A NEW SENSITIVITY AND RELIABILITY ANALYSIS FRAMEWORK FOR STRUCTURAL AND GEOTECHNICAL SYSTEMS

Q. Gu[1], M. Barbato[2] and J.P. Conte[3]

[1] *Engineer, AMEC Geomatrix Consultants Inc., 510 Superior Ave., Suit 200, Newport Beach, CA, 92663, USA*
[2] *Assistant Professor, Department of Civil & Environmental Engineering, Louisiana State University, 3531 Patrick F. Taylor Hall, Nicholson Extension, Baton Rouge, Louisiana 70803, USA.*
[3] *Professor, Department of Structural Engineering, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093 - 0085, USA.*
Email: qgu@ucsd.edu, mbarbato@lsu.edu, jpconte@ucsd.edu

ABSTRACT:

This paper presents recent advances in response sensitivity, probabilistic response and reliability analyses of structural and geotechnical systems. These developments are integrated into general-purpose nonlinear finite element (FE) software frameworks and provide the structural engineers with analytical and computational tools to propagate uncertainties through advanced large-scale nonlinear simulations and obtain probabilistic estimates of the predicted system response performance. The Direct Differentiation Method (DDM) for accurate and efficient computation of FE response sensitivities is extended and applied to large-scale nonlinear Soil-Foundation-Structure-Interaction (SFSI) systems. Extensions include numerical algorithms for response sensitivity analysis of FE models with multi-point constraints, force-based and three-field mixed elements, as well as various nonlinear material constitutive models, including a pressure independent multi-yield-surface J_2 plasticity material model used to simulate the clay soil nonlinear behavior. Response sensitivity analysis results are shown for structural and SFSI systems. Examples of probabilistic response as well as time-invariant and time-variant reliability analyses are provided. Importance sampling and orthogonal plane sampling techniques are adopted and implemented into the considered FE software frameworks for accurate computation of failure probabilities. A new visualization technique, the Multidimensional Visualization in the Principal Planes (MVPP), is developed to visualize limit state surfaces in a neighborhood of the design points, giving insight into inaccuracy of the First-Order Reliability Method (FORM) for highly nonlinear systems. Based on the MVPP technique results, a novel hybrid method, the Design Point - Response Surface - Simulation (DP-RS-Sim) method is developed for both time-invariant and time-variant reliability analysis. Application examples are provided to show that the DP-RS-Sim method can provide failure probability estimates more accurate than FORM at a small increment of the computational cost.

KEYWORDS: Finite element method; reliability analysis; response sensitivity analysis; direct differentiation method; soil-foundation-structure-interaction; multi-yield-surface plasticity model.

1. INTRODUCTION

A challenging task for structural engineers is to provide a structure with the capability of achieving a target performance over its design life-time. In order to fulfill this task successfully, all pertinent sources of the existing aleatory and epistemic uncertainties must be rigorously accounted for during the design process. Thus, proper methods are required for propagating uncertainties from model parameters describing the geometry, the material behaviors and the applied loadings to structural response quantities used in defining performance limit-states. These methods need also to be integrated with methodologies already well-known to engineers, such as the Finite Element (FE) method.

This paper presents recent developments in response sensitivity, probabilistic response and reliability analyses of structural and geotechnical systems within general-purpose software frameworks for nonlinear FE response analysis, i.e., FedeasLab (Filippou and Constantinides 2004) and OpenSees (Mazzoni et al. 2005). Current advances are highlighted which cover relevant gaps between FE response sensitivity computation using the Direct Differentiation Method (DDM) and state-of-the-art FE response analysis. Necessary extensions of the DDM are presented for accurate and efficient computation of FE response sensitivities of structural and/or Soil-Foundation-Structure-Interaction (SFSI) systems. Response sensitivity analysis results are used to gain insight into the effect and relative importance of model parameters on the predicted system response and to efficiently perform simplified probabilistic response analysis based on the First-Order Second-Moment (FOSM) method. Taking advantage of the presented advances in FE response sensitivity analysis, existing time-invariant and time-variant reliability methods, such as First- and Second-Order Reliability Methods (FORM and SORM) and mean outcrossing rate computation, are applied for reliability analysis of realistic structural/geotechnical systems to illustrate the methodology used and its capabilities. Importance sampling and orthogonal plane sampling techniques are adopted and implemented into the considered FE software frameworks for accurate computation of failure probabilities. A new visualization technique, the Multidimensional Visualization in the Principal Planes (MVPP) method, is developed to study the topology of Limit State Surfaces (LSSs) close to the Design Point (DP), providing insight inaccuracy of FORM for highly nonlinear systems especially in time-variant reliability analysis. Based on the MVPP results, a new hybrid method that combines the DP search, the response surface method and simulation techniques, referred to as DP-RS-Sim method, is developed for both time-invariant and time-variant reliability analyses, in order to obtained failure probability estimates with improved accuracy compared to other classical reliability methods (Barbato et al. 2008b; Gu 2008e). The presented probabilistic response and reliability analyses are based on two types of FE response analysis used extensively in earthquake engineering, namely pushover and time-history analysis.

