PROCEDURE TO ACCOUNT FOR NON-LINEAR EFFECTS IN EMPIRICAL TRANSFER FUNCTIONS

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ABSTRACT :
A method to account for non-linear effects in the response of layered soil deposits is presented. This method is applied to modify the linear transfer function, in fact the amplitudes of the transfer function, called empirical transfer function. Actually, reductions and shifts to low frequencies present in the amplitudes of transfer functions under non linear conditions are regarded. These reductions and shifts are done separately. First, changes in both, shear waves velocity and material damping as a function of the peak ground acceleration in rock, are established. Then a spectral function of non linearity is computed. This function is the ratio between the inelastic transfer function, disregarding the shifts to low frequencies (increments in wave number), and the elastic one. The shifts in the frequency domain are explicitly introduced by means of a proper frequency scaling. The method is exact for homogeneous single layers. The method is done in seven steps described in the body of this work. Several results, as non linear transfer function, spectral ratios and synthetic seismograms are presented to show how easy and useful the method is.

KEYWORDS:
Soil deposit, seismic response, non-linear effect, transfer function

1. DYNAMIC GROUND RESPONSE

The effects of the local soil conditions can considerably affect the seismic motion and, for this reason, the structural response. These also called site effects produce significant space variations of the ground motion, including amplifications and/or attenuations of their intensity as well as modifications of their duration and frequency content, which have a determining influence in the structural response. Higher dynamic amplifications that the ground motion undergoes usually appear where the rigidity contrast of grounds are very pronounced. This generally happens near the free surface, especially in areas of sedimentary deposits or alluvial valleys. The horizontal interfaces between layers and the lateral irregularities produce a phenomenon of multiple diffraction of seismic waves, that generates constructive and destructive interferences, and that as well, originate amplifications and attenuations, respectively. In order to evaluate the effects of amplification of the seismic motion in ground deposits it would be desirable to know the site transfer function. The experimental determination consists in the spectral ratio between the motion in the surface and the one in the bedrock. Fourier Amplitude Spectra (FAS) of both motions are used. In the absence of seismic records, the transfer function usually is determined with the one-dimensional wave propagation model. Only in special cases it is made use of models of greater complexity. This theoretical model is the most adequate in seismic engineering for the study of the site effects. In particular, its simplicity allows including, approximately, the effects related to the nonlinear behavior of the dynamic properties of the ground subjected to intense earthquakes.

The response of a soil deposit subjected to seismic excitation is function of several factors that are related to geometry and the material dynamic properties. For practical aims, nevertheless, this complexity can be reduced if the site effects are exclusively related to two parameters that measure the most relevant features of the soil deposit. These are the dominant period of vibration and the effective velocity of propagation of the site. For this, it is common to make use of an approach that consists of replacing the stratigraphic profile by a simple layer with equal effective velocity and dominant period to those of real stratigraphy. This idealization is adapted for layered media noticeably horizontal that respond essentially like a homogenous mantle.

The model of waves propagation commonly used to quantify the seismic motion amplification in soft soils with respect to which it would be had in firm ground, is a ground deposit formed by a single layer on a halfspace, excited by the vertical incidence of shear waves. Suppose, as a reference system, that this model has the vertical coordinate z, positive downwards, with the free surface of the layer in $z = -H$, and the interface between the layer and the halfspace in $z = 0$. In addition, suppose that ρ_s and ρ_0 are the mass densities of the soil and the rock, respectively, and that β_s and β_0 are the shear wave propagation velocities of the ground and the rock, respectively. The impedance contrast is given by $\eta = \rho_s\beta_s/\rho_r\beta_r$, and the wave number is $k = \omega/\beta$. If the excitation or incident wave field is V_o, the transversal displacement, v_s, in the layer can be written as

$$v_s = 2V_o \cos\left[k_s\left(z+H\right)\right]\left[\cos(k_sH) + i\,\eta\,\text{sen}(k_sH)\right]^{-1} e^{i\omega t} \tag{1.1}$$

