PROBABILISTIC EVALUATION OF SEISMIC DEMAND FOR INELASTIC MULTI-STORY REGULAR FRAMES USING MULTIVARIATE NON-LINEAR REGRESSIONS

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ABSTRACT

The prediction of structural demand is of great importance for seismic performance based design. The evaluation of the building's structural performance includes parameters which characterize the intensity of the ground motion (*IM*s) correlated with engineering demand parameters (*EDP*s). The paper presents a study on the variability reduction of the *EDP*s that are expressed as a function of a vector of *IM*s. The multivariate nonlinear regression is used for estimation of the correlation between *IM*s and *EDP*s. Because a single *IM* parameter proved not to be enough to estimate different *EDP*s with sufficient accuracy, a set of two *IM* parameters were considered: the spectral acceleration at the fundamental period of vibration, $SA(T_1)$, and the ratio between the spectral acceleration at the second period of vibration and the spectral acceleration at the fundamental period of vibration, $SA(T_2)/SA(T_1)$. The second parameter was introduced in order to capture the influence of the second vibration mode. The considered *EDP*s are: Maximum Interstory Drift Ratio, Roof Drift Ratio, Average Inter-story Drift Ratio and Peak Inter-story Drift Ratio. The study was conducted on a family of 2D one-bay regular multi-story frame structures having some main characteristics: (i) the mass is constant at all floor levels; (ii) the bay is two times larger than the story height; (iii) the moment of inertia is the same for the columns in a story and the beam above them; (iv) the first mode shape is a straight line; (v) the fundamental period is equal with $0.1N$ and/or $0.2N$; (vi) frames are designed so that simultaneous yielding is attained under a parabolic, linear and uniform load pattern; (vii) hysteretic behavior at the component level is modeled using a bi-linear model with 3% strain hardening in the moment-rotation relationship; (viii) for the non-linear time history analysis, 5% Rayleigh damping is assigned for the first 2 modes of vibration. The efficiency of using a vector of *IM*s to evaluate the *EDP*s for regular structures having a significant influence of the higher modes on their total response is shown.

KEYWORDS: PBEE , Drift Ratios, Non-linear Regression.

1. INTRODUCTION

Research efforts conducted at the PEER Centre have developed a new generation performance assessment methodology formalized on a probabilistic basis and composed of four sequential steps: hazard assessment, structural/non-structural component analysis, damage evaluation, and loss analysis or risk assessment. The product from each of these four steps is characterized by a generalized variable: Intensity Measure (*IM*), Engineering Demand Parameter (*EDP*), Damage Measure (*DM*), and Decision Variable (*DV*). In papers [3] and [6] are summarized the principal steps of this methodology, also adopted for the ATC-58 project. The variables are expressed in terms of conditional probabilities of exceedance (e.g., $p(EDP|IM)$). The general framework is described by the following equation:

$$v(DV) = \iiint G(DV \mid DM)\, dG(DM \mid EDP)\, dG(EDP \mid IM)\, d\lambda(IM) \tag{1.1}$$

This equation is obtained based on the total probability theorem where $v(DV)$ is the mean annual frequency of exceeding a specific value of DV. In this context, DV relates to collapse, loss of lives, direct material losses, and business interruption. $G(DV \mid DM)$ is the probability of exceeding a certain value of DV conditioned on DM (fragility function of DV given DM). The DMs correspond to damage states associated with repairs to structural, non-structural components or contents. $dG(DM \mid EDP)$ is the derivative of the conditional probability of a damage state being exceeded given a value of the EDP. $EDPs$ of interests in this study are story drift indexes. The term $dG(EDP \mid IM)$ is the derivative of the conditional probability of exceeding a value of an EDP given the IM. IM is a ground motion intensity measure, such as peak ground acceleration, spectral acceleration computed for the first mode period and others. Finally, the expression $d\lambda(IM)$ corresponds to the derivative of the seismic hazard curve based on *IM*.

