MODAL SPECTRAL CALCULATION OF FLOOR RESPONSE SPECTRA FOR SECONDARY SYSTEMS

R. A. Ortiz[1], E. F. Cruz[2], D. Valdivia[3], J. Nazal[4]

[1] *M. Sc., Seismic Engineer, EQCO Earthquake Engineering Consultants, Santiago, Chile*
[2] *Ph. D., Principal, EQCO Earthquake Engineering Consultants, Santiago, Chile*
[3] *M. Sc., Principal, EQCO Earthquake Engineering Consultants, Santiago, Chile*
[4] *Seismic Engineer, EQCO Earthquake Engineering Consultants, Santiago, Chile*
Email: ecruz@eqcoeng.com

ABSTRACT:

The calculation of the dynamic response of a mechanical system considered as a secondary system attached to a structure can be a very time consuming task for complex systems. This paper deals with the calculation of the response spectra at the attachment points between the primary and the secondary system, which will be called "Floor Response Spectra (FRS)." The computed floor response spectra at the attachment points of the primary system are intended to be used as input for a second step analysis of the mechanical system. Here in, a modal spectral procedure for the calculation of FRS on the primary system is presented. The method developed considers the natural vibration frequencies, mode shapes, and modal participation factors of the primary structure to describe the primary system and combines them with a physical representation of a single degree of freedom secondary system considering a spectral procedure with non-classical damping where the crossover terms of the coupling of the two systems are neglected to obtain the FRS. Parametric studies of the importance of the coupling terms and of the correlation of the response between the primary and the secondary systems are presented. A comparison of the results of this approach with response spectrum and time history analysis results obtained using the complete model (primary plus secondary systems together) are presented. As a practical application the procedure is used for the analysis of a steam turbine - generator and its foundation for a thermo-electrical power plant where the steam turbine–generator and the reinforced concrete foundation are considered as the secondary and the primary system, respectively.

KEYWORDS:

Response Spectrum Analysis, Secondary System Response, Non Classical Damping

1. INTRODUCTION

The response calculation for structural secondary system verification could be a very time consuming task for complex systems. Present day commercial software allows that the designers construct a very detailed model for the structural system, and then the inclusion of another detailed model for the secondary system. This excess of details could make the final analysis model unmanageable. A very simple procedure is presented for the calculation of Floor Response Spectrum (FRS), in order to be used for the verification of secondary systems.

The procedure starts with the development of a model that includes the detailed primary system characteristics and a very simplified model of the secondary system. The simplified model must be representative of the behavior of the secondary system in terms of masses, frequencies, and mode shapes. The match of these properties is very important for a complete representation of the dynamic interaction.

This procedure can be applied based on the output of any commercial software, and uses the natural frequencies, the modal displacements in the connection nodes (mode shapes) and the assumed damping ratios. All this data is combined for the calculation of the representative mass, damping, and stiffness matrices at the modal level, and then the equations are extended for the inclusion of a single degree of freedom (SDOF) system attached at the connection node. For the representation of the damping characteristic of the SDOF a non-classical procedure is applied for the FRS calculation.

2. THEORETICAL BACKGROUND

Starting from the classical formulation of the equation of motion of a dynamic system of N degrees of freedom (both the primary and the secondary systems can be represented):

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=-\mathbf{M}\mathbf{r}\ddot{u}_g(t) \tag{2.1}$$

where **M, C** and **K** are the mass, damping, and stiffness matrices respectively, **r** is the influence vector for the excitation, $\ddot{u}_g(t)$ is the ground acceleration and $\mathbf{u}(t)$ are the displacement degrees of freedom. Additionally, the equation of motion for a single degree of freedom system (the secondary system) attached to one of the degrees of freedom of the primary system can be represented as the following differential equation:

$$m_v\ddot{z}(t)+c_v\dot{z}(t)+k_v z(t)=-m_v\ddot{u}_{ai}(t) \tag{2.2}$$

where m_v, c_v, and k_v are the mass, damping, and stiffness respectively, $\ddot{u}_{ai}(t)$ is the absolute acceleration in the connection joint between the primary and the secondary systems in the corresponding direction and $z(t)$ is the displacement of the single degree of freedom relative to the attachment point.