2. NON-LINEAR BEHAVIOR OF THE SOIL

To account for nonlinear behavior of materials in the studies of dynamic amplification of the ground motion, is necessary to fit the more relevant dynamic properties of the soil with the strain levels expected for the design limit states, in particular, the collapse prevention. In the most of the tests it is verified that the shear stiffness modulus and the internal damping are the parameters more affected by the non-linearity of the soil. In addition, these parameters are those of greater influence in the ground seismic response. A direct way to include the non-linearity of the soil is to prescribe the cyclical relations stress-strain with base in the experimental evidences. It is necessary to introduce two concepts: skeleton curve and unload-reload criterion. The skeleton curve is the stress-strain relation that is obtained during the initial charge. For infinitesimal deformations it must reflect the linear behavior. In fact, the tangent to this curve in the origin has as slope the initial or maximum stiffness modulus. As the deformation increases, the slope of the tangent to the skeleton curve is reduced. With the unload-reload criterion the stress-strain path that must be followed during the unloading and the reloading is described. The skeleton curve and the unload-reload criterion constitute a simple mechanism for the construction of the cyclical stress-strain relations. Typical variations of the non-linearity in the stiffness modulus and the internal damping can be expressed directly as a function of the cyclical shear strain. When using the equivalent linear method (Seed and Idriss, 1970) settles down that the model of linear response is applicable for nonlinear behavior of the soil if the stiffness modulus is approximated as the module that is secant to the origin in the skeleton curve. Besides, the damping ratio is taken proportional to the area of the hysteresis cycle (a measurement of the capacity of energy dissipation of the material. Figure 1, left part, shows a typical stress-strain relationship of Masing type (Newmark and Rosenblueth, 1971). Right part shows that the stiffness modulus (referred as μ) and the damping ratio (ζ) are function of the magnitude of the strain: the rigidity reduces and the damping increases as increases the strain. The values of the stiffness modulus and the damping ratio for the cyclical deformation γ_c, can be considered with the formulas of Hardin and Drnevich (1972):

$$\mu_c/\mu_{max} = \left(1+\gamma_c/\gamma_r\right)^{-1} \tag{2.1}$$

$$\zeta_c = \zeta_{max}\left(1-\mu_c/\mu_{max}\right) \tag{2.2}$$

where μ_c is the secant stiffness modulus for the cyclical deformation γ_c, ζ_c is the damping ratio for the cyclical deformation γ_c, μ_{max} is the maximum stiffness modulus for small deformations, ζ_{max} is the maximum damping ratio for great deformations, γ_r is the reference strain for small deformations equal a τ_{max}/G_{max}, and τ_{max} is the maximum shearing stress or strength of the soil. According to equation 2.2, ζ_c must be interpreted like the damping that arises by nonlinear effects, since for infinitesimal deformations $\mu_c = \mu_{max}$ and therefore $\zeta_c = 0$. When considering the viscous damping of the material, it is had that

$$\zeta_{total} = \zeta_v + \zeta_c \tag{2.3}$$

where ζ_{total} is the total damping and it is formed by the viscous (ζ_v) and hysteretic (ζ_c) dampings of the soil. The curves that are obtained with equations 2.1, 2.2 and 2.3 do not completely reflect the nonlinear behavior of the soil. However, it is accepted that the values derived for the stiffness modulus and the damping ratio represent the average values that would be had for the deformation imposed by a harmonic excitation. There are several empirical relations adjusted to consider the parameters γ_r and ζ_{max} that are required for the application of equations 2.1 and 2.2. For sand and gravel the following equations in terms of the shear wave velocity were obtained (Seed and Idriss, 1970):

$$\log\gamma_r = 0.29 - 1.43\log\beta_s \tag{2.4}$$

$$\zeta_{max} = 0.2 + 0.1\left(\beta_s - 200\right)/800 \tag{2.5}$$

Whereas for saturated clays the following equations in terms of the plasticity index I_p were obtained:

$$\gamma_r = (5.24 I_p - 48.65) \times 10^{-5} \tag{2.6}$$

$$\zeta_{max} = 0.25 \tag{2.7}$$

When the curves of dynamic shear modulus and critical damping for specific soils are known, these will be preferred instead of the empirical relations. The equivalent linear method consists of starting off with the elastic properties of the soil and examining the deformations that are had in a dynamic analysis under an excitation. Later it is made use of equations 2.1 and 2.2 or specific empirical relations are used and then, the values of shear wave velocity $\beta_s = (\mu_s / \rho_s)^{1/2}$ and damping ζ_s compatible with the obtained level of deformation of the initial linear analysis are adopted. It is continued with these values of velocity and damping in a second linear analysis and the deformations are again examined in order to determine the dynamic parameters of the soil compatible with the level of deformations. Typically, after eight iterations one arrives at equivalent values of velocity and damping that reflect the nonlinear behavior of the soil subjected to the deformations imposed by the excitation.