This study focuses on the development of *IM – EDP* relationships based on simulations using nonlinear time history analysis of two-dimensional, non-deteriorating regular moment-resisting frame structures subjected to strong ground motions generated by Vrancea earthquakes. In order to quantify *IM – EDP* relationships, nonlinear time history analyses were performed using a set of 20 ground motions recorded in different locations in Bucharest during the last three significant Vrancea earthquakes. The *EDP*s of interest are those that correlate best with *DV*s corresponding to direct material losses and downtime: (*i*) the *Maximum Inter-story Drift Ratio* (*MIDR*) which represents the maximum value of the ratio between the relative story displacement and the story height; (*ii*) the *Roof Drift Ratio* (*RDR*) which represents the ratio between the maximum displacement at the top of the structure and the total structure height; (*iii*) the *Peak Inter-Story Drift Ratio* (*PIDR*) which represents the maximum value of the ratio between the relative story displacement and the story height, computed for all the stories; (*iv*) the *Average Inter-Story Drift Ratio* (*AIDR*) which represents the average value of all peak inter-story drift ratios.

EARTHQUAKE	FOCAL DEPTH (km)	M_W	STATION	COMP.	PGA (cm/s2)	PGV (cm/s)	PGD (cm)
March 4, 1977	109	7.5	INC	NS	202.3	67.95	16.19
				EW	181.28	29.92	9.01
August 30, 1986	133	7.0	INC	NS	109.12	11.31	2.56
				EW	96.96	15.51	3.75
			PRS	NS	150.10	23.78	3.99
				EW	116.73	10.50	3.19
			MET	NS	71.71	14.75	3.12
				EW	40.67	4.82	1.01
			OTP	NS	219.83	26.17	4.13
				EW	123.64	11.15	1.88
			PND	NS	96.16	15.03	2.83
				EW	89.43	8.15	1.44
			TIT	NS	83.78	7.52	1.37
				EW	87.54	15.38	3.23
May 30, 1990	91	6.7	INC	NS	98.91	16.97	2.91
				EW	66.21	6.35	1.06
			DRS	NS	97.92	5.43	0.95
				EW	112.50	13.82	1.95
			TIT	NS	67.44	9.63	1.65
				EW	56.84	6.66	1.28

Table 1 *Earthquake ground motions characteristics*

The investigations were performed on a generic frame models family similar with the models developed by Medina and Krawinkler, [5]. The most important characteristics of these 2*D* one-bay regular multi-story generic frames used in present study are: (*i*) the mass is constant at all floor levels; (*ii*) the bay is two times larger than the story height; (*iii*) the moment of inertia is the same for the columns in a story and the beam above them; (*iv*) the first mode shape is a straight line; (*v*) the fundamental period is equal with 0.1*N* and/or 0.2*N*, where *N* represent the stories number; (*vi*) frames are designed so that simultaneous yielding is attained under a parabolic, linear and uniform load pattern; (*vii*) hysteretic behaviour at the component level is modelled using a bi-linear model with 3% strain hardening in the moment-rotation relationship; (*viii*) for the non-linear time history analysis, 5% Rayleigh viscous damping ratio is assigned for the first 2 modes of vibration.

2. RESULTS REPRESENTATION

Several studies have shown that the spectral acceleration, $SA(T_1)$ is close related to structural response ([7]). In present paper, the control parameter used to “scale” the ground motion intensity for a given structure strength, or to “scale” the structure strength for a given ground motion intensity, is the parameter $[SA(T_1)/g]/\gamma$ where $SA(T_1)$ represents the 5% damped spectral acceleration for the fundamental vibration mode and γ represents the base shear coefficient $\gamma = V_y/W$, with V_y being the yield base shear strength. The use of $[SA(T_1)/g]/\gamma$ as a relative intensity measure can be viewed in two ways; either keeping the ground motion intensity constant while decreasing the base shear strength of the structure (the *R* factor perspective) or keeping the base shear strength constant while increasing the intensity of the ground motion (the Incremental Dynamic Analysis perspective, [8]). $SA(T_1)$ proved to be not very efficient for tall, long period buildings and when considering near-source ground motions ([4]). For such cases, in this study, a vector valued intensity measure was also analyzed by using a second parameter, defined by the ratio between the spectral acceleration computed for the second mode natural period and the fundamental mode spectral acceleration, $SA(T_2)/SA(T_1)$.

