A NEW HYBRID RELIABILITY ANALYSIS METHOD: THE DESIGN POINT - RESPONSE SURFACE - SIMULATION METHOD

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ABSTRACT :

Classical reliability methods such as First- and Second-Order Reliability Methods (FORM and SORM) have been important breakthroughs toward feasible and reliable integration of probabilistic information and uncertainty analysis into advanced design methods and modern design codes. These methods have been successfully used in solving challenging reliability problems. Nevertheless, caution should be used in the applications of these methods since their limitations and shortcomings in terms of applicability and accuracy are known and documented. Current research trends highlight the importance of structural reliability analysis methodologies that are able to provide improved estimates of the failure probability without excessive increase in computational cost when compared with ordinary FORM/SORM analyses. In this work, a new hybrid reliability analysis method, denoted as Design Point – Response Surface – Simulation (DP-RS-Sim) method is proposed and illustrated. This method innovatively combines the design point (DP) search used in FORM/SORM analyses with the response surface method and appropriate simulation techniques. The need for this combination has emerged from the results obtained through visualization of the limit state surfaces (LSSs) typically used in finite element reliability analysis. In particular, the visualization results show that these LSSs are often highly nonlinear in the neighborhood of their DPs. As application example, the time-invariant reliability analysis of a reinforced concrete frame structure subjected to horizontal pushover loads is considered. DP-RS-Sim-based estimations of the probability of limit state exceedance (expressed in terms of displacement thresholds) by the benchmark structure are compared with FORM, SORM, crude Monte Carlo and Importance Sampling results in terms of accuracy and computational cost. It is shown that the new DP-RS-Sim method can provide accurate failure probability estimates at low computational cost compared to other structural reliability methods.

KEYWORDS: Structural Reliability, Limit State Surface, Response Surface Method, Monte Carlo Simulation, Nonlinear Finite Element Analysis.

1. INTRODUCTION

Classical reliability methods such as First- and Second-Order Reliability Methods (FORM and SORM) have been significant breakthroughs toward feasible and reliable methods for integrating probabilistic information and uncertainty analysis into advanced design methods and modern design codes. These methods have been widely used with success in solving challenging reliability problems. Nevertheless, caution should be used in their applications since limits and shortcomings in terms of applicability and accuracy are known and documented, e.g., (1) existence of multiple design points (DPs) (Der Kiureghian and Dakessian 1998; Au et al. 1999), (2) nonlinearity of the limit state surface (LSS) due to non-Gaussianess of the input process for random vibration problems (Der Kiureghian 2000), (3) nonlinearity of the LSS due to nonlinearity in the system (Barbato 2007).

Accuracy of FORM and SORM approximations is strongly dependent on the nonlinearity of the LSS defining the structural reliability problem at hand. When the LSS are defined implicitly through the response of nonlinear FE models of real-world structural systems and depend on a large number of random variables, the study of the topology of these LSSs is itself a very complex problem. A recently developed visualization method, namely the Multidimensional Visualization in the Principal Planes (MVPP) method (Barbato 2007, Barbato et al. 2008a), provides accurate and efficient tools to study the topology of a LSS surface near its DP(s). MVPP results suggest that strong nonlinearities of the LSS in a neighborhood of the DP could produce significant inaccuracies in FORM- and SORM-based failure probability estimates (Barbato 2007).

Current research trends highlight the importance of structural reliability analysis methodologies able to provide improved estimates of the failure probability without an excessive increase in computational cost when compared with ordinary FORM/SORM analyses. In this work, a new hybrid reliability analysis method, referred to as Design Point – Response Surface – Simulation (DP-RS-Sim) method is proposed and illustrated. The DP-RS-Sim method combines in an innovative way the DP search used in FORM/SORM analyses with the Response Surface (RS) method and appropriate simulation techniques. The application example consists of a reinforced concrete (RC) frame structure subjected to gravity and horizontal pushover loads. DP-RS-Sim-based estimations of the probability of limit-state exceedance (expressed in terms of displacement thresholds) by the benchmark structure are compared with FORM, SORM, crude Monte Carlo and Importance Sampling results in terms of accuracy and computational cost.