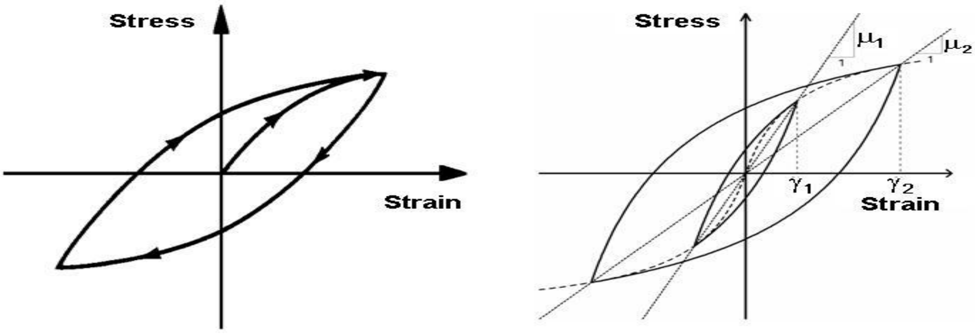

Figure 1. Left: Curved skeleton and load-reload criterion for material behavior of Masing type.
Right: Stress-strain behavior for two levels of deformation

3. EQUIVALENT LINEAR MODEL FOR A HOMOGENEOUS LAYER

In agreement with the procedure described for the application of the equivalent linear method, it is required to know the level deformations inside a continuum medium. The relation between stress and strain in these media is given by the law of Hooke, and the relation between the strain and displacement is given by the Cauchy tensor. Following these relations and considering eq 1.1, it is had that the stress field τ_s in a layer is given by:

$$\tau_s = -2V_o \mu_s k_s \mathrm{sen}\left[k_s (z+H)\right]\left[\cos(k_s H) + i\,\eta\,\mathrm{sen}(k_s H)\right]^{-1} e^{i\omega t} \tag{3.1}$$

and the angular deformation or strain in the medium is given by

$$\gamma_s = -V_o k_s \mathrm{sen}\left[k_s (z+H)\right]\left[\cos(k_s H) + i\,\eta\,\mathrm{sen}(k_s H)\right]^{-1} e^{i\omega t} \tag{3.2}$$

4. PARAMETRIC EXAMINATION OF THE NONLINEAR RESPONSE OF A LAYER

In order to realize a parametric examination of the nonlinear behavior of a layer, particularly of the changes in the transfer function, it is convenient to normalize the layer response in terms of its dominant period. Equation 1 is the displacement field at any depth in the layer in terms of the argument, $k_s H$ that can be written as $\omega T_s (1 - i\zeta_s)/4$. Then, it is had that in the surface of the layer ($z = -H$), the displacement is:

$$v_s = 2V_o \left[\cos\left(\frac{\omega T_s}{4}(1 - i\zeta_s)\right) + i\,\eta\,\mathrm{sen}\left(\frac{\omega T_s}{4}(1 - i\zeta_s)\right)\right]^{-1} e^{i\omega t} \tag{4.1}$$

On the other hand, realizing this same representation in equation 3.2, it leads to the following expression for the deformations inside the layer

$$\gamma_s = -V_o k_s \mathrm{sen}\left[\frac{\omega T_s}{4}\left(1 + \frac{z}{H}\right)(1 - i\zeta_s)\right]\left[\cos\left(\frac{\omega T_s}{4}(1 - i\zeta_s)\right) + i\,\eta\,\mathrm{sen}\left(\frac{\omega T_s}{4}(1 - i\zeta_s)\right)\right]^{-1} e^{i\omega t} \tag{4.2}$$

It is clear that, in agreement with equation 4.1, the displacement in the surface depend on the period of the layer, the damping and the impedances contrast. The strains in an arbitrary depth z depends on these parameters along with the wave propagation velocity β_s, implicit in the wave number k_s. In a parametric study of the nonlinear behavior of a layer, it would be necessary to consider the dominant period as well as the wave propagation velocity, supposing that the strength or the reference strain, implicit in equations 2.1 and 2.2, are constant parameters in the analysis. It was found that there is a relation between the plasticity index and the velocity such that the effects of the non-linearity in the velocity and in the damping are constant, independent of the value of β_s. This is shown in figure 2. The uniform hazard spectrum for firm ground of Mexico City for a return period of 125 years was used as an excitation.