3. DRIFT DEMAND EVALUATION USING A SCALAR INTENSITY MEASURE

Roof Drift Ratio (*RDR*). The maximum roof drift angle, *RDR*, resulted from a nonlinear time history analysis is a global parameter that can be used to relate *MDOF* response to *SDOF* elastic spectral information. For small relative intensities (small $[SA(T_1)/g]/\gamma$ values), the median normalized roof drift ratio $RDR/[SD(T_1)/H]$ is approximately equal to the fundamental mode participation factor, PF_1 defined as the fundamental mode participation factor obtained when the first mode shape is normalized to be equal to one at the roof. Median $RDR/[SD(T_1)/H]$ values decrease up to a relative intensity approximately $[SA(T_1)/g]/\gamma \cong 4$, which imply that in this range, as the relative intensity increases, the values of the ratio between the inelastic roof displacement and the elastic one are decreasing. The dispersion values tend to increase with the level of inelastic behavior as shown in Figure 3. In conclusion, the $RDR/[SD(T_1)/H]$ values are dominated by the first mode. The relationship between the median of the normalized roof drift ratio, relative intensity level and fundamental period is presented in Figure 5 and in Figure 6. Figure 5 shows the variation of the median $RDR/[SD(T_1)/H]$ values with period for different number of stories and $[SA(T_1)/g]/\gamma$=1, 1.5, 2, 3 while Figure 6 shows the same variation for $[SA(T_1)/g]/\gamma$=4, 5, 6, 7. Given the period (T_1=0.6; 1.2; 1.8 seconds), frames with different number of stories have similar median $RDR/[SD(T_1)/H]$ values.

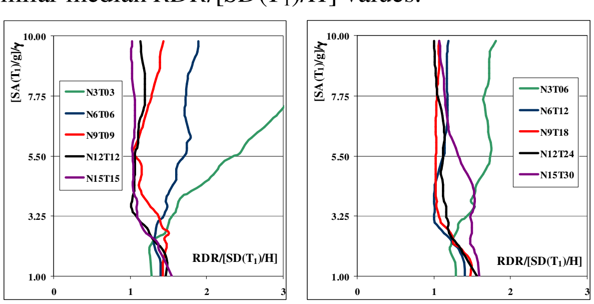

Figure 2 Median $RDR/[SD(T_1)/H]$ values

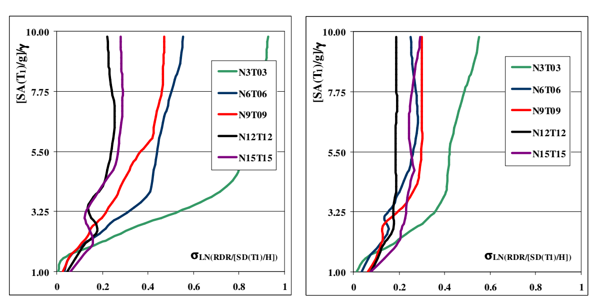

Figure 3 $\sigma_{LN\{RDR/[SD(T1)/H]\}}$ values

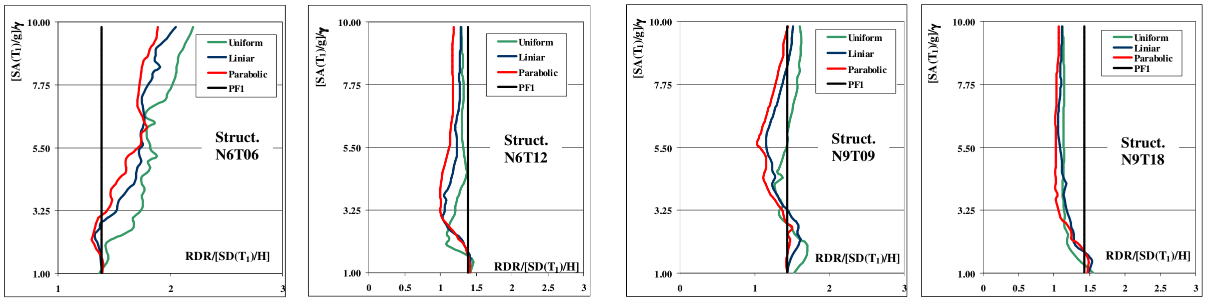

Figure 4 Median $RDR/[SD(T_1)/H]$ and PF_1 values for a : (*i*) uniform; (*ii*) linear; (*iii*) parabolic lateral force distribution

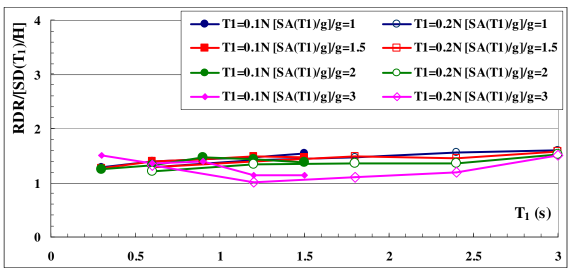

Figure 5
Dependence of median $RDR/[SD(T_1)/H]$ on T_1, for $[SA(T_1)/g]/\gamma$=1; 1.5; 2; 3.