2. THE DP-RS-SIM METHOD

Information about the topology of the LSS(s) near the DP(s) can be used effectively in order to improve on the FORM approximation accounting for nonlinearities in the limit state function (LSF). Indeed, the use of the MVPP method to study the topology of typical LSSs indicates that the inaccuracy in FORM/SORM approximations of nonlinear LSS(s) can be a major source of error in estimating time-invariant and time-variant failure probabilities for nonlinear inelastic structural systems. Based on this new insight, a novel hybrid reliability method, namely the DP-RS-Sim method, has been developed (Barbato 2007; Gu 2008).

2.1. Basic Features of the DP-RS-Sim Method

The DP-RS-Sim method combines: (1) the concept of DP, (2) the Response Surface (RS) method to approximate in analytical (polynomial) form the LSF near the DP, and (3) a simulation technique (Sim) to be applied on the RS representation of the actual LSF. The innovative integration of these three methods together with the insight gained through the MVPP method provides several beneficial properties:

1. The DP is an optimal center point for generating a RS approximation of a LSF. This fact is well known and documented in the literature (Yao and Wen 1996; Carley et al. 2004) and, therefore, several approximate methods have been proposed to find a suitable center point as close as possible to the DP (Bucher and Bourgund 1990; Rajashekhar and Ellingwood 1993; Breitung and Faravelli 1996; Yao and Wen 1996; Zhao et al. 1999). In the work of Huh and Haldar (2002), the use of the DP obtained through FORM analysis is directly employed.
2. In general, the application of the RS method is limited to problems defined in terms of a small number of variables (5-7 at most), due to the fact that the number of samples required to define the RS approximation increases exponentially with the number of dimensions (or basic random variables). Several techniques have been proposed to decrease the total number of parameters to be explicitly considered in the definition of the RS approximation, neglecting some parameters or lumping parameters in groups (e.g., Schotanus 2002). The proposed method does not require eliminating parameters at the modeling stage but is able to capture the nonlinearities in the LSF by using a relatively small number of transformed parameters.
3. Simulation techniques are very general and able to take into account the existence of multiple DPs and multiple failure modes (system reliability) without additional approximations. However, when sampling requires a FE analysis of a large nonlinear model of a complex real-world structural system, the computational cost of generating a large number of samples can be unfeasible and inhibit the use of simulation techniques in FE reliability analysis. The capability of accounting for multiple DPs and multiple failure modes is retained by the proposed method, while the relative computational cost of a single simulation is reduced dramatically and consists of a simple polynomial evaluation, making possible the generation of millions of samples in a very short time on a regular personal computer.

Since the proposed method is based on simulation techniques for estimating the failure probability, in principle, it is suitable for both component and system time-invariant reliability problems and for component mean outcrossing rate computations with only minor variations.

2.2. DP-RS-Sim Method for time-invariant component reliability analysis

The main steps of the DP-RS-Sim method for time-invariant component reliability analysis involving a LSS with a single DP are:

1. *DP search (step common to FORM, SORM and MVPP method).*

The DP is defined as the most likely failure point in the standard normal space, i.e., the point on the LSS that is closest to the origin. The DP is an optimum point at which to approximate the LSS and also an optimum center point for a RS model. Finding the DP is a crucial step for approximate semi-analytical methods (e.g., FORM, SORM and importance sampling) to evaluate the time-invariant failure probability (Au and Beck 1999; Breitung 1984; Der Kiureghian et al. 1987).