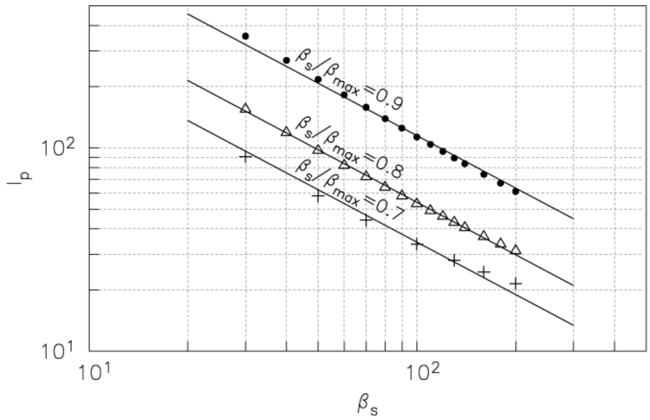

Figure 2 Plasticity index I_p versus wave propagation velocity β_s for constant relations of β_s/β_{max}

4.1. Response and Fourier Amplitude Spectra of uniform hazard for firm ground

Nine Uniform Hazard Response Spectra (UHRS) (5% of damping) for firm ground of Mexico City were used, corresponding to return periods of 10, 20, 30, 50 75, 100, 125.175 and 250 years (Ordaz et al, 2003). The aim is to find tendencies in the changes that are had in the dominant period of a layer as well as in the damping, when varying the intensity of the excitation. These spectra as well as the response spectrum corresponding to a great subduction earthquake (M=8.1) registered in firm ground, are shown in figure 3, left part. Note that for structural periods $T_e > 0.7s$, this spectrum produces intensities with return periods between 75 and 125 years, approximately. For smaller structural periods the contribution of other seismic sources different from the subduction is clear. In the right part, the FAS of the selected UHRS as well as the FAS corresponding to the observed subduction earthquake are shown. For frequencies of the excitation $f < 1.4Hz$, this spectrum produces intensities with return periods between 75 and 125 years, approximately. For greater frequencies the contribution of other seismic sources is also clear.

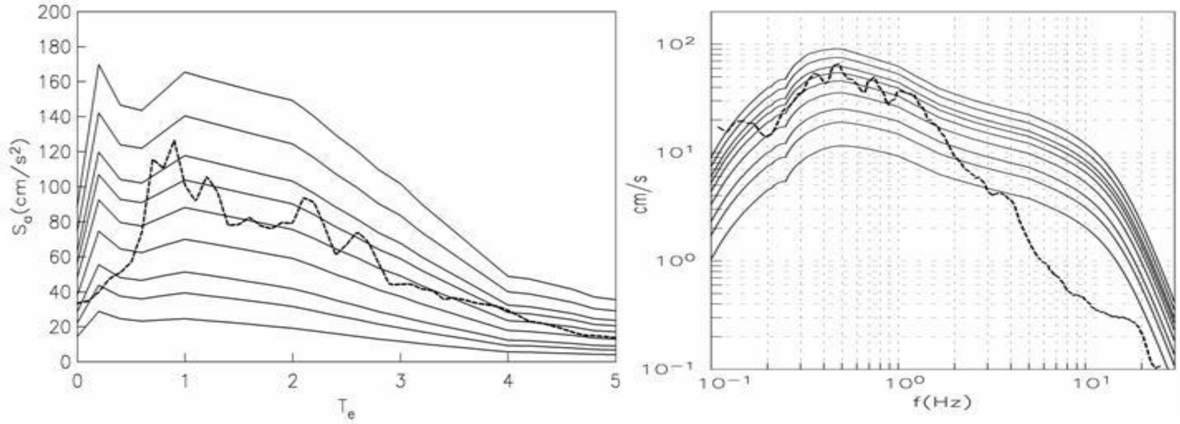

Figure 3. Left: Response spectra (RS) of uniform hazard (solid line) for return periods of 10 (minimum amplitudes), 20, 30, 50, 75, 100, 125, 175 and 250 (maximum amplitude) years. The RS corresponding to the September 19th, 1985, Michoacán earthquake, registered in firm ground, is shown with dashed heavy line. Right: FAS associated to the uniforms hazard (UHRS) (solid line) for the same return periods of the left part. With the dashed heavy line the FAS corresponding to the September 19th, 1985, Michoacán earthquake, registered in firm ground, is shown with dashed heavy line.

