while the coupling beams were modeled using one-component elements. This model was used by Emori and Schnobrich (1981) to model the shears wall of a 10-storey frame-wall building. Linear shear deformations were assumed in the analysis. The models were found to satisfactorily represent the nonlinear behaviour of the studied structure.

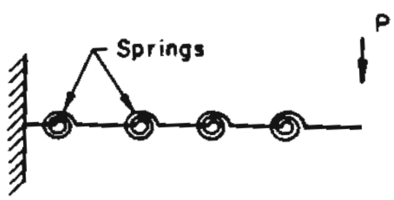
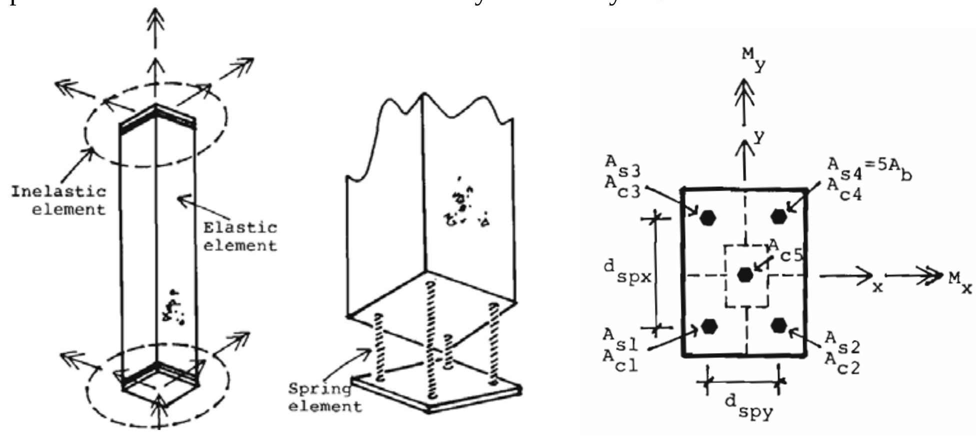


Figure 6 Multiple spring model (proposed by Takayanagi and Schnobrich 1976).

5.4 Multi-axial spring model (MS model)

This model was proposed by Lai et al. (1984) to simulate the axial-flexure interaction of RC columns. The proposed model consisted of an elastic linear member with two multi-axial spring elements (MS elements) of zero dimensions located at the two member ends as shown in Figure 7. The MS element consisted of 5 concrete and 4 steel springs, each spring was assumed to be uniaxially stressed and its behaviour was governed by the hysteretic stress-strain characteristics of the simulated material (concrete or steel). The main input for this model was the material (concrete/steel) constitutive laws rather than the load-deformation relationship of the whole member. Multi-linear curves were used to represent the stress-strain or (force-deformation) relationship for concrete and steel springs. The spring deformations were conformed to the plane section assumption. The MS element was simplified by Jiang and Saiidi (1990) to have only 4 composite springs, each spring represented the combined behaviour of concrete and steel materials. The behaviour of the composite springs was defined using unsymmetric load-deformation hysteresis model. Li and Otani (1993) increased the number of springs in the MS element to 16 concrete and 9 steel springs. They differentiate between the core concrete and shell concrete properties to account for the effect of concrete confinement. They reported that the higher number of springs would lead to a higher accuracy, and in that case the time needed for computation did not increase significantly.

This model has been extended by increasing the number of springs to be used for the analysis of RC walls. The MS model was used by Galal (2008) to investigate the response of RC walls when subjected to lateral loads. The analytical model was verified using the shake table dynamic tests of CAMUS walls (Combescure 2002). In this study the RC walls were modeled using CANNY wall element (Li 2006) which is based on the MS model. This model allows accounting for axial-flexural interaction, concrete cracking and stiffening in tension, confinement effect in compression, and other phenomena that are related to RC walls. The model includes a nonlinear shear spring that controls the shear deformations of the wall. The analytical model was efficient in representing the experimental results for both the static and dynamic analyses.



(a) (b) (c)

Figure 7 Multi-axial spring model by Lai et al. (1984): a) Member model, b) Inelastic element, c) Effective concrete and steel springs.