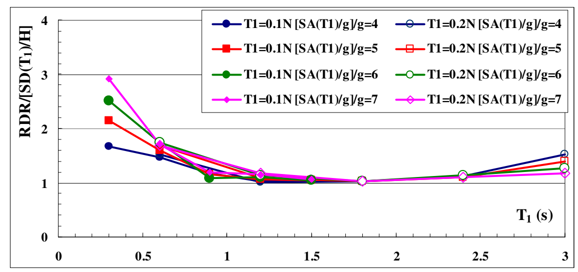

Figure 6
Dependence of median $RDR/[SD(T_1)/H]$ on T_1, for $[SA(T_1)/g]/\gamma$=4; 5; 6; 7.

Average Inter-Story Drift Ratio (*AIDR*). *AIDR* is an important measure of the structural damage if damage is about linearly proportional to drift. Plots of the relationship between median $AIDR/[SD(T_1)/H]$ values and $[SA(T_1)/g]/\gamma$ are shown in Figure 7. Like for *RDR* case, the dispersion values for normalized *AIDR* tend also to increase with the level of inelastic behaviour as shown in Figure 8.

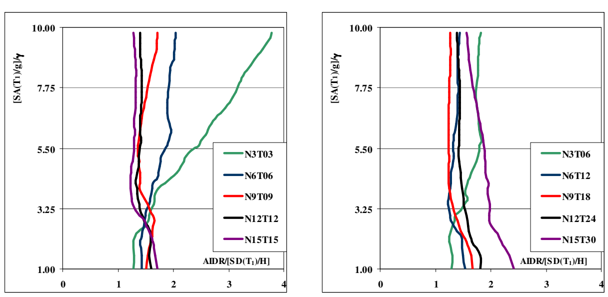

Figure 7 Median $AIDR/[SD(T_1)/H]$ values

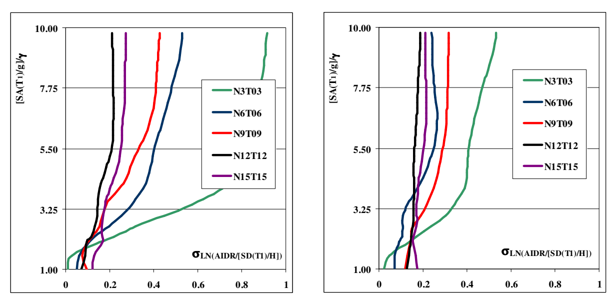

Figure 8 $\sigma_{LN\{AIDR/[SD(T1)/H]}$ values

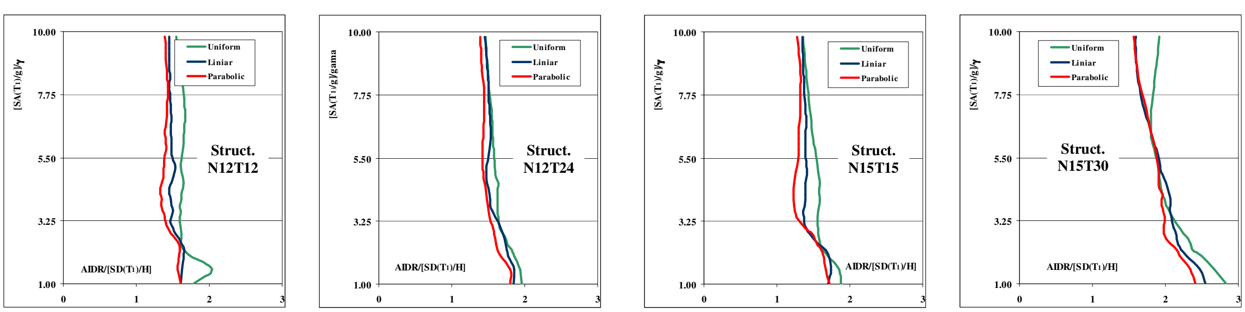

Figure 9 Median $AIDR/[SD(T_1)/H]$ values for a : (*i*) uniform; (*ii*) linear; (*iii*) parabolic lateral force distribution

The similarities between these two drift ratios imply that they are well correlated. This correlation is outlined in Figure 10 in which values of the ratio between median *AIDR* and *RDR* are evaluated for different relative intensities. For case of rigid (T_1=0.1N) frame structures this ratio remains almost constant with $[SA(T_1)/g]/\gamma$. For case of flexible (T_1=0.2N) frame structures this ratio shows slightly larger values and larger variations with increased $[SA(T_1)/g]/\gamma$ values. These larger values are obtained because of the presence of the superior modes of vibration influence. Although for a chosen fundamental period this ratio shows a negligible dependence on the number of stories, a larger one can be observed on the T_1 values, as it can be seen in Figure 13 and Figure 14.