2. FINITE ELEMENT RESPONSE SENSITIVITY COMPUTATION

FE response sensitivities represent an essential ingredient for gradient-based optimization methods required in various subfields of structural and geotechnical engineering such as structural optimization, reliability analysis, system identification, and FE model updating (Ditlevsen and Madsen 1996; Kleiber et al. 1997). In addition, FE response sensitivities are invaluable for gaining insight into the effects and relative importance of system and loading parameters on system response. If r denotes a generic scalar response quantity, the sensitivity of r with respect to the geometric, material or loading parameter θ is the partial derivative of r with respect to the modeling parameter θ, i.e., $\partial r/\partial\theta$, evaluated at $\theta = \theta_0$, where θ_0 denotes the nominal value taken by the sensitivity parameter θ for the FE response analysis.

Several methodologies are available for response sensitivity computation, such as the Finite Difference Method (FDM), the Adjoint Method (AM), the Perturbation Method (PM) and the Direct Differentiation Method (DDM) (Kleiber et al. 1992, 1997; Zhang and Der Kiureghian 1993; Conte et al. 2001, 2003, 2004; Gu and Conte 2003). The FDM is the simplest method for response sensitivity computation, but is computationally expensive and can be negatively affected by numerical noise. The AM is extremely efficient for linear and nonlinear elastic structural models, but is not competitive with other methods for path-dependent (i.e., nonlinear inelastic) problems. The PM is computationally efficient but generally not very accurate. The DDM, on the other hand, is general, efficient and accurate and is applicable to any material constitutive model. These advantages can be obtained at the one-time cost of differentiating analytically the space- and time-discrete equations describing the structural response and implementing these “exact” derivatives in a FE program.

In the DDM, the consistent FE response sensitivities are computed at each time step, after convergence is achieved for the response computation. This requires the exact differentiation of the FE algorithm for the response calculation with respect to each sensitivity parameter. Consequently, the response sensitivity calculation algorithm affects the various hierarchical layers of FE response calculations, namely: (1) the structure level, (2) the element level, (3) the integration point (section for frame/truss elements) level, and (4) the material level. Details on the derivation of the DDM sensitivity equation can be found in the literature (Kleiber et al. 1997; Conte 2001). Hereafter, some newly developed algorithms and recent extensions are presented which close gaps between FE response sensitivity computation using the DDM and state-of-the-art FE response-only analysis.

2.1. DDM-based response sensitivity algorithm for force-based and three-field mixed elements

Recent years have seen significant advances in the nonlinear analysis of frame structures. Advances were led by the development and implementation of force-based elements (Spacone et al. 1996), which are superior to classical displacement-based elements in tracing material nonlinearities such as those encountered in reinforced concrete beams and columns. In a classical displacement-based element, the cubic and linear Hermitian polynomials used to interpolate the transverse and axial frame element displacements, respectively, are only approximations of the actual displacement fields in the presence of non-uniform beam cross-section and/or nonlinear material behavior. On the other hand, force-based frame element formulations stem from equilibrium between section and nodal forces, which can be enforced exactly in the case of a frame element. The force-based elements are shown to enable, at no significant additional computational costs, a drastic reduction in the number of elements required for a given level of accuracy in the simulated response of a FE model of frame structures (Spacone et al. 1996).

