NUMERICAL SIMULATION OF THE INELASTIC SEISMIC RESPONSE OF RC STRUCTURES WITH ENERGY DISSIPATORS

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ABSTRACT :

The nonlinear dynamic response of RC buildings with dissipators is studied using advanced computational techniques. A fully 3D geometric and constitutive nonlinear model is used for the description of the dynamic behavior of structures. Each material point of the cross section is assumed to be composed of several simple materials with their own constitutive laws. A specific element based on the beam theory is proposed for the dissipators. Several numerical tests are carried out to validate the proposed model.

KEYWORDS: Nonlinear analysis, beam model, finite elements.

1. INTRODUCTION

Conventional seismic design practice permits designing RC structures for forces lower than elastic ones, on the premise that the design assures significant ductility. Frequently, the dissipative zones are located near the beam-column joints and, due to cyclic inelastic incursions, structural elements can suffer a great amount of damage. New techniques based on adding devices with the objective of dissipating the energy exerted by the earthquake and alleviating the ductility demand on primary structural elements have contributed to improve the seismic behavior of buildings. In the case of *passive energy dissipating devices* (EDD) an important part of the energy input is absorbed and dissipated, therefore, concentrating the nonlinear phenomenon without the need of an external energy supply.
Most of he design methods proposed for RC structures are based on the assumption that the behavior of the bare structure remains elastic, while the energy dissipation relies on the control system. However, experimental and theoretical evidence show that inelastic behavior can also occur in the structural elements of controlled building during severe earthquakes. Considering that most of the elements in RC buildings are columns and beams, one-dimensional formulations for structural elements appear as a solution combining both numerical precision and reasonable computational costs. Some formulations of this type have been extended for considering geometric nonlinearities and considering inhomogeneous distributions of materials on arbitrarily shaped beam cross sections.

Formulations for beams considering both constitutive and geometric nonlinearity are rather scarce; most of the geometrically nonlinear models are limited to the elastic case and the inelastic behavior has been mainly restricted to plasticity. Recently, Mata *et.al.*have extended the geometrically exact formulation for beams due to Reissner and Simo to an arbitrary distribution of composite materials on the cross sections for the static and dynamic cases. EDDs usually have been described in a global sense by means of force-displacement or moment-curvature relationships attempting to capture the energy dissipating capacity of the devices.

In summary, a modern numerical approach to the structural seismic analysis of RC buildings should take into account the following aspects:

(i) Geometric nonlinearity due to the changes in the configuration experienced by flexible structures.

(ii) Constitutive nonlinearity. Inhomogeneous distributions of inelastic materials can appear in many structures. The estimation of the dissipated energy should be considered in a manner consistent with the thermo-dynamical basis of the constitutive theory.

(iii) Control techniques, allowing to improve the dynamics of structures by means of incorporating EDDs.

At the authors's knowledge, there is not an unified approach covering all these aspects in a manner consistent with the principles of the continuum mechanics .

In this work, a fully geometric and constitutive nonlinear formulation for rod elements is extended to the case of flexible RC structures equipped with EDDs. A fiber approach is used for arbitrary distributions of materials on the cross sections. EDDs are considered as beams without rotational degrees of freedom. Thermodynamically consistent constitutive laws are used for concrete and steel. In particular, a damage model able to treat the degradation associated to the tensile and compressive components of stress in an independent manner is presented. The mixing rule is employed for the treatment of the resulting composite. A specific nonlinear hysteretic force-displacement relationship is provided for describing the mechanical behavior of several types of EDDs. Numerical examples cover several complex phenomena such as the inelastic *P*-Δ effect and inelastic dynamic structural torsion.

2. FINITE DEFORMATION FORMULATION FOR STRUCTURAL ELEMENTS

The geometrically exact formulation for rods due to Reissner and Simo is expanded for considering a curved reference configuration. In this section, a brief summary of results relevant for the development of constitutive laws able to be incorporated in the beam theory are presented.

Let $\{\hat{E}_i\}$ and $\{\hat{e}_i\}$ be the spatially fixed *material* and *spatial* frames[1], respectively. The straight reference beam is defined by $\hat{\varphi}_{00}=S\,\hat{E}_1$ with S in $[0,L]$ its arch-length coordinate. The beam cross sections are described by the coordinates ξ_β directed along $\{\hat{t}_{0\beta}\}$ and the position vector of any material point is $\hat{X}=S\,\hat{E}_1+\xi_\beta\hat{E}_\beta$.
A beam with initial curvature is considered with a spatially fixed curve $\hat{\varphi}_{00}$. Each point on this curve has attached an orthogonal frame $\hat{t}_{0i}=\boldsymbol{\Lambda}_0\,\hat{E}_i$ where $\boldsymbol{\Lambda}_0$ in $SO(3)$. The beam cross section $\boldsymbol{A}$ is defined considering ξ_β directed along $\{\hat{t}_{0\beta}\}$. The position vector of a material point on the curved reference beam is $\hat{x}=S\,\hat{\varphi}_1+\boldsymbol{\Lambda}_0\,\xi_\beta\,\hat{E}_\beta$.
The motion deforms points from $\hat{\varphi}_0$ to $\hat{\varphi}_0(S,t)$ (at time *t*) adding a translational displacement $\hat{u}_0$ and the local orientation frame is simultaneously rotated from $\boldsymbol{\Lambda}(S,t)$ to $\boldsymbol{\Lambda}_0$ by means of the *incremental rotation tensor* $\boldsymbol{\Lambda}=\boldsymbol{\Lambda}_n\boldsymbol{\Lambda}_0$ (see Fig. 1).
The position vector of a material point on the current beam is