4.2. Preliminary calibrations

Let suppose a homogenous mantle characterized by its dominant period, its internal damping and its impedance contrast with the halfspace. Equation 4.1 describes the motion that would be had in the surface under vertical incidence of shear waves as a function of these parameters. It is required to know the velocity β_s in order to apply the equivalent linear method. For Mexico City, some empirical relations that allow knowing the velocity, the impedance contrast and the damping of the layer as functions of the dominant period have been calibrated (Avilés and Perez-Rocha 1998, Aguilar et al 2003). Using these expressions, it was found what plasticity index is required to have a reduction of 20% in β_s under the excitation corresponding to 125 years of return period. After incurring the iterations of the equivalent linear method would be had that $\beta_s/\beta_{max} = 0.80$. The layer periods T_s =1.0, 1,5, 2,0, 2,5, 3 and 4 were selected for this parametric analysis. The peak ground acceleration and the effects of the non-linearity in the velocity (β_s/β_{max}) and in the damping (ζ_c) due to the 9 excitations of figure 3, were determined and displayed in figure 4. It could be pointed out that it is possible to predict the changes in the shear wave velocity and the internal damping in a soil deposit based on the peak rock acceleration. Results of a regression considering all the values calculated for layer periods are shown with solid line and are written as follows

$$\beta/\beta_{max} = 1 - 0.296\, a_o^{r\,0.85} \quad \text{and} \quad \zeta_c = 0.129\, a_o^{r\,0.79} \tag{4.3}$$

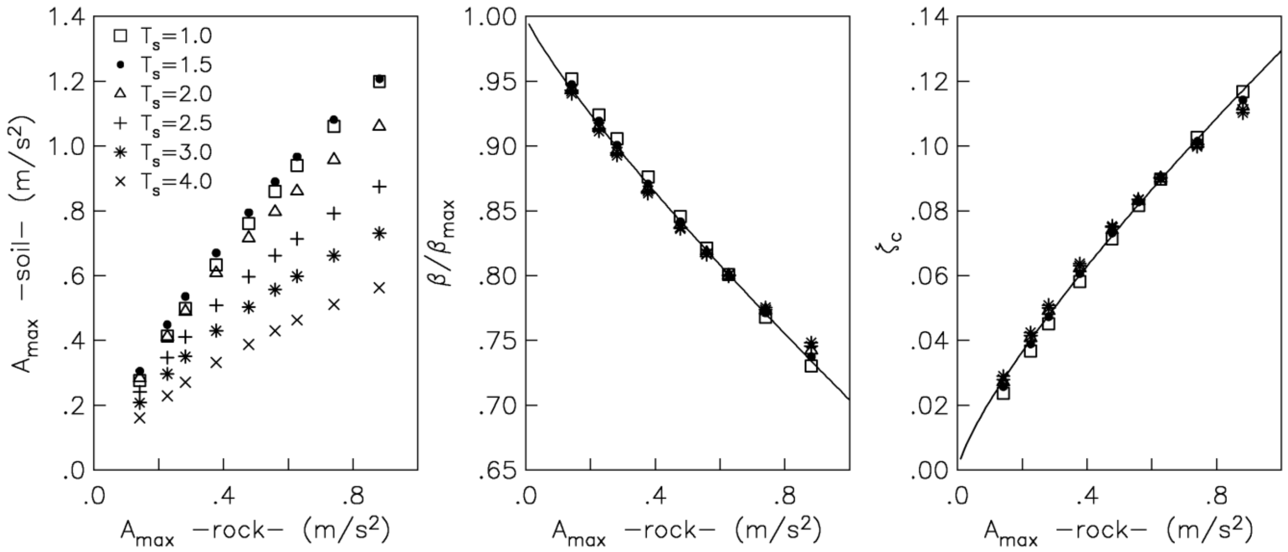

Figure 4 Variation of the response in several soil deposits (characterized by their dominant period) with nonlinear behavior excited by a variety of motions of uniform hazard for different return periods, given in fig 3.