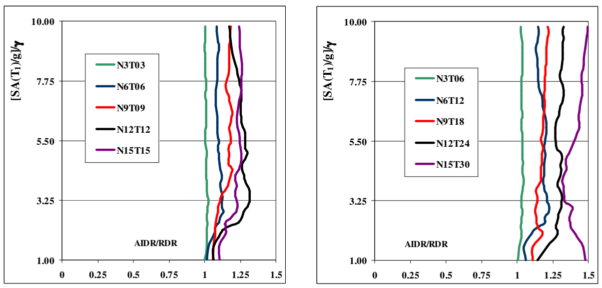

Figure 10 Median *AIDR*/*RDR* values

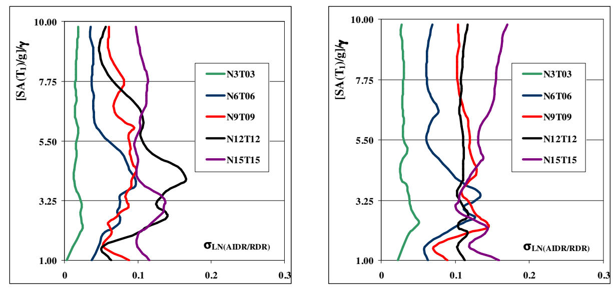

Figure 11 $\sigma_{LN\{AIDR/RDR}$ values

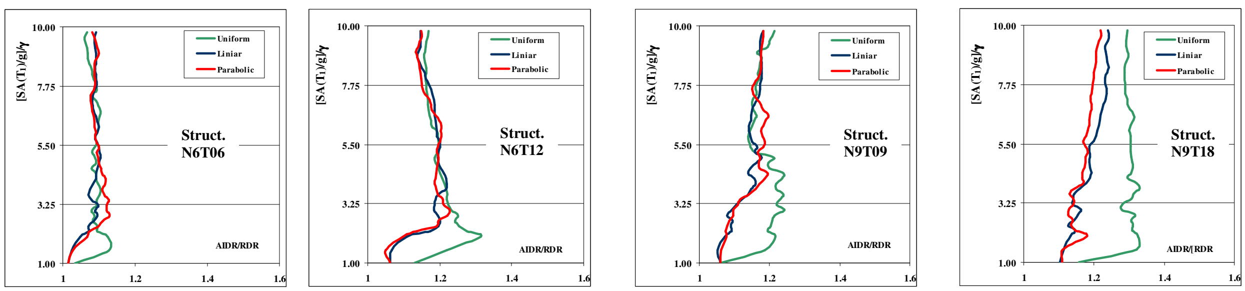

Figure 12 Median *AIDR*/*RDR* values for a : (*i*) uniform; (*ii*) linear; (*iii*) parabolic lateral force distribution

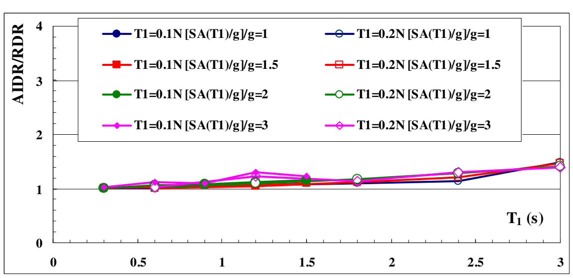

Figure 13
Dependence of median *AIDR*/*RDR* on T_1, for $[SA(T_1)/g]/\gamma$=1; 1.5; 2; 3.

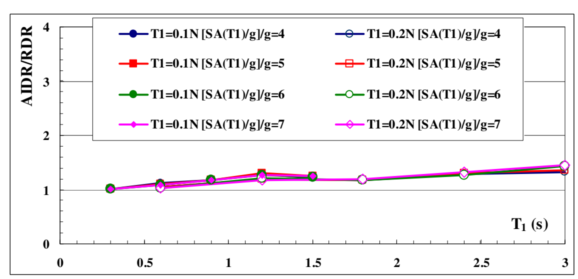

Figure 14
Dependence of median *AIDR*/*RDR* on T_1, for $[SA(T_1)/g]/\gamma$=4; 5; 6; 7.