The total acceleration at the attachment point can be represented as the sum of the relative acceleration of node *I*, and the ground acceleration as is shown in equation (2.3).

$$\ddot{u}_{ai}(t)=\ddot{u}_i(t)+\ddot{u}_g(t) \tag{2.3}$$

Considering that the analysis will be carried out using modal analysis, the damping effect will be taken out from the equations in order to develop a simplified expression. Next the equations are considered together in the following matrix expression:

$$\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & m_v\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{u}}(t)\\ \ddot{z}(t)\end{bmatrix}+\begin{bmatrix} & & & & & 0\\ & & & & & \vdots\\ & & \mathbf{K}+\mathbf{A}_r^T k_v\mathbf{A}_r & & & -k_v\\ & & & & & 0\\ & & & & & \vdots\\ 0 & \cdots & -k_v & 0 & \cdots & k_v\end{bmatrix}\begin{bmatrix}\mathbf{u}(t)\\ z(t)\end{bmatrix}=\begin{bmatrix}-\mathbf{M}\mathbf{r}\\ 0\end{bmatrix}\ddot{u}_g(t)+\begin{bmatrix}\mathbf{0}\\ -m_v\end{bmatrix}\ddot{u}_{ai}(t) \tag{2.4}$$

where $\mathbf{A}_r$ is the matrix representing the cinematic transformation from the secondary degree of freedom to the primary system degree of freedom. Using equation (2.3) into equation (2.4), the combined equation of motion considering the ground acceleration as input is:

$$\underbrace{\begin{bmatrix} & \mathbf{M} & & & \mathbf{0} \\ 0 & \cdots & m_v^i & 0 & \cdots & m_v \end{bmatrix}}_{\overline{\mathbf{M}}}\begin{bmatrix}\ddot{\mathbf{u}}(t)\\ \ddot{z}(t)\end{bmatrix}+\underbrace{\begin{bmatrix} & & & & & 0\\ & & & & & \vdots \\ & \mathbf{K}+\mathbf{A}_r^T k_v \mathbf{A}_r & & & & -k_v\\ & & & & & 0\\ & & & & & \vdots\\ 0 & \cdots & -k_v & 0 & \cdots & k_v\end{bmatrix}}_{\overline{\mathbf{K}}}\begin{bmatrix}\mathbf{u}(t)\\ z(t)\end{bmatrix}=\begin{bmatrix}-\mathbf{Mr}\\ -m_v\end{bmatrix}\ddot{u}_g(t) \qquad (2.5)$$

Note that the resulting mass matrix is non-symmetric and includes the coupling effect between the primary and the secondary system masses through the term m_v^i coming from the inertia force corresponding to the absolute acceleration of the degree freedom *i*. We can now consider a linear transformation defined with the expression $\mathbf{u}(t)=\Phi\mathbf{q}(t)$, where the columns of the matrix Φ are the mode shapes or eigen-vectors of the system **M\K** (primary system)**,** and the equations can be written in the following form:

$$\overline{\mathbf{M}}\cdot\begin{bmatrix}\Phi\ddot{\mathbf{q}}(t)\\ \ddot{z}(t)\end{bmatrix}+\overline{\mathbf{K}}\cdot\begin{bmatrix}\Phi\mathbf{q}(t)\\ z(t)\end{bmatrix}=\begin{bmatrix}-\mathbf{Mr}\\ -m_v\end{bmatrix}\cdot\ddot{u}_g(t) \qquad (2.7)$$

Pre-multiplying by $\begin{bmatrix}\Phi^T & \mathbf{0}\\ \mathbf{0} & 1\end{bmatrix}$, dividing by $\begin{bmatrix}\hat{\mathbf{M}} & \mathbf{0}\\ \mathbf{0} & m_v\end{bmatrix}$, and taking advantage of the orthogonality conditions satisfied by the mode shapes of the primary structure, the equation can be re-written as:

$$\begin{bmatrix} & & I & & & \mathbf{0}\\ \phi_{i1} & \cdots & \phi_{ii} & \cdots & \phi_{iN} & 1\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{q}}(t)\\ \ddot{z}(t)\end{bmatrix}+\begin{bmatrix} & & & & & 0\\ & & & & & \vdots\\ & \Omega_n^2+\hat{\mathbf{M}}\backslash\Phi^T\mathbf{A}_r^T k_v\mathbf{A}_r\Phi & & & & -\omega_n^2\\ & & & & & 0\\ & & & & & \vdots\\ 0 & \cdots & -\omega_n^2 & 0 & \cdots & \omega_n^2\end{bmatrix}\begin{bmatrix}\mathbf{q}(t)\\ z(t)\end{bmatrix}=\begin{bmatrix}\mathrm{T}_n\\ -1\end{bmatrix}\ddot{u}_g(t) \qquad (2.8)$$