5 SPECTRAL MODEL OF NONLINEAR BEHAVIOR

Let suppose the transfer function given in eq 4.1 with $T_s = 4H/\beta_s$, and $(1-i\zeta) = (1+i\zeta)^{-1}$. Let β_s^o, ζ_s^o and η_s^o be the shear wave velocity, the damping ratio and the impedances contrast for strains within the elastic interval. Analogously, a transfer function for the nonlinear condition can be written by replacing the values $\beta_s^{nl}, \zeta_s^{nl}$ and η_s^{nl}. This substitution will consequently bring a shift towards smaller frequencies, due to the reduction in the velocity, and a reduction in the spectrum amplitudes due to two concepts: the increment in the value of the damping and the reduction of the impedances contrast (the rock remains elastic). In order to separate these effects, the shifting in the frequency and the amplitude reduction, it is proposed to use a transfer function that reflects only the spectral reductions, and later to consider explicitly the reduction in the velocity. That is, substitute ζ_s^o and η_s^o by ζ_s^{nl} and η_s^{nl} in the linear transfer function. The ratio between this two transfer function, linear and pseudo nonlinear, is

$$G(\omega)=\left[\cos\left(\frac{\omega H_s}{\beta_s^o(1+i\zeta_s^{nl})}\right)+i\,\eta_s^{nl}\operatorname{sen}\left(\frac{\omega H_s}{\beta_s^o(1+i\zeta_s^{nl})}\right)\right]\left[\cos\left(\frac{\omega H_s}{\beta_s^o(1+i\zeta_s^{o})}\right)+i\,\eta_s^{o}\operatorname{sen}\left(\frac{\omega H_s}{\beta_s^o(1+i\zeta_s^{o})}\right)\right]^{-1} \quad (5.1)$$

If the empirical transfer for elastic conditions is expressed by means of $E_s^o(\omega)$, with the reductions by non-linearity it would be expressed as

$$\tilde{E}_s^{nl}(\omega)=E_s^o(\omega)\times G(\omega) \quad (5.2)$$

Finally, to include the shifting-to-low-frequencies a frequency scaling given by the factor $\tilde{\beta}=\beta/\beta_{max}=\beta_s^{nl}/\beta_s^o$ is done, that is to say

$$E_s^{nl}(\omega)=\tilde{E}_s^{nl}(\omega\times\tilde{\beta}) \quad (5.3)$$

If $E_s^o(\omega)$ is the transfer function of a homogenous layer the solution is exact. The steps to modify a transfer function by nonlinear behavior are enlisted as follows:

1) Identify the dominant period of the site T_s in the elastic empirical transfer function $E_s^o(\omega)$.
2) Determine the values of the thickness of the deposit, the internal damping, and the impedance contrast.
3) Determine the peak rock acceleration that it is had for the period of interest return.
4) Determine the values of β/β_{max} and ζ_c corresponding to the seismic intensity given by a_o^r with eq 4.3.
5) Determine the values of the shear wave velocity β_s^{nl}, the damping ratio $\zeta_s^{nl}=\zeta_s^o+\zeta_c$, and the impedances contrast η_s^{nl} for the condition of non-linearity imposed by a_o^r.
6) Determine the spectral function ratio of non-linearity with eq 5.1
7) Obtain the nonlinear transfer function incorporating the reductions and shifting given by eqs 5.2 and 5.3, respectively.

This procedure for two homogenous layers of 2 and 4 seconds period is shown in the top and the bottom of figure 5, respectively. The spectral functions of non-linearity considering $a_o^r=0.5\,m/s^2$ are shown in left column. The elastic transfer functions and the elastic ones multiplied by the corresponding nonlinear spectral function, that is to say the reduced functions, are shown in the middle column. Finally, the elastic transfer functions and the final transfer functions after applying to the reductions and shifting by the non-linearity, appears in the right column.