Maximum Inter-Story Drift Ratio (*MIDR*). An evaluation of the relationship between *MIDR* and $[SA(T_1)/g]/\gamma$, is represented in Figure 15. It can be seen that *MID*R variation doesn't follow the same patterns as the ones observed for *RDR* and *AIDR*. Median *MIDR*/$[SD(T_1)/H]$ values increase with relative intensity having the largest values at around $[SA(T_1)/g]/\gamma$=2. This increase is followed by a rapid decrease that coincides with the migration of the *MIDR* from the top to the bottom stories. Further, the dispersion of *MIDR*/$[SD(T_1)/H]$ values is greater than the one observed for *RDR*/$[SD(T_1)/H]$ and *AIDR*/$[SD(T_1)/H]$. The dispersion of *MIDR*/$[SD(T_1)/H]$ values represented as a function of $[SA(T_1)/g]/\gamma$ is shown in Figure 16. The dispersion increases with period, and in some cases, values for the standard deviation of the natural log of the *MIDR*/$[SD(T_1)/H]$ data can be very big when the models experience small levels of inelastic behaviour. This observation suggests that the *MIDR* over the height is sensitive to the effect of the higher modes of vibration particularly at small relative intensities. Figure 18 and Figure 19 show relationships between the *MIDR*/$[SD(T_1)/H]$, fundamental period, and number of stories. Median *MIDR*/$[SD(T_1)/H]$ values increase with the level of intensity and fundamental period.

Differences of more than a factor of 4 are observed in Figure 18 between $MIDR/[SD(T_1)/H]$ values corresponding to periods of 0.3 and 3.0 seconds. For both small (Figure 18) and large (Figure 19) relative intensities and a given fundamental period (0.6 sec, 1.2 sec, and 1.8 seconds), the stiffer frames ($T_1 = 0.1N$) experiences median $MIDR/[SD(T_1)/H]$ values that are up to 20% larger than those obtained for the flexible frames ($T_1 = 0.2N$). The results show a clear dependence of median $MIDR/[SD(T_1)/H]$ values on the number of stories. For a given relative intensity, median $MIDR/[SD(T_1)/H]$ values increase with the fundamental period.