where, I is the N by N identity matrix, ϕ_{ii} is the component *ii* of the matrix Φ, $\hat{\mathbf{M}}=\Phi^T\mathbf{M}\Phi$, $\Omega_n^2=\hat{\mathbf{M}}\backslash\Phi^T\mathbf{K}\Phi$ and ω_n is the natural frequency of the secondary system. The second term in equation (2.8) can be modified by choosing a k_v value that is very small compared with the eigen-values of the system **M\K** making "zero" the term $\hat{\mathbf{M}}\backslash\Phi^T\mathbf{A}_r^T k_v\mathbf{A}_r\Phi$, additionally the cross terms $-\omega_n^2$ can be neglected considering that the forces coming back from the additional mass representing the secondary system are small. With these two assumptions, the final equation of motion for the analysis of the combined system can be written as:

$$\begin{bmatrix} & & I & & & \mathbf{0}\\ \phi_{i1} & \cdots & \phi_{ii} & \cdots & \phi_{iN} & 1\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{q}}(t)\\ \ddot{z}(t)\end{bmatrix}+\begin{bmatrix}\Omega_n^2 & \mathbf{0}\\ \mathbf{0} & \omega_n^2\end{bmatrix}\begin{bmatrix}\mathbf{q}(t)\\ z(t)\end{bmatrix}=\begin{bmatrix}\mathrm{T}_n\\ -1\end{bmatrix}\ddot{u}_g(t) \qquad (2.9)$$

Since the "mass matrix" of this system is non-symmetric a classical damping modal analysis is impossible and a non-classical damping method shall be used in order to determine the spectral response of the degree of freedom *i* (Sinha and Igusa, 1995). A diagonal matrix with the corresponding modal damping ratios for the first N positions is used to represent the primary structure damping and for the additional degree of freedom in the analysis the desired value of damping for the secondary system is directly added.

3. CALCULATION OF FRS FOR A STEAM TURBINE-GENERATOR FOUNDATION SYSTEM

In order to test the quality of the results from the proposed procedure the method is applied to a real system corresponding to a steam turbine-generator (STG) foundation system, where the foundation and a very simplified model of the stem turbine-generator equipment are considered as the primary structure and a single degree of freedom at the top table of the foundation is considered as the secondary system. The purpose of the inclusion of the turbine-generator simplified model in the analysis is to allow including in the response of the secondary system the possible dynamic interaction effects between the equipment and the foundation.

In what follows, the results of the calculation of the FRS are compared to the results obtained from the analysis of the complete system considering three different earthquake excitation sources: two artificial records obtained from the design acceleration spectrum used in the Response Spectrum Analysis and a real Chilean record from the March 3, 1985 earthquake (Melipilla). The maximum accelerations for the three records are listed in Table 3.1.

Table 3.1 Earthquake PGA

Earthquake	PGA (g)
Artificial 1	0.64
Artificial 2	0.66
Melipilla	0.67

A set of 6 nodes of the top table (shown in Figure 2) were selected to evaluate the results, comparing the FRS results obtained with the procedure presented here in against the results form time history analysis using the earthquake records. The time history analysis was carried out by linear modal integration, and the acceleration histories obtained from the selected nodes were used to evaluate the response spectra at the nodes. For the time history analysis and the FRS calculations the damping ratio used was 3%. The comparison of the response spectra for the three records with the design spectra considered is shown in Figure 1.

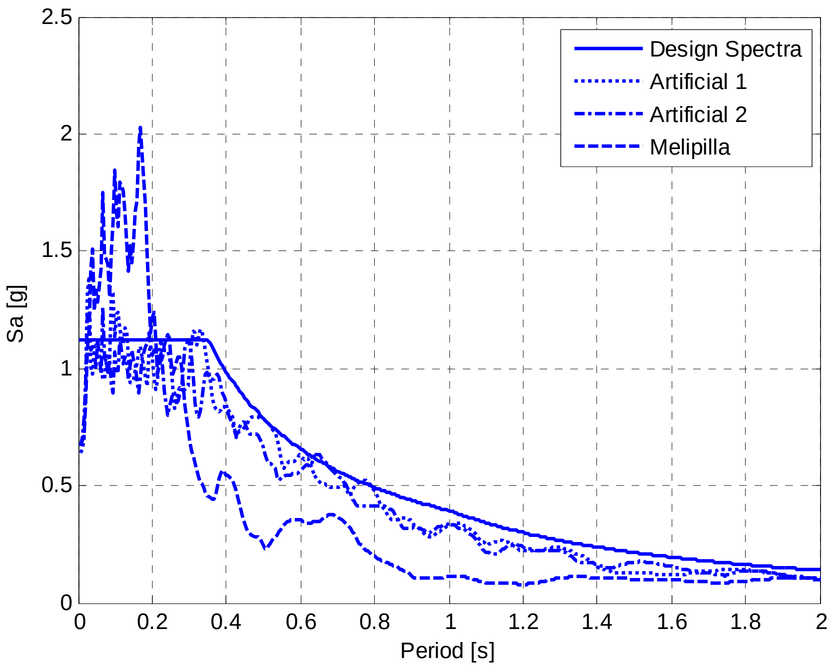


Figure 1: Design Acceleration spectrum and used earthquake records spectra.

