FORCE-BASED VERSUS DISPLACEMENT-BASED FORMULATIONS IN THE CYCLIC NONLINEAR ANALYSIS OF RC FRAMES

A.A. Correia[1], J.P. Almeida[1] and R. Pinho[2]

[1] *PhD Student, ROSE School, Pavia, Italy*
[2] *Assistant Professor, Structural Mechanics Department, University of Pavia, Italy*
Email: aaraujo@roseschool.it, jpacheco@roseschool.it, rui.pinho@unipv.it

ABSTRACT:

The current work deals with the comparison between the numerical results of two different frame element formulations, namely the classical displacement-based element versus the force-based approach. In the former an approximation is made for the displacement field throughout the frame element length, from which strains, stresses and stress resultants are computed. The fact that this displaced shape is only approximate is responsible for most of the problems that these elements present when inelastic behaviour is expected.

On the other hand, force-based elements satisfy exactly the equilibrium conditions by using an exact description for the stress resultants' field throughout the frame element length. It implies a somewhat more complex iterative procedure to solve nonlinear problems. Nevertheless, this increase in complexity is clearly justified by the overwhelming benefit that the force-based solution holds for any material response. From the earthquake engineering analysis viewpoint, where large inelastic demands are expected, such characteristic plays a major role on behalf of the flexibility approach, as will be subsequently observed.

To validate and evaluate both formulations, numerical and experimental results of cyclic tests on bridge piers are compared, including strain localization phenomena. The superiority of force-based formulations in comparable conditions is established.

KEYWORDS: force-based, displacement-based, frame element, reinforced concrete, distributed inelasticity, localization

1. INTRODUCTION

Historically, the vast majority of finite elements used for the nonlinear analysis of framed structures have been based on displacement formulations (also known as stiffness formulations). These finite elements rely on an approximation for the displacement field throughout the frame element length, from which strains, stresses and stress resultants are computed. The easiness of implementation and the need of an iterative process only at the structural level are the main advantages for using these stiffness-based formulations in the nonlinear analysis of structures. Given the approximate nature of the displacement interpolation functions, the displacement field on the element is exact only if the frame element is prismatic, with linear elastic behaviour, and the loading consists only of nodal loads. If any of these premises is not satisfied, the results obtained by this formulation will be stiffer than the real ones. Hence, the advantages of displacement-based formulations tend to disappear when inelastic behaviour is modelled. This issue is of special relevance in the seismic analysis of structures.

Inversely, force or flexibility-based formulations use an approximation for the stress resultants' field throughout the frame element length, which strictly satisfies equilibrium conditions and is exact, independently of the nonlinearity in the material behaviour. Hence, the difficulties arising in displacement-based formulations are inexistent in this framework. The main disadvantage of this approach is the need of a three-level iterative procedure: structure, element and cross-section. However, recent work has shown that this iterative procedure can be transformed in a two level or even a single level iterative procedure, without loss of accuracy [Neuenhofer and Filippou, 1997]. But even this issue of a smaller computational cost of the displacement-based formulations is opposed by the need of adopting more elements per frame element in order to obtain similar results to the force-based formulations. Another advantage of the latter is the easiness with which span loading may be considered.

To validate and evaluate both formulations, the theoretical background of which is not thoroughly presented due to space limitations, numerical and experimental results of cyclic tests on bridge piers are herewith compared, including strain localization phenomena. The superiority of force-based formulations in comparable conditions is established.

2. COMPARISON BETWEEN THE FLEXIBILITY AND STIFFNESS APPROACHES

2.1. Experimental Results

The validation of both formulations was carried out through the comparison of the numerical response estimates with experimental results from the Kawashima Laboratory of the Tokyo Institute of Technology. Several results of experimental tests for the study of the cyclic behaviour of bridge piers are available at the website of the Kawashima Laboratory (http://seismic.cv.titech.ac.jp). These experiments involved the simultaneous application of vertical and horizontal loads to reinforced concrete specimens. Figure 1 depicts the experimental set-up and the corresponding simplified structural model.