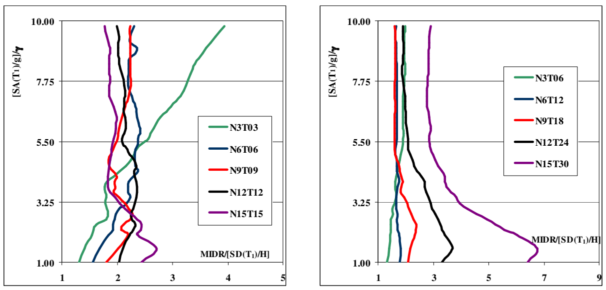

Figure 15 Median $MIDR/[SD(T_1)/H]$ values

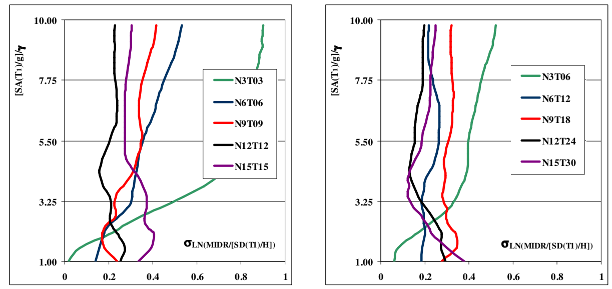

Figure 16 $\sigma_{LN\{MIDR/[SD(T1)/H]\}}$ values

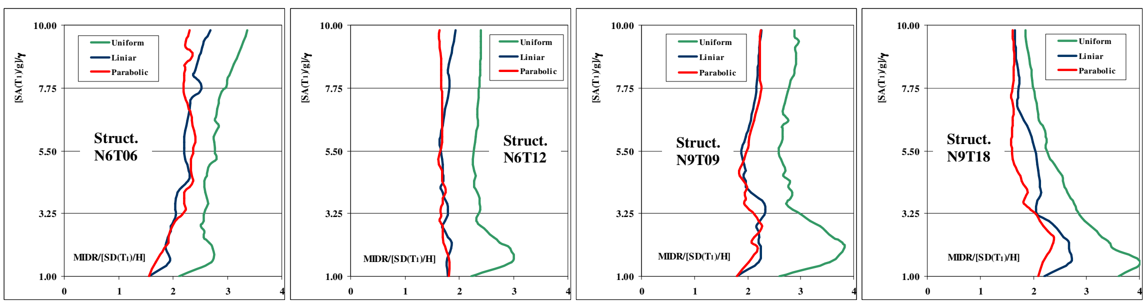

Figure 17 Median $MIDR/[SD(T_1)/H]$ values for a : (*i*) uniform; (*ii*) linear; (*iii*) parabolic lateral force distribution

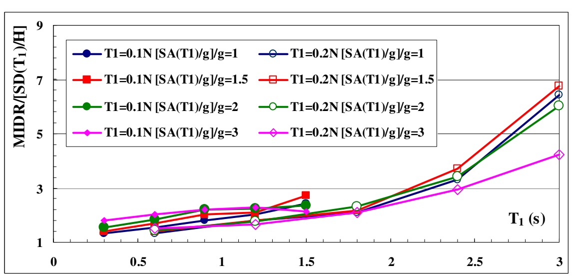

Figure 18
Dependence of median $MIDR/[SD(T_1)/H]$ on T_1, for $[SA(T_1)/g]/\gamma$=1; 1.5; 2; 3

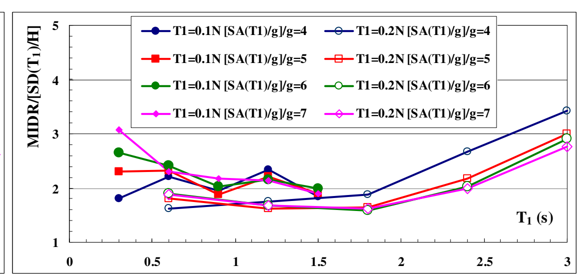

Figure 19
Dependence of median $MIDR/[SD(T_1)/H]$ on T_1, for $[SA(T_1)/g]/\gamma$=4; 5; 6; 7

4. DRIFT RATIOS EVALUATING USING A VECTOR VALUED INTENSITY MEASURE

Since a single ground motion parameter may not be sufficient to describe the response of a structure, one could consider using more than such a parameter. An important goal that can be achieved using a vector-valued *IM* is increased estimation accuracy. In general, by increasing the number of *IM* parameters, more information can be transferred between the ground motion hazard and structural response stages of the analysis. Bazzuro and Cornell ([1]) carried out vector valued regression analysis using spectral acceleration values at two different natural frequencies, and this was demonstrated to be beneficial for the response predictions of flexible structures, having response dominated by multiple modes. In the following, instead of using a scalar ground motion parameter, it is proposed to use a vector of ground motion parameters that can improve the prediction of the structural response measurements.

Multivariate Regression Analysis. To evaluate the predictive power of vector-valued intensity measures, each response measure needs to be regressed on the sets of ground motions parameters. Suppose our interest is in the prediction of a response parameter Y, from an r-dimensional intensity measure vector, **X**. A multivariate regression model can be described as follows:

$$Y = b_0\left(X_1^{b_1} \times X_2^{b_2} \times ... \times X_r^{b_r}\right) \cdot \varepsilon \tag{4.1}$$

where each X_j is a predictor variable that is a part of the vector **X**; b_0 and b_j (j=1..r) are regression coefficients; ε is a residual term associated with each response variable.

Regressions Results. The variations of *MIDR*, *RDR* & *AIDR* as a function of $[SA(T_1)/g]/\gamma$ and $\{[SA(T_1)/g]/\gamma; SA(T_2)/SA(T_1)\}$ for N12T12P structure are presented in the Figure 20 and Figure 21.