The DDM-based response sensitivity computation algorithm has been recently extended to force-based frame elements (Conte et al. 2004; Scott et al. 2004). The benefit of using force-based instead of displacement-based frame elements has been found even more conspicuous for accurate and efficient computation of structural response sensitivities to material and loading parameters than for response-only computations. This benefit in terms of improved accuracy at lower computational cost increases with the complexity of the structural system being analyzed. As application example, a statically indeterminate 2D single-story single-bay steel frame (see inset in Figure 1(a)) with distributed plasticity (modeled by using a Von Mises J_2 plasticity constitutive law) subjected to a horizontal force P at roof level is presented here (Barbato and Conte 2005). Figure 1(a) and (b) compare the normalized sensitivity to the kinematic hardening modulus of the horizontal roof displacement obtained from FE analyses employing different meshes of force-based and displacement-based frame elements, respectively. It is found that convergence of the FE response to the exact solution is much faster when force-based elements are employed and this trend is even stronger for FE response sensitivities.

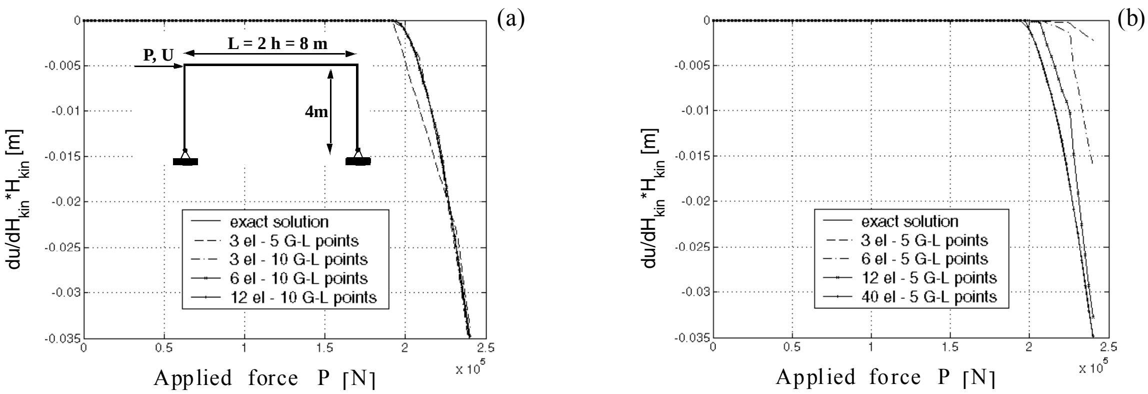

Figure 1. Sensitivities of roof displacement to kinematic hardening modulus for a one-bay one-story steel frame modeled by different FE meshes using (a) force-based frame elements and (b) displacement-based frame elements

A large body of research has been devoted to mixed FE formulations in the last 30 years. Several finite elements based on different variational principles have been developed (Washizu 1975; Belytschko et al. 2000) and relationships among them have been established. The DDM algorithm for a three-field mixed formulation based on the Hu-Washizu functional has been derived and presented elsewhere (Barbato et al. 2006). This formulation stems from basic principles (equilibrium, compatibility and material constitutive model equations), considers both material and geometric nonlinearities, is valid for both quasi-static and dynamic FE analysis and incorporates material, geometric and loading sensitivity parameters.

2.2. Extension of the DDM to Soil-Foundation-Structure-Interaction (SFSI) systems

The seismic excitation experienced by structures (buildings, bridges, etc.) is a function of the earthquake source (fault rupture mechanism), travel path effects, local site effects, and SFSI effects. Irrespective of the presence of a structure, the local soil conditions (stratification of subsurface materials) may change significantly, through their dynamic filtering effects, the earthquake motion (seismic waves) from the bedrock level to the ground surface. The complex and still poorly understood interactions between subsurface materials, foundations, and the structure during the passage of seismic waves is further significantly complicated by clouds of uncertainties associated with the various components of a SFSI system as well as the seismic excitation. A comprehensive and efficient analytical methodology is necessary for studying the propagation of uncertainties in nonlinear dynamic analysis of SFSI systems for performance-based earthquake engineering.

Recently, DDM-based response sensitivity analysis has been extended to SFSI systems, through the development of the DDM algorithm for a pressure independent multi-yield-surface J_2 plasticity material model used to describe the nonlinear behavior of clay soil. This model was first developed by Iwan (1967) and Mroz (1967), then applied by Prevost (1977) to soil mechanics and later modified and implemented into OpenSees (Elgamal et al. 2003). In contrast to the classical J_2 plasticity model with a single yield surface, the multi-yield-surface J_2 plasticity model employs a series of nested yield surfaces (Gu et al. 2008a) to achieve a better representation of the material plastic behavior under cyclic loading conditions (see Figure 2b). The extension of DDM to this soil model requires the computing of its consistent tangent modulus and stress sensitivities (Gu et al. 2008a, 2008b). Furthermore, the DDM was extended to FE models with multipoint constraints, such as the ones required to enforce a simple shear condition in the soil (Gu et al. 2008c).