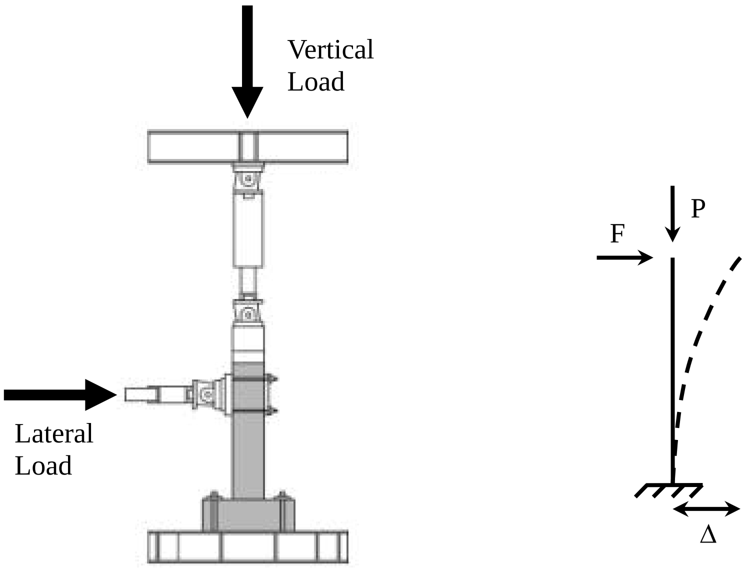


Figure 1 Experimental set-up and simplified structural model.

The experimental specimen used for the present work is identified with the number TP-011; the general geometrical characteristics, as well as the reinforcement detailing, are presented in Figure 2. The cylinder strength of concrete is 20.6 MPa and the yield strength of the longitudinal reinforcement is 367 MPa. The vertical load is constant and equal to 160 kN, while the history of imposed lateral displacements is indicated in Figure 3; such displacements take into account the footing sliding and rotation. Within the available results we can also find the equivalent lateral force (which produces the same moment in the base of the pier than the real loads, considering the footing rotation and slide, as well as the second order effects). Figure 3 also includes the curve lateral equivalent force – lateral drift.

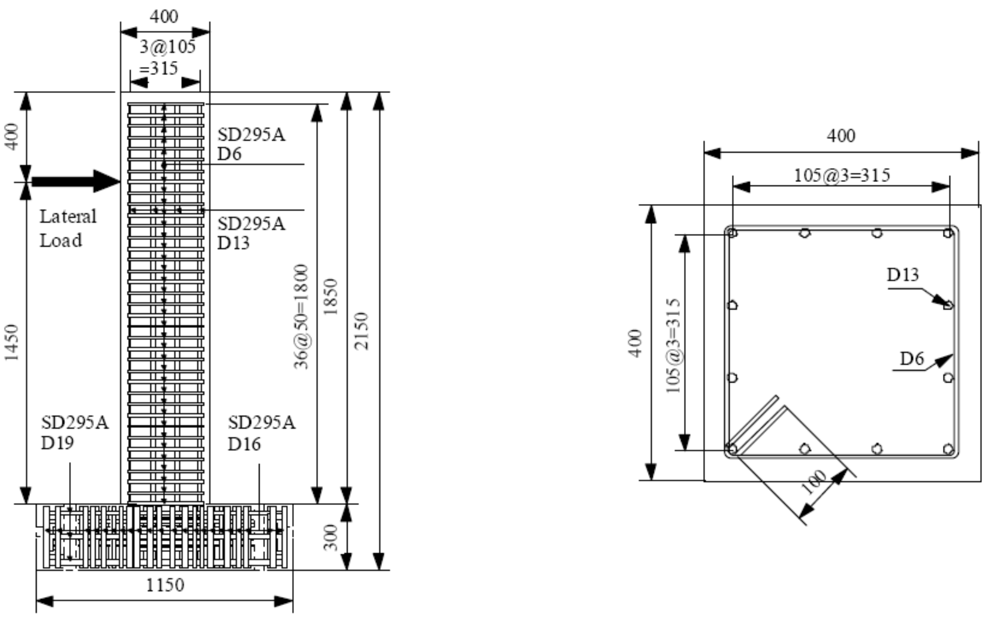


Figure 2 Geometrical characteristics and reinforcement detailing (in mm).