The DP, **y***, is the solution of a nonlinear constrained optimization problem, in which the constraint function depends on the random modeling parameters both explicitly and implicitly, through the response of a FE structural model. This nonlinear constrained optimization problem is efficiently solved by employing gradient-based optimization algorithms (Gill et al. 1981; Liu and Der Kiureghian 1991) coupled with algorithms for accurate and efficient computation of the gradient of the constraint function (i.e., FE response sensitivities). Herein, FE response sensitivities are computed through the Direct Differentiation Method (Zhang and Der Kiureghian 1993; Kleiber et al. 1997; Conte el al. 2003).

2. *Change of the reference system (step common to SORM and MVPP method).*

After the DP is obtained, a new reference system in the standard normal space is defined so that the n-th axis (with n = number of random variables) is oriented in the direction corresponding to the DP vector **y*** and the new origin coincides with the DP. This new reference system is defined by a rotation matrix **R** obtained through an orthonormalization technique (Barbato 2007; Gu 2008). This reference system is very convenient for fitting a RS model to the LSF near the DP.

3. *Determination of the principal directions (PDs) of interest (step common to SORM with curvature fitting, see Breitung 1984, and MVPP method).*

Each principal plane (PP) is defined by the direction of the DP vector (i.e., the n-th axis in the new reference system) and one of the eigenvectors (Principal Directions: PD) of the normalized and reduced Hessian matrix **A**:

$$\mathbf{A} = \frac{\mathbf{H}_{red}}{\left\| \nabla_y G\big|_{y^*} \right\|} \tag{2.1}$$

in which $\mathbf{H}_{red}$ = reduced Hessian computed at the DP in the standard normal space, defined so that $[\mathbf{H}_{red}]_{ij} = [\mathbf{R} \cdot \mathbf{H} \cdot \mathbf{R}^T]_{ij}$, i, j = 1, 2, ..., n-1, $\mathbf{H}$ = Hessian matrix of the LSF at the DP in the standard normal space, and $\left\| \nabla_y G\big|_{y^*} \right\|$ = Euclidean norm of the gradient of the LSF at the DP. The PDs are sorted in decreasing order of magnitude (absolute value) of the corresponding eigenvalues. Only the first few PDs are computed (using any algorithm for finding the eigenvalues/eigenvectors of a real-valued symmetric square matrix) and the corresponding PPs are obtained. Only these few PDs are needed to represent the nonlinearities in the LSF.

4. *Decomposition of the LSF in a linear and a nonlinear part.*

It has been observed (Barbato 2007; Gu 2008) that often the values of the principal curvatures of the LSS at the DP decrease very fast for increasing order of the PDs. The MVPP results confirm that even strongly nonlinear LSSs concentrate their nonlinearity in only few PDs, while the LSSs are almost linear in the subspace defined by the remaining variables. Therefore, it is useful to separate from the contributions of all variables to the LSF the contribution of variables defining a subspace of the standard normal space in which the LSF at the DP is strongly nonlinear. The contribution of the other remaining variables to the LSF can be linearized with little or negligible loss of accuracy. Evaluation of the linear part requires only knowledge of the gradient of the LSF computed at the DP in the standard normal space

5. *RS approximation of the nonlinear part of the LSF.*
The nonlinear part of the LSF needs to be approximated using the RS method. Any of the existing methods can be applied. Herein, the recently proposed Multivariate Decomposition Method (Xu and Rahman 2004, 2005; Rahman and Xu 2004; Rahman and Wei 2006; Wei and Rahman 2007) is adopted. Both univariate and bivariate decomposition methods have been employed (Barbato 2007).

6. *Computation of P_f through simulation.*
The estimate of the time-invariant failure probability can be obtained using crude Monte Carlo simulation or any other more advanced variance reduction simulation technique (e.g., importance sampling) applied on the analytical RS approximation of the actual LSF. In this work, importance sampling is employed with sampling distribution taken as the standard normal joint PDF centered at the DP.