The FE model of a benchmark SFSI system is shown in Figure 2. A detailed description of this benchmark problem can be found somewhere else (Gu 2008e). The shear stress-strain response of the soil provided in the inset of Figure 2(a)) shows that the system experiences significant nonlinear behavior. Figure 2(c) shows the normalized sensitivities of the first interstory drift in the x-direction, Δ_1, to the four material parameters affecting most significantly Δ_1, i.e., the shear strengths, $\tau_{max,3}$ and $\tau_{max,4}$, of the two bottom soil layers, and the young's modulus E_{col} and yield strength $\sigma_{Y,col}$ of the reinforcement steel of the columns.

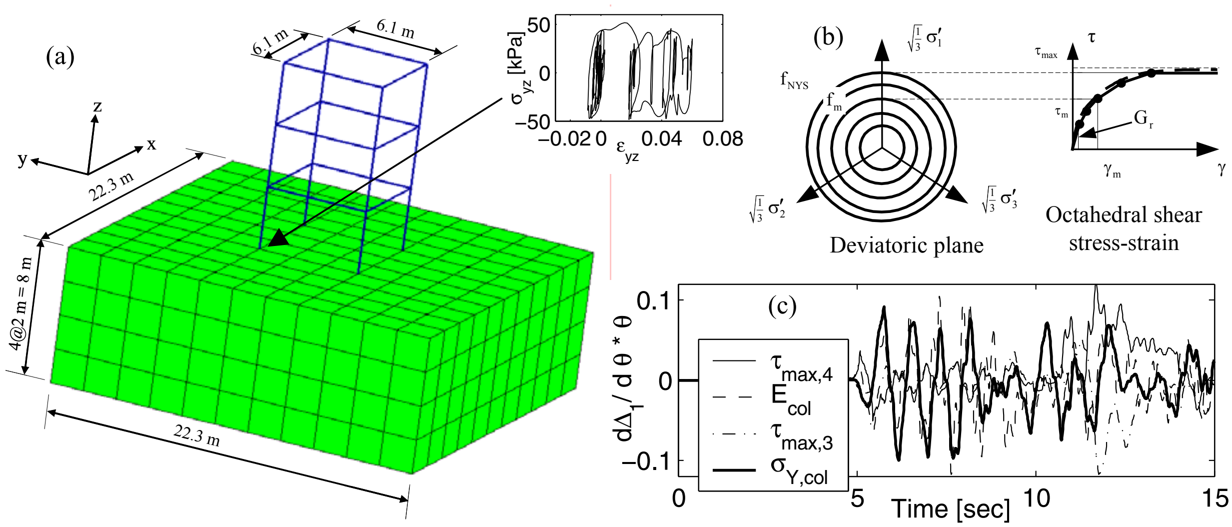

Figure 2. Benchmark SFSI system: (a) geometry, FE mesh and soil response, (b) soil material constitutive model, and (c) relative importance measure of different material parameters on first interstory drift (normalized sensitivities).

3. FINITE ELEMENT PROBABILISTIC RESPONSE ANALYSIS OF SFSI SYSTEMS

Probabilistic response analysis consists of computing the probabilistic characterization of the response of a specific system, given as input the probabilistic characterization of the material, geometric and loading parameters. An approximate method of probabilistic response analysis is the First-Order Second-Moment (FOSM) method, in which mean values (first-order statistical moments), variances and covariances (second-order statistical moments) of the response quantities of interest are estimated by using a mean-centered, first-order Taylor series expansion of the response quantities in terms of the modeling parameters described as random variables (Barbato et al. 2008a). Thus, this method requires only the knowledge of the first- and second-order statistical moments of the random parameters. It is noteworthy that statistical information about the random parameters is often limited to first and second moments and, therefore, probabilistic response analysis methods more advanced than FOSM analysis cannot be fully exploited.