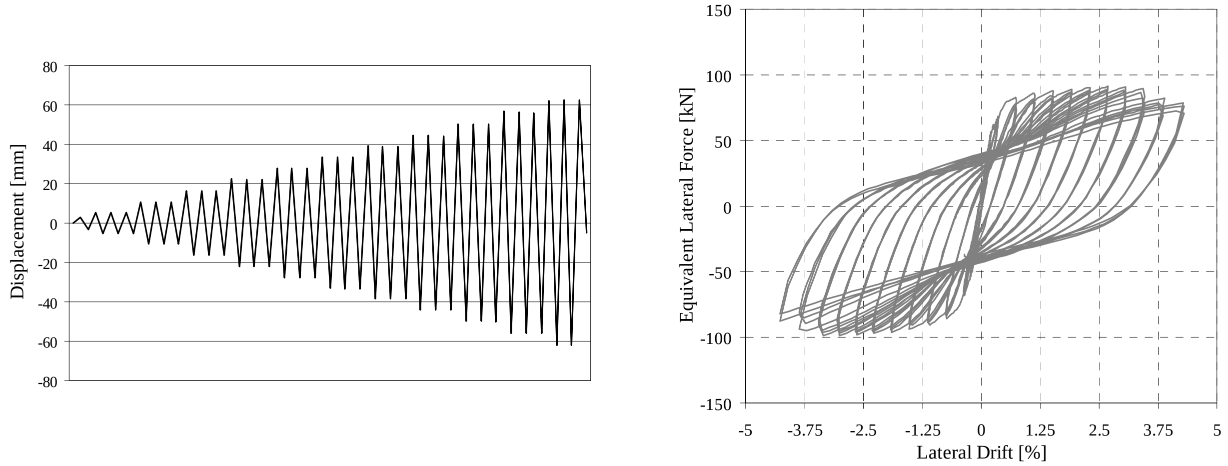


Figure 3 History of imposed lateral displacements and history of equivalent lateral force vs. lateral drift.

2.2. Comparison between Experimental and Numerical Results

The numerical model was developed with the same geometrical and loading characteristics previously presented. The 1.45m height pier was modelled by two finite elements, the bottom one having 0.45m. Two integration sections per element were used (Gauss quadrature), each one containing around 250 integration points.

In what concerns the constitutive relations, for the concrete in compression the well known model of Mander et al. [1988] was adopted, with the improvements later introduced by Martínez-Rueda and Elnashai [1997]. A linear behaviour for the concrete in tension was assumed, followed by an abrupt reduction after the tension resistance. This latter parameter was taken as $f_{t\,0} = 0.34\, f_{c\,0}^{\;1/2}$, where $f_{c\,0}$ is the unconfined concrete strength in compression ([Vinagre, 1997], [Lin and Scordelis, 1975]). The concrete Young's modulus is estimated, according to Priestley et al. [1996], as $E_{c\,0} = 4700\, f_{c\,0}^{\;1/2}$. In order to account for the effect of confinement due to the presence of stirrups, the compression strength and the corresponding strain were modified through the following confinement factor (k_c):

$$f_{cc} = k_c \; f_{c0}$$
$$\varepsilon_{cc} = \varepsilon_{c0} \left[1 + 5\left(k_c - 1\right)\right] \tag{2.1}$$

The unconfined concrete strain (ε_{c0}) corresponding to the maximum compression strength was taken as 0.002, while the value for the confinement factor was 1.161 for the confined concrete and 1.0 for the concrete cover.