$$\hat{x}(S,\xi_\beta,t)=\hat{\varphi}(S,t)+\xi_\beta\hat{t}_\beta(S,t)=\hat{\varphi}+\boldsymbol{\Lambda}\xi_\beta\hat{E}_\beta. \tag{1}$$

The deformation gradient is the gradient of the deformation mapping of Eq. (1) and determines the strain measures at any material point of the beam cross section. The deformation gradient (relative to the curved reference beam) is

$$\mathbf{F}_n=\mathbf{F}\mathbf{F}_0^{-1}=g_0^{-1}[\hat{\varphi}_{,S}-\hat{t}_1+\widetilde{\boldsymbol{\omega}}_n\xi_\beta\hat{t}_\beta]\otimes\hat{t}_{01}+\boldsymbol{\Lambda}_n, \tag{2}$$

where $g_0=Det[\boldsymbol{F}_0]$ and $\tilde{\boldsymbol{\omega}}=\boldsymbol{\Lambda}_{n,S}\boldsymbol{\Lambda}^t$ is the spatial curvature tensor relative to the curved reference beam. In Eq. (2), the term $\hat{\gamma}=\hat{\varphi}_{,S}-\hat{t}_1$ corresponds to the reduced spatial strain measure of shearing and elongation.

[1] The indices i and b range over {1,2,3} and {2,3}, respectively; and summation convention holds.

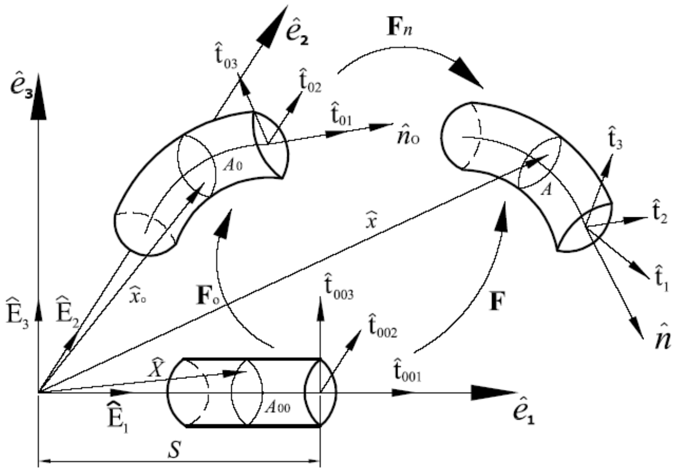

Figure 1. Configurational description of the beam.

$\hat{P}_1$ is *First Piola Kirchhoff* (FPK) stress tensor energetically conjugated pair to $\hat{\varepsilon}=\hat{\gamma}+\tilde{\varpi}\hat{t}_{\beta}$. The corresponding material forms are given by means of the pullback by Λ. Additionally, the spatial form of the stress resultant and the stress couple vectors are

$$\hat{n}(S)=\int_{\mathcal{A}}\hat{P}_1\mathrm{d}\mathcal{A},\qquad \hat{m}(S)=\int_{\mathcal{A}}(\hat{x}-\hat{\varphi})\times\hat{P}_1\mathrm{d}\mathcal{A}.$$

(3)

According to the developments given in Ref. [2], the classical form of the equilibrium equations for rods are

$$\begin{aligned}\hat{n},_S+\hat{n}_p&=\mathcal{A}_{\rho 0}\ddot{\hat{\varphi}}+\underbrace{\tilde{\boldsymbol{\alpha}}_n\hat{\mathcal{S}}_{\rho 0}+\tilde{\boldsymbol{v}}_n\tilde{\boldsymbol{v}}_n\hat{\mathcal{S}}_{\rho 0}}_{D_1},\\ \hat{m},_S+\hat{\varphi},_S\times\hat{n}+\hat{m}_p&=\boldsymbol{\mathcal{I}}_{\rho 0}\hat{\alpha}_n+\tilde{\boldsymbol{v}}_n\boldsymbol{\mathcal{I}}_{\rho 0}\hat{v}_n+\underbrace{\hat{\mathcal{S}}_{\rho 0}\times\ddot{\hat{\varphi}}}_{D_2},\end{aligned}$$

(4)

where $\hat{n}_p$ and $\hat{m}_p$ are the external *body force* and *body moment* per unit of reference length at time *t*, A_{ρ_0}, $\hat{S}_{\rho_0}$ and $\boldsymbol{I}_{\rho_0}$ are the cross sectional mass density, the first mass moment density and the second mass moment density per unit of length of the curved reference beam, respectively.