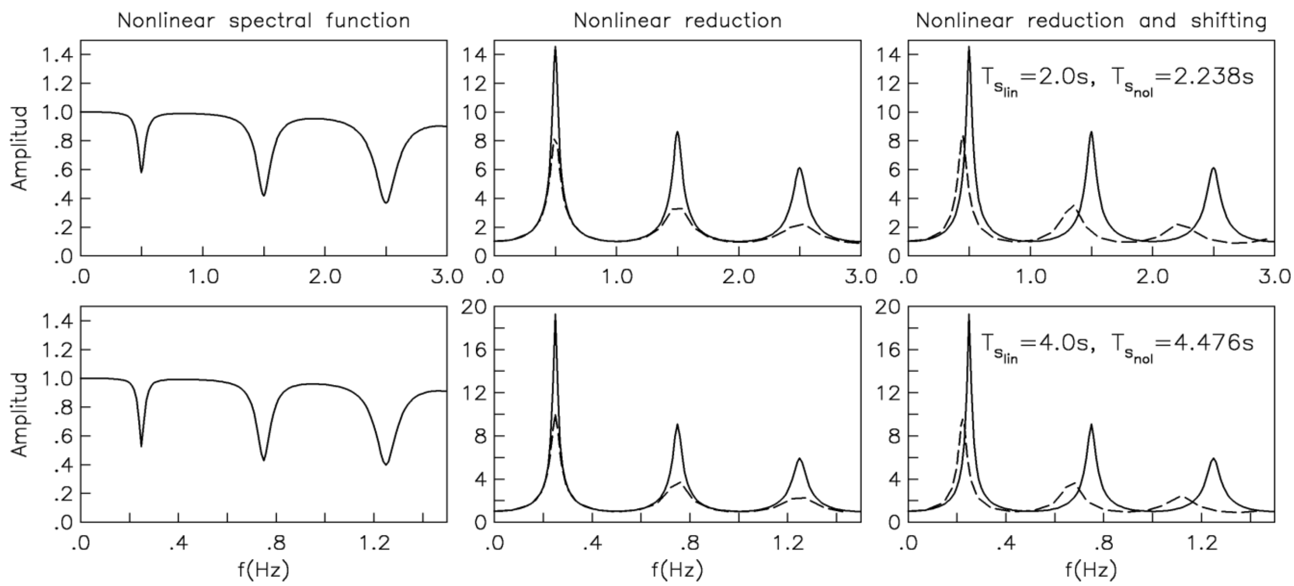

Figure 5. Account for the effects of non-linearity in a spectral analysis of two sites. Top: T_s=2s, Bottom: T_s=4s. Left: nonlinear spectral functions considering $a_o^r=0.5\,m/s^2$. Middle: elastic transfer functions (solid line) and elastic ones affected by the corresponding nonlinear spectral function (dashed lines). Right: elastic transfer functions (solid line) and final transfer functions (dashed line) after reduction and shifting by non-linearity.

5,2 Example

In order to illustrate this method a real empirical transfer function of a site with $T_s = 2s$ was selected. Figure 6, left part, shows the transfer functions that would be had for the nine levels of non-linearity given by the nine excitations given by their FAS of figure 3. Note that these functions are losing their typical characteristic of narrow band as the effects of the non-linearity are increased. It has been observed in rigorous numerical experiments with stratigraphy of greater complexity (Perez-Rocha, 1990). Right part of figure 6 shows the UHRS (5% of structural damping) that would have for each excitation considering the nonlinear effects introduced in the transfer functions of left part. Spectra corresponding to 10 (minimum amplitude), 30, 100 and 250 (maximum amplitude) years are indicated with heavy lines. Note that as the return period grows the peak spectral value appears in greater structural periods. Finally, synthetic accelerogramas for each one of the return periods of 10, 30, 100 and 250 years indicated with heavy lines in figure 6 are shown in fig 7. The method used to obtain accelerograms is the one proposed by Boore (1983).