The model of Giuffré, Menegotto and Pinto ([Giuffrè and Pinto, 1970], [Menegotto and Pinto, 1973]) was applied for the longitudinal reinforcement, along with the subsequent improvements introduced by Filippou et al. [1983]. In order to account for the cyclic degradation of steel strength depicted by the experimental results without changing the steel model, a negative value of the parameter a_3 was considered. The steel Young's modulus was taken equal to 200 GPa, while the hardening and cyclic behaviour parameters were calibrated in order to better reproduce the experimental results: b = 0.015, R_0 = 20, a_1 = 18.5, a_2 = 0.15, a_3 = -0.025 and a_4 = 15.

The iterative procedure developed by Taucer et al. [1991] and Spacone et al. [1996] was adopted for the force-based element. Additionally, a corotational formulation was used to account for the geometrically nonlinear effects. Figure 4 depicts the results obtained with the two formulations, along with the experimental data.

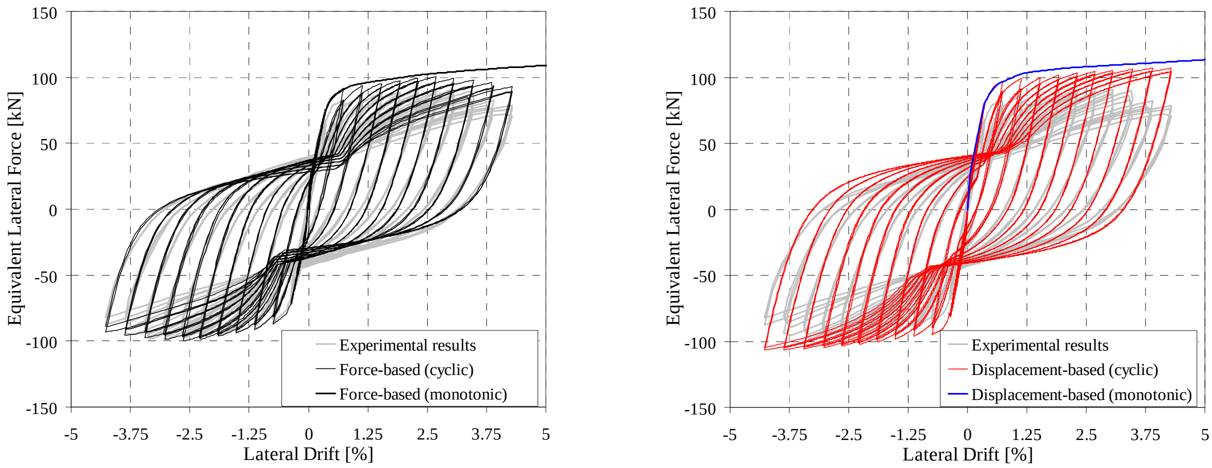
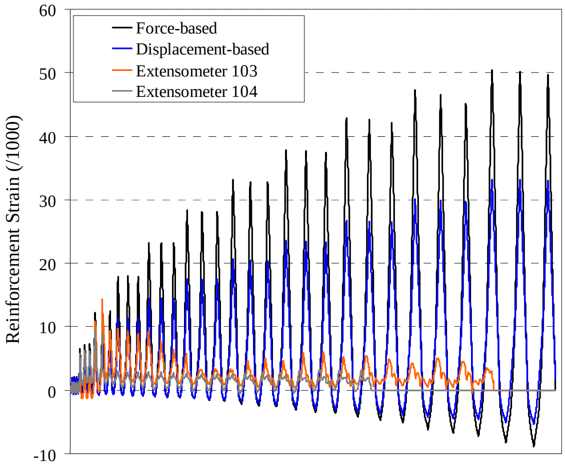


Figure 4 Equivalent lateral force *vs* lateral drift: comparison between the experimental and numerical results.