The structural model for the analysis contains 325 "frame", 2402 "shell" and 43000 "solid" elements. In the foundation "solid" elements were used for the modeling of big volumes of soil and concrete, "shell" elements were used to model the concrete slabs (at the top table and the mezzanine floor), and "frame" elements were used to model the pedestals. The simplified turbine-generator model was built using "frame" type elements and it was adjusted to capture the inertia properties, the modes shapes, and the natural frequencies of the system. The analysis was carried out for excitation along the Y axis which is along the short side of the foundation. Table 3.2 shows the frequencies and effective mass ratios of the first 9 modes of the model.

Table 3.2 Periods and Effective Mass ratios for the first natural vibration modes of the model

Mode	1	2	3	4	5	6	7	8	9
Period (s)	0.41	0.33	0.22	0.20	0.15	0.15	0.12	0.11	0.10
Effective Mass X Dir	0.0%	77.5%	0.0%	0.0%	20.4%	0.0%	0.0%	0.0%	0.0%
Effective Mass Y Dir	69.0%	0.0%	0.4%	0.0%	0.0%	26.6%	0.0%	1.3%	0.0%

Additionally it should be noted that the periods of all the natural vibration modes involving deformation of the STG sub-model are below 0.1 seconds. The results of the two types of analysis were obtained for 6 nodes located at the top table of the foundation as shown in Figure 2. Node 1 and Node 2 are located in the area of the generator supports. Node 3 and Node 6 are located at both sides of the low pressure turbine (LPT), Node 5 is located at the support connection between the medium pressure turbine (MPT) and the LPT, and Node 4 is located at the end of the high pressure turbine (HPT).

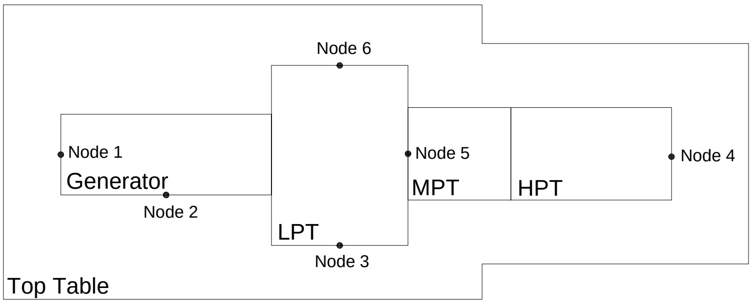
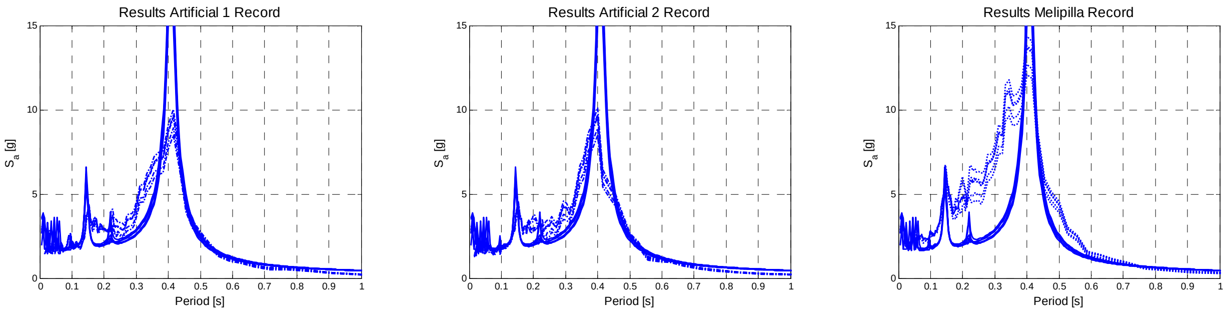