3. APPLICATION EXAMPLE

The benchmark structure considered in the application example consists of a two-story two-bay reinforced concrete frame on rigid base, a model of which is shown in Fig. 3.1a. This frame structure is modeled using displacement-based Euler-Bernoulli frame elements with distributed plasticity, each with four Gauss-Legendre integration points along its length. Section stress resultants at the integration points are computed by discretizing the frame sections into layers (i.e., the 2-D equivalent of fibers for the 3-D case). The concrete is modeled using a smoothed Popovics-Saenz model with zero tension stiffening for the envelope curve (Balan et al. 1997, 2001; Kwon and Spacone 2002; Zona et al. 2005). This model is obtained from the model presented in Zona et al. (2004) smoothing the unloading/reloading branches with third-order polynomials to preserve the smoothness of the monotonic envelope also in the cyclic behavior. Different material parameters are used for confined (core) and unconfined (cover) concrete in the columns. Typical cyclic stress-strain responses for both confined and unconfined concrete models are shown in Fig. 3.1b. The constitutive behavior of the reinforcement steel is modeled using the Menegotto-Pinto constitutive model with kinematic hardening (Menegotto and Pinto 1973; Barbato and Conte 2006). A typical cyclic stress-strain response of the Menegotto-Pinto steel model is plotted in Fig. 3.1c.

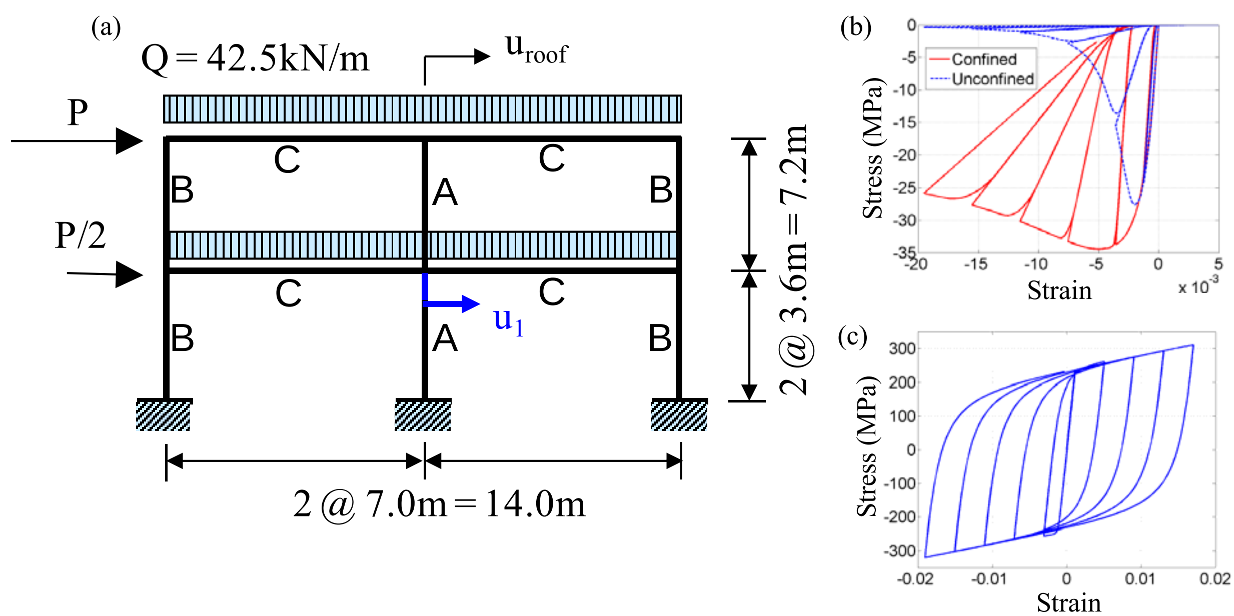

Figure 3.1 Benchmark structure: (a) geometry, (b) typical cyclic stress-strain response for the concrete constitutive model, and (c) typical cyclic stress-strain response for the steel constitutive model