It can be observed that the results obtained with the flexibility formulation are very close to the experimental ones. The maximum resistance, the loading and unloading stiffnesses and the strength degradation are very satisfactorily modelled, especially for negative drift values. On the other hand, the results of the displacement formulation overestimate the real strength and stiffness shown by the experimental results. Such difference is a direct consequence of the imposition of an element deformed shape that does not correspond to the beam real deformation, in the displacement approach, versus the imposition of a force field that is exact (if one does not consider the second order effects), in the force-based formulation.

The experimental data include the output of some extensometers attached to the reinforcement steel at the base of the pier. Those results are compared with the values of the strains obtained numerically in the bottom Gauss integration section of the bottom 0.45m element; this section is located at a height of around 9.5cm from the specimen base. Extensometers 103 and 104 are the closest to this section, being located at a height of 7.5cm. Figure 5 represents the comparison between the numerical and experimental values.



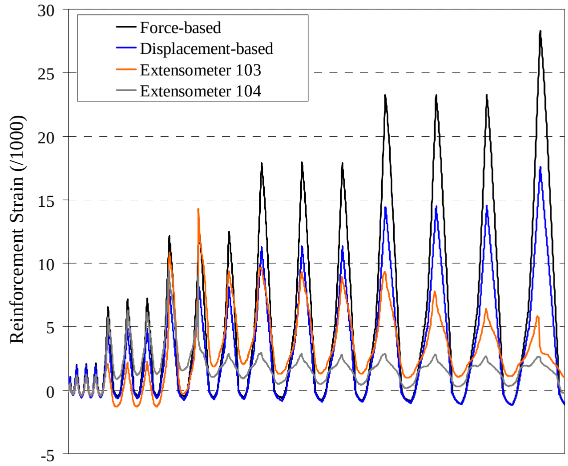


Figure 5 Comparison between experimental and numerical longitudinal reinforcement strains at a height of around 8.5cm from the base of the pier (zoomed in on the right side).

Up to a value of strain of 2‰ the cyclic experimental results and the numerical outputs with both formulations are similar. However, when the strain increases to around 7‰ the differences between the stiffness and the flexibility formulations start becoming apparent. Such disagreements intensify with the increase in lateral drift, portraying displacement-based strains manifestly smaller than their force-based counterparts.

Some comments relative to the experimental results are required. First of all, one should notice that the output from both extensometers is not reliable beyond a strain value of about 10‰. Secondly, it is noted that the data provided by both extensometers are not coincident. Strains from extensometer 104 coincide with those from the equilibrium formulation up to 10‰, even if featuring a residual strain after about 7‰. On the other hand, extensometer 103 (which is located at the same height of 104, in neighbouring reinforcements) coincides with the flexibility approach in the mentioned initial range of strains up to 2‰ and in the first cycle at 10‰, but (unexplainably) presents lower values in the previous and following cycles.

The comparison of the numerical strains produced by both methods and the observation that those yielded by the force approach are larger than the displacement-based ones leads to the supposition of a more pronounced concentration of plastic deformations near the base for the nonlinear flexibility method (in relation to that of the classical stiffness method). To support such observation the curvature in both integration sections of the bottom 0.45m element was computed, for each formulation – see Figure 6.

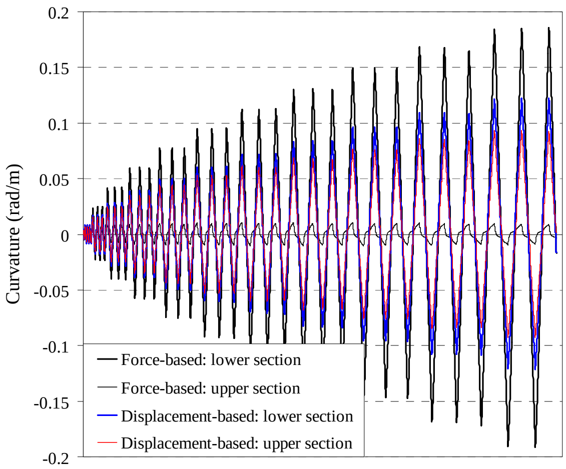


Figure 6 Evolution of numerical curvature in both integration sections of the bottom element, for each formulation.