5.5 Truss models

The truss model has been used in evaluating the shear capacity of RC structural members such as deep beams or shear walls. This model assumed that the wall will act as a statically determinate truss. The model consists of diagonal concrete compression struts, horizontal tension ties (representing the shear reinforcement), and two boundary elements at the wall ends to carry the moment acting. Figure 8(a) shows the truss model used by Oesterle at al. (1984) for analysis of shear response of RC shear walls. Other models based on the same analogy were used to calculate the capacity of RC walls, such as the Softened-Strut-and-Tie model shown in Figure 8(b). The model was used by Yu and Hwang (2005) to predict the shear capacity of RC squat walls. It is worth noting that, although such models are able to predict the capacity of RC elements, they can not capture the cyclic or the hysteretic behaviour of these elements.

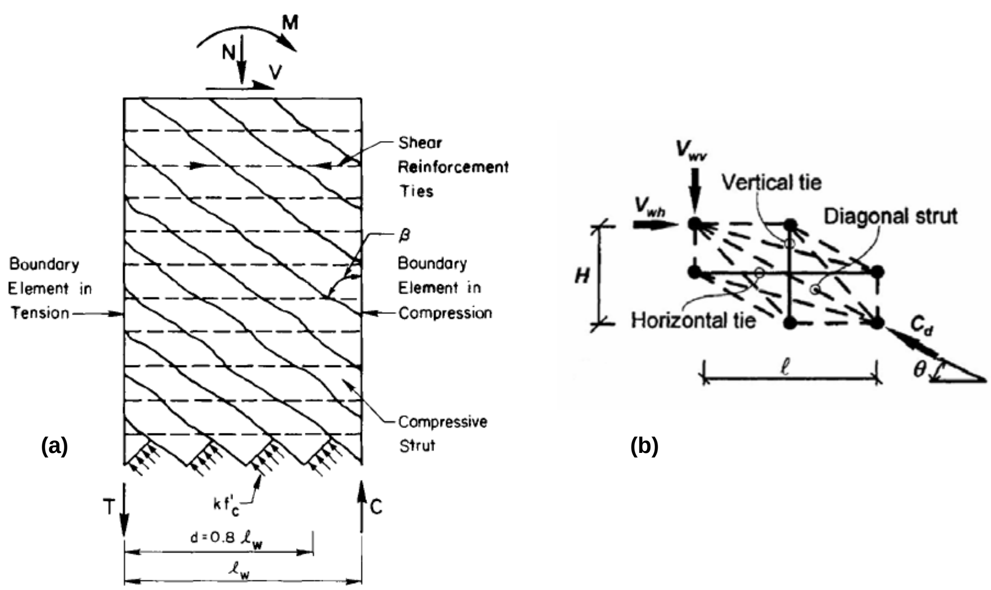


Figure 8 (a) Truss model used by Oesterle et al. (1984), (b) Softened-Strut-and-Tie model (Yu and Hwang 2005).

5.6 Combined models

5.6.1. Three Vertical Line Element (TVLE) model

This model was first introduced by Kabeyasawa et al. (1982). The model consisted of three vertical line elements connected to each other by rigid bars at the top and the bottom wall ends, two edge links with axial springs representing the boundary elements, and the central one-component element with three springs to control the vertical, horizontal, and rotational deformations of the wall as shown in Figure 9. The main problems about this model were the lack of deformation compatibility between the wall and the boundary elements, and the difficulty in defining the properties of the springs. This model was modified later by removing the rotational spring at the central element, and by providing coupling between the axial-flexure behaviour. The modified model was used by Kunnath et al. (1990), Linde and Bachmann (1994) and by Kim and Foutch (2007) to simulate the behaviour of RC shear walls.

Another modification was done by Vulcano and Bertero (1986) to reduce the complexity in defining the hysteretic properties of the model springs. The axial spring of the boundary element was replaced by two axial elements connected in series named as axial-element-in-series model (AESM) as shown in Figure 10. The upper element is a one-component element that represent the axial stiffness of the boundary element where the bond between steel and concrete still exists, while the lower element is a two-component element (steel and concrete springs) that represent the axial stiffness of the boundary element where the bond is lost. This model should simulate the actual hysteretic response of the materials and their interaction (e.g. concrete cracking, bond deterioration, etc.). Although the model was able to predict the flexural behaviour of the tested wall that was dominated by flexural failure, it could not simulate the actual shear deformations of the wall, which indicates that this model is not suitable for walls dominated by the shear deformations. In this model the deformation compatibility between the wall and the boundary element was still not enforced.