The approximate first- and second-order response statistics can be readily obtained when response sensitivities evaluated at the mean values of the random parameters are available. Only a single FE analysis is needed in order to perform a FOSM probabilistic response analysis, when the FE response sensitivities are computed using the DDM. Probabilistic response analysis can also be performed by using Monte Carlo Simulation (MCS). In this study, MCS is used to assess the accuracy of the FOSM approximations when used for nonlinear FE-based probabilistic response analysis of a 2D SFSI system consisting of a reinforced concrete building on layered soil modeled with random/uncertain material parameters and subjected to quasi-static pushover (see Figure 3(a)). The frame is modeled using displacement-based, fiber-section Euler-Bernoulli frame elements with distributed plasticity. The reinforcement steel behavior is modeled by the 1D J_2 plasticity model, while the concrete behavior is modeled using the Kent-Scott-Park model with zero tension stiffening. Different material parameters are used for the confined and unconfined concrete. The foundation footings are assumed linear elastic and the soil is modeled using the multi-yield surface J_2 plasticity model with multiple sets of material parameters. The Nataf model (Ditlevsen and Madsen 1996) was used to generate realizations of the random modeling parameters. The details on FE modeling of the system and the probabilistic characterization of the random parameters can be found elsewhere (Barbato et al. 2008a; Gu and Conte 2008d).

Figure 3(b) compares the estimates of the mean value ± 1 standard deviation of the roof displacement in the x-direction, u_{top}, for a quasi-static pushover analysis with an upper-triangular pattern of applied horizontal forces, obtained using FOSM and MCS. Figure 3(c) provides the estimates of the standard deviation of u_{top} obtained by using MCS and FOSM with sensitivities computed through DDM and backward/forward finite differences (BFD and FFD, respectively) and their average. It is found that a DDM-based FOSM analysis can provide at a very low computational cost (only a single FE analysis) estimates of the first- and second-order response statistics which are in good agreement with much more expensive MCS estimates (3000 FE analyses in this example) when the structural systems experience low-to-moderate nonlinearities.

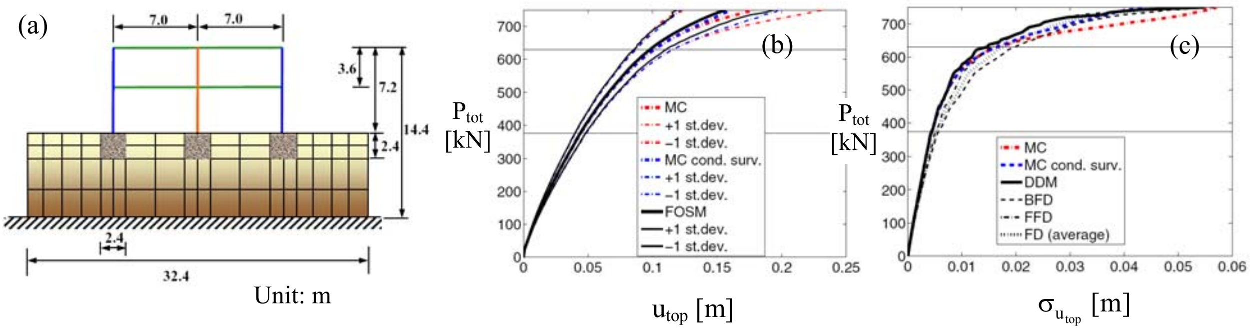

Figure 3. Probabilistic response analysis: (a) SFSI FE model, (b) roof displacement mean ± 1 standard deviation obtained from FOSM and MCS, and (c) roof displacement standard deviation estimates from FOSM and MCS.

4. FINITE ELEMENT RELIABILITY ANALYSIS

In general, a structural reliability problem consists of computing the probability of failure P_f of a given structure, which is defined as the probability of exceedence of a specified limit state (or damage state) when the loading(s) and/or structural properties and/or parameters in the limit state functions (LSFs) are uncertain quantities and modeled as random variables. This paper focuses on component reliability problems, i.e., the problem is described by a single LSF $g = g(\mathbf{r}, \boldsymbol{\theta})$, where $\mathbf{r}$ denotes a vector of response quantities of interest and $\boldsymbol{\theta}$ is the vector of all basic random variables used to define the system. The LSF g is chosen such that $g \leq 0$ defines the failure domain/region. Thus, the time-invariant component reliability problem takes the following mathematical form

$$P_f = P[g(\mathbf{r}, \boldsymbol{\theta}) \leq 0] = \int_{g(\mathbf{r}, \boldsymbol{\theta}) \leq 0} \mathrm{p}_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{4.1}$$

where $\mathrm{p}_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$ denotes the joint probability density function (PDF) of random variables $\boldsymbol{\theta}$. In time-variant reliability problems, the objective is computing the time-variant failure probability, $P_f(\mathrm{T})$, corresponding to the probability of at least one outcrossing of the LSF during the time interval [0, T]. An upper bound of $P_f(\mathrm{T})$ is given by