As predicted, for the flexibility formulation the values of the curvature are very high in the base of the pier and considerably smaller above that zone. This is unlike what happens in the stiffness formulation, for which the values of the curvature in both sections do not differ much.

When a flexibility approach is used the curvature in the plastic hinge region can be very high since the latter is not imposed a priori, but is instead dependent (through a highly nonlinear constitutive relationship) on the value of the moment in the corresponding section. Obviously, for global compatibility reasons at the element level, the curvature in the other integration section must be necessarily reduced. On the other hand, the stiffness approach does not manage to simulate such concentration of curvatures because the latter is constrained to have a smooth and pre-determined variation throughout the element. Again, considering elementary compatibility requirements, this implies that the curvature of the top section (in the bottom element) will not drastically differ from that of the bottom. This is also the reason why the model is stiffer and the moments (or equivalent lateral forces) are larger for the same values of lateral drifts, in comparison with the force approach. Summarising, one can say that the plastic hinge is more concentrated in the force-based approach than in the displacement-based formulation. Later it will be shown that this characteristic is related to the location of the first integration point for the flexibility model and with the dimension of the first element (consequently, with mesh discretization) for the stiffness method. Note that the plastic concentration phenomenon modelled by the flexibility approach is a mathematical consequence of the developed model, and is not necessarily in agreement (even if it may seem) with the real plastic hinge features of the tested specimen, as discussed hereafter.

The previous discussion, along with a critical review of the results obtained by the majority of other similar research efforts (see for instance Coleman and Spacone [2001]), allows one to conclude that the majority of state-of-the-art models for practical engineering applications are still not suitable to predict, with the desired level of accuracy, important local quantities such as strains and curvature distribution. Such observation is of special relevance in the context of performance-based evaluation. This is in contrast with the satisfactory degree of precision attained for element wise predictions. Additionally, the apparent computational advantage of the stiffness-based element is found to be no longer a valid argument for its preference. Such observation is justified with the fact that for flexibility-based elements a one-level iteration procedure is also found to be possible and successful [Neuenhofer and Filippou, 1997]. This is especially true when the material nonlinear behaviour is of relevance, for which case the displacement-based element gives approximate results (mesh subdivision becomes inevitable, thus ruining its initial numerical appeal). On the contrary, force-based elements are able to model material nonlinearity using a single element per structural member. Besides, the ease of inclusion of span loading is an extra advantage of the flexibility element formulation.

2.3. Localization Effects

From the previous comparison the greater suitability of the so-called nonlinear flexibility elements for earthquake engineering applications was hinted at. One of the main concerns with this type of elements is the occurrence of localization for certain types of sectional constitutive behaviour. In fact, the numerical integration of the element integrals leads to deformation localization, typically observed at the end integration points where the bending moments reach their maximum. Strain localization also affects stiffness-based frame elements, for which these issues have been widely studied, but the displacement interpolation functions force localization within a single element instead of one integration point. Figure 7 shows the effect of decreasing length of the lower element of the pier model, by comparing the curvature time-history for three different cases. This plot illustrates that in displacement-based elements the localization phenomena are related to the length of the element and that the curvature at the base of the pier (in absolute value) increases as the length of the critical elements is reduced, getting closer to the results obtained with the force-based approach (Figure 6).

Strain localization effects within the frame flexibility formulation context are related to the length associated with the integration points within one element and the type of sectional constitutive response, having been addressed in a structured way by Coleman and Spacone [2001]. It was shown that for a strain-hardening sectional behaviour there are no localization problems, and the choice of the number of integration points throughout the element only affects the numerical accuracy of the output. However, for a perfectly plastic sectional response the curvature demand prediction becomes non-objective (i.e., increasing the number of integration points does not lead to a convergence of the results). If the sectional constitutive relation depicts strain-softening then there is also a loss of objectivity at the element level.