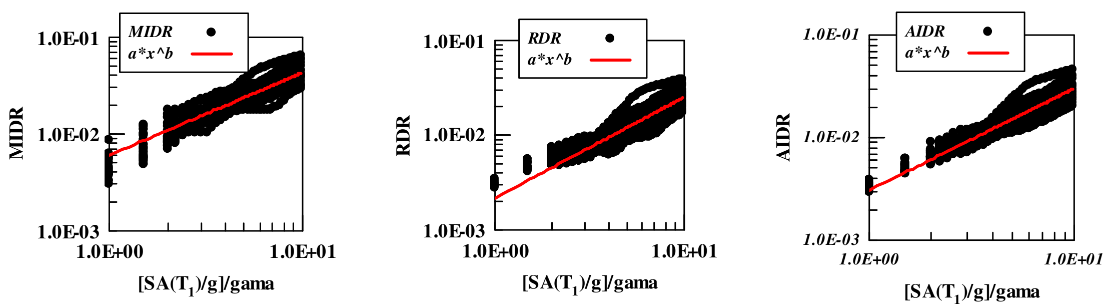

Figure 20 *MIDR*, *RDR* & *AIDR* as a function of $[SA(T_1)/g]/\gamma$ for N12T12P structure

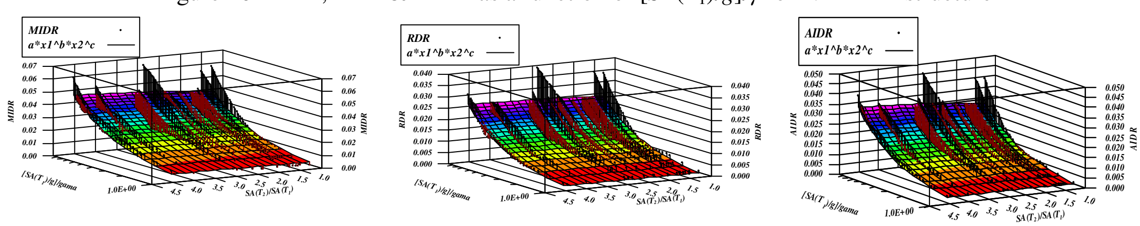

Figure 21 *MIDR*, *RDR* & *AIDR* as a function of $\{[SA(T_1)/g]/\gamma$ & $SA(T_2)/SA(T_1)\}$ for N12T12P structure.

The standard deviations of the residuals for *MIDR*, *AIDR* and *PIDR* are reduced significantly because of the addition of the second intensity measure in the regression analysis, but there is almost no beneficial effect on the standard deviation of the residuals for *RDR*. This supports the statement that *RDR* is mostly influenced by the first mode since the ratio $SA(T_2)/SA(T_1)$ does not improve the regression fit of *RDR*.

5. CONCLUSIONS

1. The normalized Roof Drift Ratio, $RDR/[SD(T_1)/H]$ is approximately equal to the first-mode participation factor, PF_1. This implies that the elastic and inelastic roof displacements are dominated by the fundamental vibration mode (PF_1 is obtained using a fundamental shape normalized to be equal to one at the top level).

2. The normalized Average Inter-Story Drift Ratio $AIDR/[SD(T_1)/H]$ shows almost similar trend to those of the $RDR/[SD(T_1)/H]$. Dispersion of $AIDR/[SD(T_1)/H]$ is comparable to that of the $RDR/[SD(T_1)/H]$ except for low levels of inelastic behavior where the influence of superior modes of vibration results in larger dispersions.

3. The ratio *AIDR*/*RDR* is a stable measure showing a relatively large independency on the fundamental period. *MIDR*/[$SD(T_1)/H$], exceed the *RDR*/[$SD(T_1)/H$] by a percentage that increases with period. For a given period and relative intensity, higher mode effects cause systems with larger number of stories to experience larger median *MIDR*/*RDR*, and hence, a less uniform distribution of *MIDR*.

4. In case of uni-variate regressions ([$SA(T_1)/g$]/γ) a strong variability of the *PIDR* estimated values can be observed. The same variability of the estimated values is observed for case of *MIDR* and *AIDR*, especially for medium height or high structures.

5. *RDR* has a particular characteristic, showing almost the same variability of the estimated values for both cases of flexible and rigid structures considered in the study, mainly because its value is influenced only by the first mode contribution.

6. In case of bi-variate regressions ({[$SA(T_1)/g$]/γ & $SA(T_2)/SA(T_1)$}) estimated values of *PIDR* show a little reduction in residuals standard deviation for small height structures. For medium height or high structures a semnificative reduction of the residuals standard deviation is observed for estimated values of *PIDR*.

7. The *MIDR* and *AIDR* residuals standard deviation has decreased values when the second vibration mode influence is introduced in regression analysis as a predictor variable.

8. The use of bi-variate regressions doesn't result in an perceptible decrease of *RDR* estimated values, the superior modes of vibration contribution being of small importance for the final *RDR* estimated values.

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