Thirteen material constitutive parameters are used to characterize the various structural materials present in the structure, namely five parameters each for the confined concrete ($f_{c,core}$: peak strength, $\varepsilon_{c,core}$: strain at peak strength, $f_{0,core}$: residual strength at a control point, $\varepsilon_{0,core}$: strain at which the residual strength is reached, $E_{c,core}$: initial tangent stiffness) and the unconfined concrete ($f_{c,core}$, $\varepsilon_{c,core}$, $f_{0,core}$, $\varepsilon_{0,core}$, $E_{c,cover}$), and three parameters for the reinforcement steel (f_y: yield strength, E_0: initial stiffness, b: post-yield to initial stiffness ratio). These material parameters are modeled as random fields spatially fully correlated, i.e., each material parameter is modeled with a single random variable. The marginal PDFs of these material parameters were obtained from studies reported in the literature based on real data (Mirza and MacGregor 1979; Mirza et al. 1979) and are presented in Barbato (2007).

After static application of the gravity loads (assumed as uniformly distributed load per unit length of beam with deterministic value Q = 42.5kN/m at each floor, which corresponds to a uniformly distributed load per unit area q = 8.5kN/m^2 assuming an inter-frame distance of L' = 5.0m), the structure is subjected to a quasi-static pushover analysis, in which an upper triangular distribution of horizontal forces is applied at the floor levels (see Fig. 3.1a). The horizontal force applied at the roof level, P, is modeled as lognormal random variable with mean μ_P = 350kN and cov = 20%, while the horizontal force applied at the first floor level is considered fully correlated with P and with value P_1 = P/2. FE response, response sensitivity and reliability analyses are performed using the FE analysis framework OpenSees (Mazzoni et al. 2005), in which three-dimensional frame elements were augmented for response sensitivity analysis (Barbato et al. 2006) and the response sensitivity algorithm for imposing multipoint constraints was implemented (Gu 2008).

Table 3.1 Time-invariant reliability analysis results

Analysis	P_f	CPU relative time
FORM	0.0203	1
$SORM_B$	0.0223	2.26
$SORM_{HR}$	0.0257	2.26
$IS_{0.05}$	0.0266	53.83
$IS_{0.01}$	0.0262	1103.51
DP-RS-$Sim_{(univ,1)}$	0.0264	2.63
DP-RS-$Sim_{(univ,2)}$	0.0269	2.90
DP-RS-$Sim_{(univ,3)}$	0.0269	3.17

A roof displacement u_{lim} = 0.144m (corresponding to a roof drift ratio of 3.0% and computed from the horizontal displacement of the top of the middle column) is considered as failure condition. Thus, the LSF is given by g = 0.144m - u_{roof}. The DP search is performed with the origin of the standard normal space as starting point using the improved HL-RF algorithm (Ditlevsen and Madsen 1996). Solutions from FORM, SORM based on the Breitung formula ($SORM_B$, Breitung 1984), SORM based on the Hohenbichler-Rackwitz formula ($SORM_{HR}$, Hohenbichler and Rackwitz 1986), importance sampling with cov = 0.05 ($IS_{0.05}$), and importance sampling with cov = 0.01($IS_{0.01}$) are compared to the failure probabilities obtained using the DP-RS-Sim method in Table 3.1. The univariate decomposition (Xu and Rahman 2004, 2005) is adopted to obtain the RS approximation of the LSF at the DP. Three different approximations are considered in the decomposition of the LSF into linear and nonlinear part, corresponding to retaining the nonlinear contribution of one (DP-RS-$Sim_{(univ,1)}$), two (DP-RS-$Sim_{(univ,2)}$) and three (DP-RS-$Sim_{(univ,3)}$) variables, respectively. A fourth order approximation over a square grid of side length equal to two units and centered at the DP is used for each of the univariate components (i.e., five points are used in the Lagrangian interpolation, with points positioned at -1.0, -0.5, 0.0, 0.5 and 1.0 on each axis). The visualization of the trace of the LSS in the first PP is shown in Fig. 3.2 together with the traces of 1st (FORM), 2nd (SORM) and 4th order (RS) approximations of the LSF. It is observed that: (1) the 4th order RS representation of the LSF provides a very good approximation of the LSS trace in the first principal plane, and (2) the DP-RS-Sim provides accurate estimates of the failure probability (considering the estimate obtained using the $IS_{0.01}$ as reference solution) even employing only one principal direction in the nonlinear part approximation of the LSF at a small increase in computational cost compared to FORM and SORM analyses.