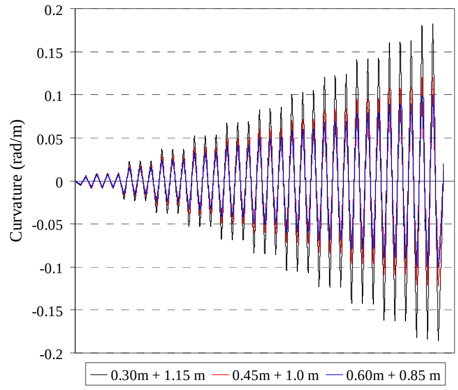


Figure 7 Evolution of numerical curvature at the pier base for different mesh sizes in the displacement-based formulation.

Figure 8 confirms such non-objectivity of the global response through the comparison of the previous pier cyclic behaviour with another model that features the same element lengths but a higher number of integration points. Some remedies have been proposed in past studies to regularize the response, both at the sectional level, where the use of a physical characteristic length (as the plastic hinge length) is usually required ([Scott and Fenves, 2006], [Adessi and Ciampi, 2007]), and at the element one, where the adoption of a constant fracture energy criterion was envisaged [Coleman and Spacone, 2001]. However, the shortcomings of such methods justify further research and scrutiny on this topic.

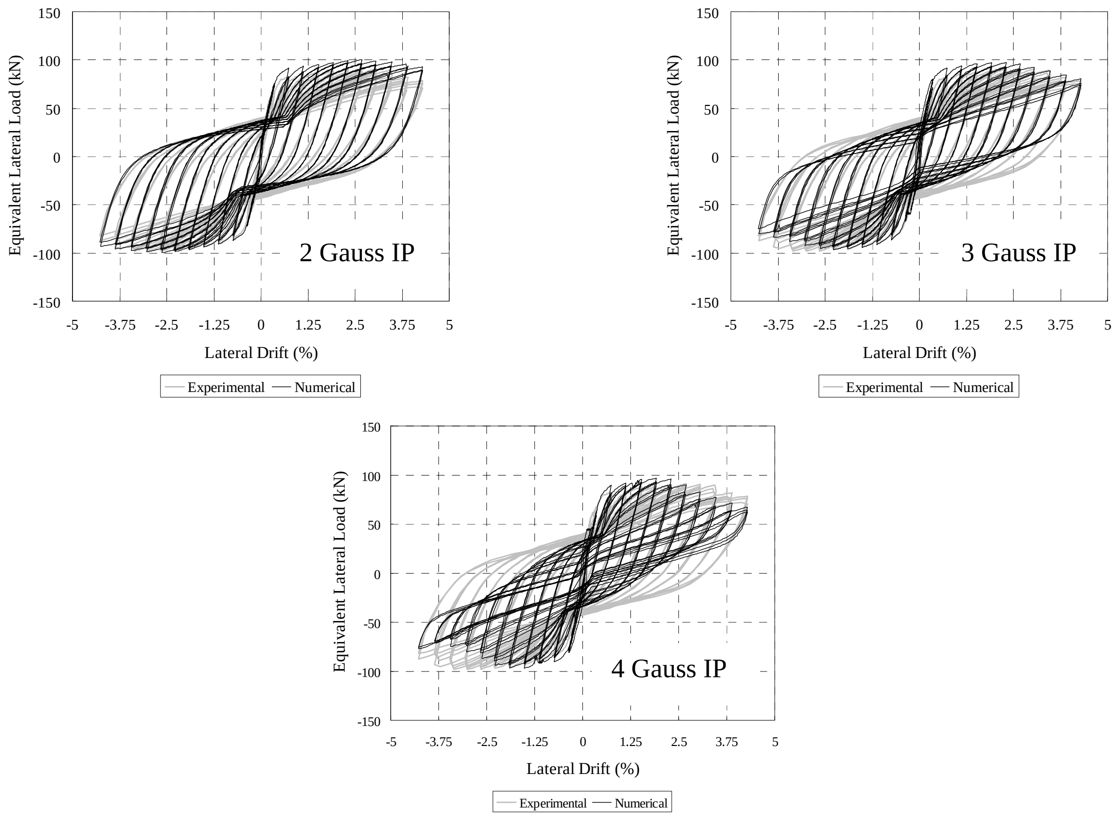


Figure 8 Non-objective response: equivalent lateral force *vs* lateral drift.