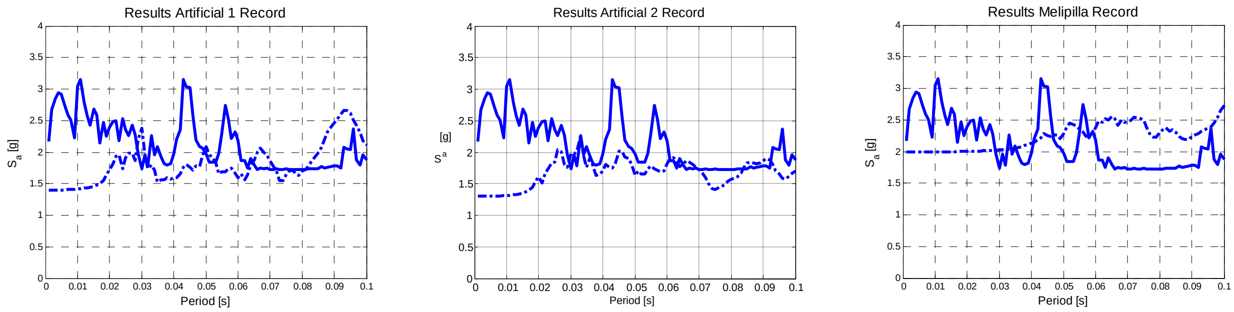
Figure 2 Location of nodes used in the analysis in the Top Table of the foundation.

In order to establish a better understanding of the procedure, the values for the FRS calculated from the Response Spectrum Analysis with the proposed procedure are called FRSS and the ones calculated from the time history analysis are called FRST. Results are presented in Figures 3 and 4. Figures 3a, 3b and 3c show the FRSS results in solid lines for each of the nodes. It can be seen that the results of FRSS are quite similar for the six nodes considered; this condition is a consequence of the symmetry in mass and stiffness distribution in the primary system model, and to some extent of the rather large stiffness of the top table area of the foundation. The FRST results are presented in dashed lines. It can be noticed from Figure 3a and 3b that FRST values are quiet similar to those of FRSS; however, a difference in the period range between 0.15 to 0.35 seconds can be observed. Figure 3c shows the comparison of the FRST results obtained for the Melipilla record with the FRSS results; in this case, the two curves have big differences in amplitude in practically all the range of periods considered. This is not unexpected as a comparison of the design acceleration spectrum with the Melipilla record spectrum shows significant differences between the two. Finally, in Figure 4, the average values of FRSS are presented in solid lines and those of FRST in dashed lines; the figure also makes a zoom to the response for those periods near the natural frequencies of the STG system (0 to 0.1 s). Figure 4a and 4b reproduce the same trends explained for Figures 3a and b, and show that the proposed procedure for computing FRS provides conservative estimates of the spectrum ordinates in a wide range of periods.



(a) Artificial 1 results (b) Artificial 2 results (c) Melipilla results

Figure 3 FRS results in solid line, time history results in dashed lines for all nodes.



(a) Artificial 1 results (b) Artificial 2 results (c) Melipilla results

Figure 4 Mean FRS results in solid line, mean time history results in dashed line.

4. CONCLUSION

For the artificial records, Figure 3a and 3b, the fit of the FRSS results with the FRST results is rather good, except for the range of frequencies where there is large amplification in the FRSS. This is the result of two factors: the spectra for the artificial records have a smaller amplitude than the design acceleration spectrum (see Table 4.1) in that range of periods, and the earthquake excitation in the FRS calculation is applied in a system with only one degree of freedom and the cross term relation was broken in order to avoid a double representation of the interaction, so the amplifications tends to the typical value of amplification of the response for a damping factor of 3% (approximately 17). Table 4.2 shows the amplification factors for the FRS values calculated from the records and calculated from the design acceleration spectrum.

Table 4.1 Spectral ordinate values for the first natural vibration mode (0.41 s).

Spectrum	Sa (g)	Ratio to Design Spectrum
Design Spectrum	0.96	100.0%
Artificial record 1	0.81	84.7%
Artificial record 2	0.80	82.6%
Melipilla	0.54	56.3%

Table 4.2 Amplifications Factors obtained from FRSS and FRST, calculated at a period of 0.41s.

Node	Artificial 1	Artificial 2	Melipilla	FRSS
1	6.13	6.99	6.46	16.83
2	6.48	7.20	6.61	17.75
3	6.78	7.36	6.77	19.28
4	7.13	7.27	7.01	20.36
5	6.83	7.12	6.65	19.49
6	6.89	7.33	6.77	19.28

Figure 4, shows that for periods lower than 0.1 seconds, which are close to the actual periods of the STG system the FRSS are close to the FRST results being conservative for the artificial records and for periods less than 0.06 s.

Given the overall quality of the results obtained, the proposed procedure for computing FRS can be considered appropriate to obtain the design spectrum for secondary systems. As since the computation of the FRS in the procedure starts from the provided design spectra for the primary system, the resulting FRSS values are consistent with the design requirements as reflected in the design spectrum for the primary system.

4. ACKNOWLEDGMENTS

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