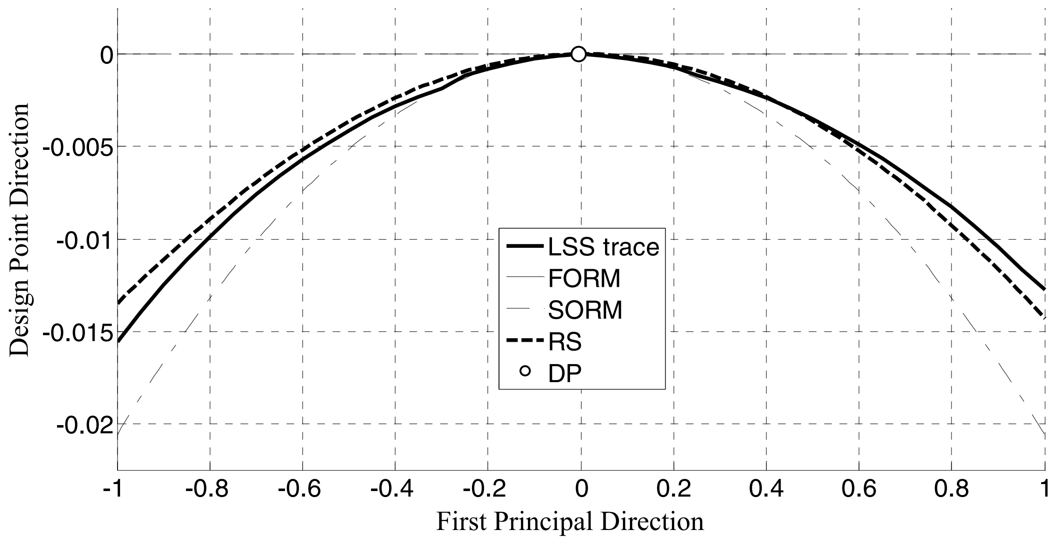

Figure 3.2 MVPP visualization of the LSS in the first principal plane.

4. CONCLUSIONS

A new hybrid finite element (FE) reliability method, denoted as DP-RS-Sim method, is developed and presented in this paper. The DP-RS-Sim method combines the concept of design point, the response surface methodology and simulation techniques to improve the accuracy of failure probability estimates obtained by classical FORM and SORM analyses. This new method is applied here to time-invariant component reliability analysis of a two-story two-bay reinforced concrete frame structure subjected to pushover analysis. The structural system is modeled by a nonlinear FE model employing realistic nonlinear constitutive models for concrete and steel materials. Material properties and applied horizontal loads are modeled as random variables, with probabilistic characterization consistent with data available in the literature. Failure probability estimates obtained using the DP-RS-Sim method are compared to FORM, SORM, and importance sampling results. For the presented application example, the DP-RS-Sim method provides accurate estimates of the failure probability at a small computational cost compared to other classical reliability analysis methods. Visualization of the limit state surface using the Multidimensional Visualization in the Principal Planes method graphically explains this improved accuracy.

The DP-RS-Sim method, in principle, can be used in time-invariant component reliability, time-invariant system reliability and time-variant component reliability analyses. Extension of the DP-RS-Sim method to system reliability is currently under study by the authors. The method capabilities and limitations need further study to be fully assessed. Nevertheless, the DP-RS-Sim method is very promising since, as shown here, it is able to provide at reasonable computational cost accurate failure probability estimates for FE reliability problems involving advanced nonlinear FE models and a large number of random variables.

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