3. CONCLUSIONS

This work showed that a displacement-based approach, despite its easiness for computational implementation, may not yield results as accurate as those obtained from a force-based approach. The latter more accurately reproduces the maximum strength, the loading and unloading stiffness and the cyclic strength degradation. Even if it also presents localization issues, one can conclude that force-based elements are more appropriate for the modelling of plastic hinge length and plastic rotation. This is undoubtedly correct for strain-hardening sectional responses, where deformations do not localize. Displacement-based elements give a very stiff response, with the corresponding plastic hinge length being very dependent on the member discretization. As a general conclusion, approaching the force field by equilibrium conditions appears to be the most adequate for simulating the inelastic behaviour of frame structures subjected to earthquake loading.

4. ACKOWLEDGMENTS

The authors wish to thank the financial support provided by the Fundação para a Ciência e Tecnologia through the funding program Programa Operacional Ciência e Inovação 2010.

REFERENCES

Addessi, D., Ciampi, V. (2007). A regularized force-based beam element with a damage-plastic section constitutive law. *International Journal for Numerical Methods in Engineering* **70**, 610-629.

Coleman, J., Spacone, E. (2001). Localization issues in force-based frame elements. *Journal of Structural Engineering* **127:11**, 1257-1265.

Filippou, F.C., Popov, E.P. and Bertero, V.V. (1983). Effects of bond deterioration on hysteretic behavior of reinforced concrete joints. Earthquake Engineering Research Centre, *Report UCB/EERC-83/19*, University of California, Berkeley, USA.

Giuffrè, A. and Pinto, P.E. (1970). Il comportamento del cemento armato per sollecitazioni cicliché di forte intensità. *Giornale del Génio Civile*.

Lin, C. and Scordelis, A.C. (1975). Nonlinear analysis of reinforced concrete shells of general form. *Journal of the Structural Division*, ASCE **101:ST3**.

Mander, J.B., Priestley, M.J.N. and Park, R. (1988). Theoretical stress-strain model for confined concrete. *Journal of Structural Engineering*, ASCE **114:8**, 1804-1826.

Martínez-Rueda, J.E. and Elnashai, A.S. (1997). Confined concrete model under cyclic load. *Materials and Structures* **30**, 139-147.

Menegotto, M. and Pinto, P.E. (1973). Method of analysis for cyclically loaded RC plane frames including changes in geometry and non-elastic behavior of elements under combined normal force and bending. *Symp. Resistance and ultimate deformability of structures acted on by well defined repeated loads*, IABSE Reports, 13, Lisboa.

Neuenhofer, A., Filippou, F.C. (1997). Evaluation of nonlinear frame finite-element models. *Journal of Structural Engineering* **123:7**, 958-966.

Priestley, M.J.N., Seible, F. and Calvi, G.M. (1996). Seismic design and retrofit of bridges. John Wiley & Sons.

Scott, M.H., Fenves, G.L. (2006). Plastic hinge integration methods for force-based beam-column elements. *Journal of Structural Engineering* **132:2**, 244-252.

Spacone, E., Filippou, F.C. (1996). Fibre-beam column model for nonlinear analysis of R/C frames: Part I. Formulation. *Earthquake Engineering and Structural Dynamics* **25**, 711-725.

Taucer, F.F., Spacone, E. and Filippou, F.C. (1991). A fiber beam-column element for seismic response analysis of reinforced concrete structures. Earthquake Engineering Research Centre, *Report No. UCB/EERC-91/17*, University of California, Berkeley, USA.

Vinagre, J. (1997). Avaliação dos efeitos de 2ª ordem em edifícios de betão armado. PhD thesis, Instituto Superior Técnico, UTL, Lisboa, Portugal.