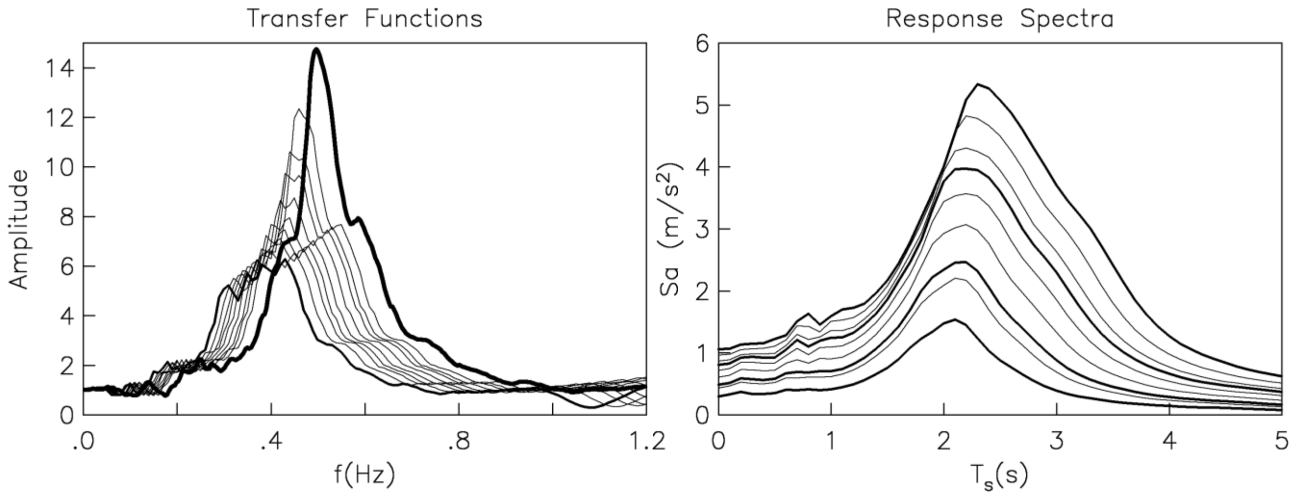

Figure 6. Left: Elastic transfer function (heavy line) and the ones affected by non-linearity under the 9 FAS of figure 3. Right: UHRS (5% of damping) for return periods of 10 (minimum amplitude), 20, 30, 50, 75, 100, 125, 175 and 250 (maximum amplitude) years. Spectra for 10, 30, 100 and 250 years are indicated heavy lines.

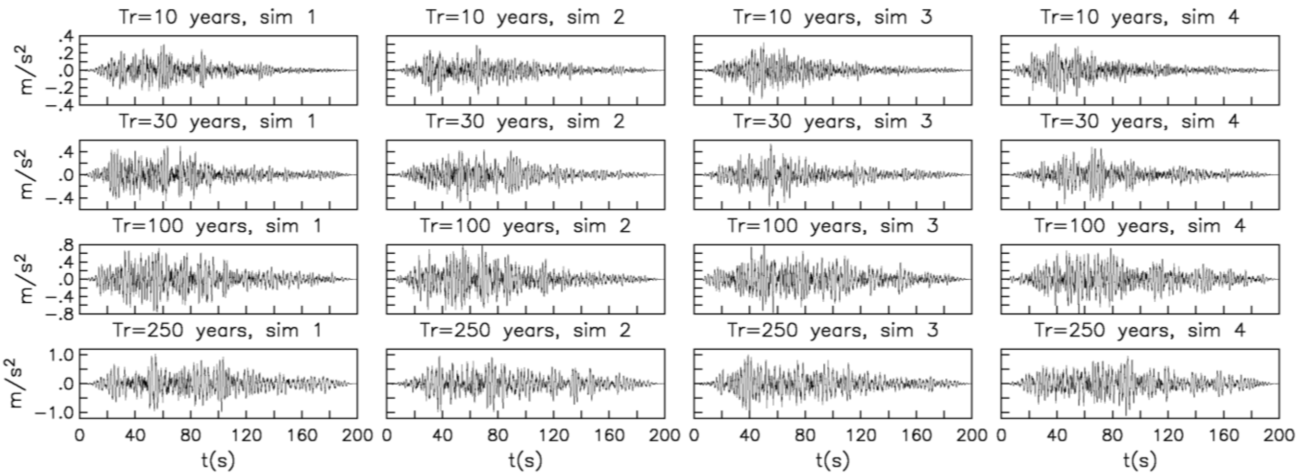

Figure 7. In columns appear four simulations of synthetic accelerograms for the selected site in fig 6. Each row shows the synthetics of each one of the regarded return periods (10, 30, 100 y 250 years).

6. CONCLUSIONS

A method to take into account the effects of the non-linearity in the dynamic response of layered soil deposits was presented. The method consists of taking into account the reductions and the shifting towards smaller frequencies than appear in the amplitude of the transfer functions when conditions of non-linearity. These reductions and shifting are made separately. Firstly the changes in the shear wave velocity and in the internal damping of the soil deposit are settled down based on the peak rock acceleration. Later a spectral function of non-linearity is computed. It is the ratio between the inelastic transfer function without considering the shifting towards smaller frequencies (increases in the wave number) and the elastic transfer function. The elastic transfer function is scaled with this function. The shifting is introduced explicitly by means of a frequency scaling of this transfer function. The method is exact for homogenous mantles. Response spectra and synthetic acelerogramas were computed with these functions. By simplicity empirical relations were used to modify the shear wave velocity and the damping ratio inside the layer. The results are satisfactory, although for realistic situations it will be necessary to establish empirical relations that reflect the behavior of the materials under different levels from non-linearity.

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