$$P_f(\mathrm{T}) \leq \int_0^{\mathrm{T}} \nu_g(\mathrm{t}) d\mathrm{t} \tag{4.2}$$

where $\nu_g(\mathrm{t})$ denotes the mean down-crossing rate of level zero of the LSF g and t represents the time. An estimate of $\nu_g(\mathrm{t})$ can be obtained numerically from the limit form relation (Hagen and Tvedt 1991)

$$\nu_g(\mathrm{t}) = \lim_{\delta \mathrm{t} \to 0} \frac{P\left[g(\mathbf{r}(\boldsymbol{\theta}, \mathrm{t}), \boldsymbol{\theta}) > 0 \cap g(\mathbf{r}(\boldsymbol{\theta}, \mathrm{t}+\delta \mathrm{t}), \boldsymbol{\theta}) \leq 0\right]}{\delta \mathrm{t}} \tag{4.3}$$

The numerical evaluation of the numerator of Eqn. 4.3 reduces to a time-invariant two-component parallel system reliability analysis. It is clear that Eqn. 4.1 represents the building block for the solution of both time-invariant and time-variant reliability problems (Der Kiureghian 1996). The problem in Eqn. 4.1 is extremely challenging for real-world structures and can be solved only in approximate ways. A well-established method consists of introducing a one-to-one mapping between the physical space of variables $\boldsymbol{\theta}$ and the standard normal space (SNS) of variables $\mathbf{y}$ (Ditlevsen and Madsen 1996) and then computing the probability of failure P_f as

$$P_f = P[\mathrm{G}(\mathbf{y}) \leq 0] = \int_{\mathrm{G}(\mathbf{y}) \leq 0} \varphi_{\mathbf{Y}}(\mathbf{y}) d\mathbf{y} \tag{4.4}$$

where $\varphi_{\mathbf{Y}}(\mathbf{y})$ denotes the standard normal joint PDF and $\mathrm{G}(\mathbf{y}) = g(\mathbf{r}(\boldsymbol{\theta}(\mathbf{y})), \boldsymbol{\theta}(\mathbf{y}))$ is the LSF in the SNS. Solving the integral in Eqn. 4.4 remains a formidable task, but this new form of P_f is suitable for approximate solutions taking advantage of the rotational symmetry of $\varphi_{\mathbf{Y}}(\mathbf{y})$ and its exponential decay in both the radial and tangential directions. An optimum point at which to approximate the LSS $\mathrm{G}(\mathbf{y}) = 0$ is the "design point" (DP), which is defined as the most likely failure point in the SNS, i.e., the point on the LSS that is closest to the origin. Finding the DP is a crucial step of semi-analytical approximate methods to evaluate the integral in Eqn. 4.4, such as FORM, SORM and importance sampling (Breitung 1984). The DP, $\mathbf{y}^*$, is found as solution of the following constrained optimization problem:

$$\mathbf{y}^* = \arg\left\{\min\left(0.5\mathbf{y}^{\mathrm{T}}\mathbf{y}\right) \middle| \mathrm{G}(\mathbf{y}) = 0\right\} \tag{4.5}$$

The most effective techniques for solving the constrained optimization problem in Eqn. 4.5 are gradient-based optimization algorithms (Liu and Der Kiureghian 1991) coupled with algorithms for accurate and efficient computation of the gradient of the constraint function G(**y**), requiring computation of the sensitivities of the response quantities. DDM is an excellent tool for this task. A well known general-purpose optimization software toolbox, Sparse Nonlinear Optimization (SNOPT), is integrated into OpenSees for solving DP search problems (Gu 2008e). SNOPT provides a number of desirable features: (1) applicability to large scale problems, (2) economical computation, (3) tolerance of discontinuities in the function gradients, and (4) flexibility. The SNOPT-based DP search algorithm is, in general, more efficient and robust than classical algorithms (e.g., HLRF). After the DP is obtained, several classical semi-analytical approximate methods, such as FORM and SORM may be used to evaluate P_f or $\nu_g(t)$. However, it is observed that these methods may give very inaccurate results when the systems are highly nonlinear. Instead, advanced sampling techniques, such as importance sampling (IS) and orthogonal plane sampling (OPS) are adopted and further improved for this purpose (Barbato et al. 2008b; Gu 2008e).