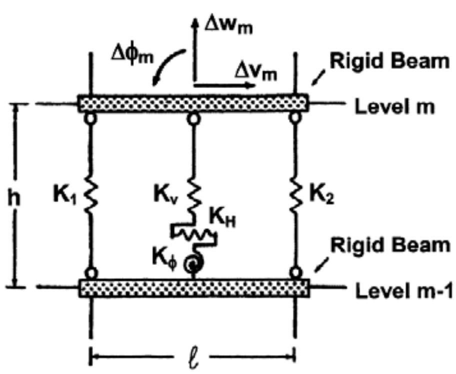


Figure 9 Three Vertical Line Element model (Kabeyasawa et al. 1983)

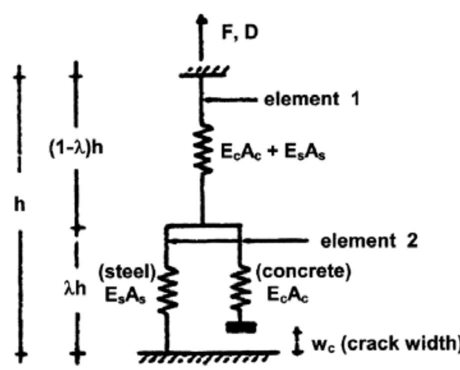


Figure 10 Axial-element-in-series model (Vulcano and Bertero 1986).

5.6.2. Multiple Vertical Line Element (MVLE) model

This model was first introduced by Vulcano et al. (1988). In this model (Figure 11), the wall element was represented by a number of uniaxial elements connected in parallel using infinitely rigid bars located at the top and bottom wall ends; two external elements simulates the wall boundary elements, while the other elements simulates the combined axial-flexure behaviour of the central panel. A horizontal spring was used to represent the inelastic shear behaviour of the wall. The authors modified the axial-element-in-series model (AESM) by having two-component model for element 1, representing the cracked concrete and steel reinforcement behaviour, instead of the one-component element in the original model as shown in Figure 12. The constitutive laws for concrete (cracked and uncracked) and steel elements were defined to describe the hysteretic response of the materials. It was concluded that the model predicted the flexural behaviour of the wall efficiently even when relatively few uniaxial elements were used (4 elements). It is worth noting that, although the proposed model considered both flexural and shear behaviour, but their responses were not coupled. Colotti (1993) modified the MVLE model to include the interaction between axial and shear responses, which led to a more accurate simulation. Simpler constitutive laws and some modifications to the MVLE were introduced by Fischinger et al.

(1990) and by Orakcal and Wallace (2006) to improve the efficiency of the model in predicting the response of RC shear walls without sacrificing the accuracy.

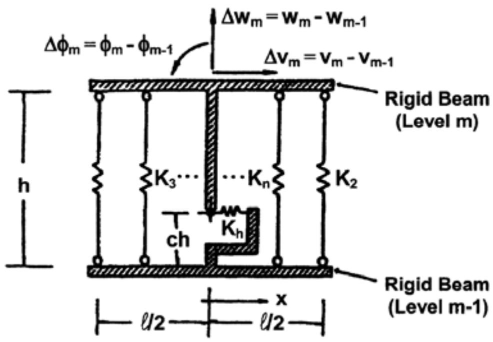


Figure 11 Multiple vertical line element model (Vulcano et al. 1988).

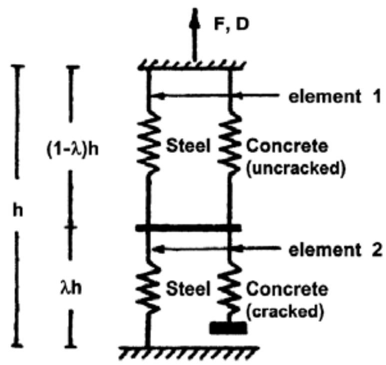


Figure 12 Modified axial-element-in-series model (Vulcano et al. 1988).

SUMMARY

Different modeling techniques that have been used by researchers in modeling of RC shear walls were discussed. These ranged from macro-models such as lumped plasticity, multi-axial spring models, truss models, combined models, up to micro-models such as finite element models and fibre models. The efficiency of each model in representing both the global and local behaviour of RC shear walls was summarized. The paper discussed different factors that affect the response of RC shear walls and should be taken into account when a numerical model is developed. Different hysteretic models used to simulate the behaviour of RC shear walls were also discussed. The paper provided a state-of-the-art on the recent advancements and challenges in the area of modeling of RC shear walls.

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