In order to analyze the inaccuracies of the FORM/SORM approximations, as well as gain physical and geometrical insight into the structural reliability problem and possibly point to more efficient and accurate computational reliability methods, a new visualization method, the Multidimensional Visualization in Principle Planes (MVPP) (Barbato et al. 2008c), is developed to visualize the LSS in the neighborhood of the DP(s). This method requires finding the trace of the LSS in the planes of principal curvatures at the DP(s) (called principal planes) in decreasing order of magnitude of the principal curvatures. The MVPP method provides important information about the topology of the LSS identifying a reduced dimension subspace (in the space of random variables) of interest and thus requiring a limited number of FE simulations to visualize the LSS (Barbato et al. 2008b; Gu 2008e). A new hybrid reliability method, referred to herein as DP-RS-Sim method and able to enhance the FORM/SORM estimates of time-invariant and time-variant failure probabilities for structural and/or geotechnical systems, is briefly presented and illustrated in the sequel. This method combines (1) the DP search, (2) the Response Surface (RS) method to approximate in analytical (polynomial) form the LSF near the DP, and (3) a simulation technique (Sim), to be applied on the RS representation of the actual LSF. The proposed method is suitable, with minor variations, for both component and system time-invariant reliability problems and for component mean outcrossing rate computations (Barbato 2007; Gu 2008e).

4.1 Time-invariant and time-variant reliability analysis of a 2D SFSI system

Reliability analysis applications are presented here based on the same benchmark SFSI system considered in Section 3. In the time-invariant reliability analysis example, after static application of the gravity loads, the structure is subjected to a quasi-static pushover analysis, in which an upper triangular distribution of horizontal forces is applied at the floor levels. The material parameters, as well as the applied horizontal loads, are considered as random variables. The LSF is defined as $g(\theta) = \Delta_{\text{limit}} - \Delta_1$, where Δ_1 is the first interstory drift and Δ_{limit} is the displacement threshold, taken as 2.5, 5.0, and 7.5cm, respectively. The SNOPT-based DP search algorithm is used to find the DPs. FORM, IS and OPS failure probability estimates are compared to crude MCS results in Figure 4(a). From these results, it is observed that for large thresholds, MCS becomes computationally prohibitive, while FORM, IS and OPS provide consistent results. Other application examples presented elsewhere (Gu 2008e) show that IS and OPS results may differ significantly from FORM results. The same 2D SFSI example is used for time-variant reliability analysis. The stochastic excitation is modeled as a white noise with power spectral density ϕ_0 = 0.0275 m^2/s^3. The LSF is the same as in time-invariant case with the displacement threshold Δ_{limit} taken as 1% of the first story height (i.e., $\Delta_{\text{limit}} = 3.6\text{cm}$, corresponding to a significantly nonlinear behaviour of the structure). The mean upcrossing rate analysis is performed with time interval 0.5s. In Figure 4(b), the MCS estimates of the expected cumulative number of upcrossings, E[N], and the failure probability, P_f, are compared with the upper bound approximation of the failure probability obtained by OPS and FORM. It is observed that OPS results are much more accurate than FORM results, considering MCS results as reference solution (for t = 3s, the relative errors between FORM and MCS results and OPS and MCS results are equal to 134% and 19.6%, respectively) (Gu 2008e).

(a) LSF: $g(\theta) = \Delta_{\text{limit}} - \Delta_1$

Δ_{limit} (cm)		2.5	5.0	7.5
Failure probability (P_f)	FORM	8.94e-2	8.19e-4	1.59e-4
	MCS (1e4 simulations)	9.03e-2	5.0e-4	0
	IS	8.62e-2	8.27e-4	1.42e-4
	OPS	8.85e-2	7.01e-4	1.05e-4

(b)

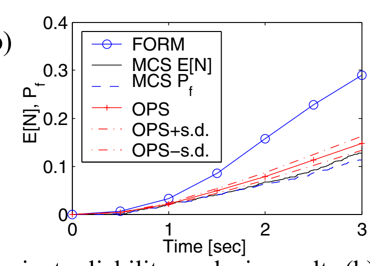

Figure 4. Reliability analysis of a 2D SFSI system: (a) time-invariant reliability analysis results (b) time-variant reliability analysis results.

4.2 MVPP and DP-RS-Sim time-variant reliability analysis for structural system

The use of the DP-RS-Sim method in the case of a time-variant reliability problem is illustrated using a shear-type single-story steel frame with height H = 3.2m, bay length L = 6.0m and made of European HE340A steel columns (Barbato 2007; Conte et al. 2007; Gu 2008e). The frame is subjected to white noise base excitation with power spectral density $\phi_0 = 0.25\mathrm{m}^2/\mathrm{s}^3$ and displacement threshold 0.048m (corresponding to a significantly nonlinear response behaviour of the structure). Figures 5(a) provides visualization of the LSSs at times t = 1.0s and t + δt = 1.001s using the MVPP method in the first principal plane. Figure 5(a) compares the traces of these LSSs obtained by using different response surface approximations, namely a 1st order (FORM), 2nd order and 8th order polynomial approximation. It is seen that the 8th order response surface approximates the actual LSSs fairly well in the first principal plane.

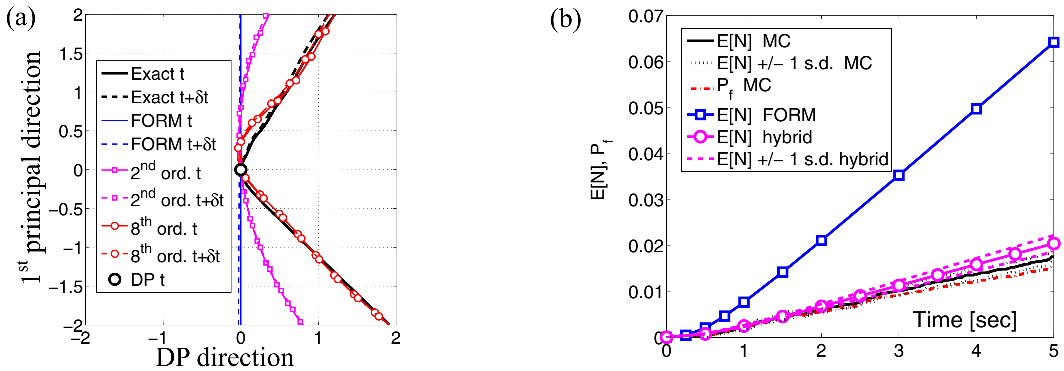

Figure 5. Time-variant reliability analysis of a one-story one-bay frame system modelled as a nonlinear hysteretic system: (a) visualization of LSSs by the MVPP method for mean up-crossing rate computation at time t = 1.0s in 1st principal plane, and (b) time-variant failure probability estimates obtained by using MCS, FORM and DP-RSM-Sim method.

The DP-RS-Sim method is applied to compute the time-variant failure probability (for T = 5.0s) of the inelastic system defined above. After the response surfaces are obtained, the probability content of the hyper-wedge defined by the intersection of the two component failure domains, Eqn. 4.3, is estimated via importance sampling with sampling distribution centred at the DP. Figure 5(b) compares the results obtained through crude MCS for the expected cumulative number of upcrossings, E[N], and the failure probability, P_f, with the upper bound approximation of the failure probability obtained through FORM and DP-RS-Sim. The results obtained show that the DP-RS-Sim method reduces significantly the error by FORM (errors at T = 5.0s are 266% and 16% for FORM and DP-RS-Sim method, respectively), providing very good estimates of E[N] with a reasonable additional computational cost compared to FORM (Barbato 2007; Gu 2008e).

5. CONCLUSIONS

This paper presents recent advances in FE response sensitivity, probabilistic response and reliability analyses of structural and/or geotechnical systems. These advances are integrated into general-purpose software frameworks for nonlinear FE response analysis. The objective is to extend analytical/numerical tools familiar to structural and/or geotechnical engineers for propagating uncertainties through advanced realistic nonlinear response analyses of structural and/or geotechnical systems and obtaining probabilistic estimates of structural performance. Extensions of the Direct Differentiation Method for accurate and efficient computation of system response sensitivities are shown. First-Order Second-Moment (FOSM) probabilistic response analysis, time-invariant and time-variant reliability analyses, and sampling techniques (IS and OPS) are presented and illustrated through structural and SFSI examples. A new Multidimensional Visualization in the Principal Plane (MVPP) technique is employed to study the topology of limit state surfaces near their design points. Based on the insight gained from the MVPP results, the new hybrid DP-RS-Sim method is developed for both time-invariant and time-variant structural reliability analyses. Extension of the DP-RS-Sim method to system reliability applications is currently under study by the authors. From the application examples shown, it is observed that the presented methodology yields, at reasonable computational cost, probabilistic results that are sufficiently accurate for engineering purposes.

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