Considering an admissible variation $(\delta\hat{\varphi},\delta\hat{\theta})$ of the pair $(\hat{\varphi},\Lambda)$ taking the dot product with Eqs. (4), integrating over the length of the curved reference beam and integrating by parts, we obtain the following nonlinear functional *G*

$$\begin{aligned}G_w(\hat{\varphi},\boldsymbol{\Lambda},\hat{h})&=\int_0^L\left[(\delta\hat{\varphi},_S-\delta\hat{\theta}\times\hat{\varphi},_S)\cdot\hat{n}+\delta\hat{\theta},_S\cdot\hat{m}\right]\mathrm{d}S\\&+\int_0^L\left[\delta\hat{\varphi}\cdot\mathcal{A}_{\rho_0}\ddot{\hat{\varphi}}+\delta\hat{\theta}\cdot(\boldsymbol{\mathcal{I}}_{\rho_0}\hat{\alpha}_n+\hat{\boldsymbol{v}}_n\boldsymbol{\mathcal{I}}_{\rho_0}\hat{v}_n)\right]\mathrm{d}S\\&-\int_0^L\left[\delta\hat{\varphi}\cdot\hat{n}_p+\delta\hat{\theta}\cdot\hat{m}_p\right]\mathrm{d}S-(\delta\hat{\varphi}\cdot\hat{n}+\delta\hat{\theta}\cdot\hat{m})\Big|_0^L,\\&=G_w^{\mathrm{int}}(\hat{\varphi},\boldsymbol{\Lambda},\hat{h})+G_w^{\mathrm{ine}}(\hat{\varphi},\boldsymbol{\Lambda},\hat{h})+G_w^{\mathrm{ext}}(\hat{\varphi},\boldsymbol{\Lambda},\hat{\Phi},\hat{h})=0.\end{aligned}$$

(5)

1.1. Energy dissipating devices

The finite deformation model for EDDs is obtained from the previously described rod model, releasing the rotational degrees of freedom and supposing that the complete mechanical behavior of the device is described in terms of the evolution of a unique material point located in the middle of the resulting bar. This point is referred as the *dissipative nucleus* (see Fig. 2).

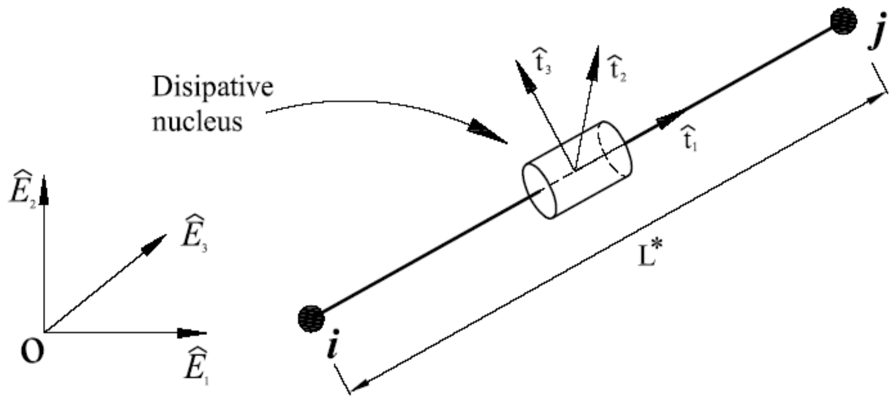

Figure 2. Energy dissipating device.

The current position of a point in the EDD bar is obtained from Eq. (1) but considering that no cross sectional description is required; thus, one can assume $\hat{x}=\hat{\varphi}$. The current orientation of the (straight) EDD bar of initial length L^0 is given by the tensor Λ^0. Assuming that the rotational degree of freedom are released, the spatial position of the *dissipative nuclei* is obtained as $\hat{\varphi}(L^0/2,t)$. The only nonzero component of the strain vector is

$$\mathcal{E}_{d_1}(t) = \hat{\Gamma}_n\Big|_{(L^*/2)} \cdot \hat{E}_1 = [(\mathbf{\Lambda}^{*T}\hat{\varphi},_S)\cdot\hat{E}_1]\Big|_{(L^*/2)} - 1 = [\hat{\varphi},_S\cdot\hat{t}_1]\Big|_{(L^*/2)} - 1, \tag{6}$$

3. CONSTITUTIVE MODELS

In this work, material points on the cross sections are considered as formed by a *composite material* corresponding to a homogeneous mixture of different simple *components*, each of hem with its own constitutive law. The resulting behavior is obtained by means of the *mixing theory.*

3.1. Degrading materials: Tension-compression damage model

In this work the *tension-compression damage* model is modified in order to allow its inclusion in the Reissner-Simo formulation for inelastic rods. They permit to consider two important features of the mechanical behavior of concrete: (i) Independent degradation of the mechanical properties for tensile or compressive loading paths (ii) Large differences in the tensile and compressive thresholds. The model is based on an adequate form of the free energy density depending on two scalar damage variables $d^{\pm}$ in [0, 1], related to the degradation mechanisms occurring under tensile (+) or compressive (-) stress concentrations.

Stress split. Let $\bar{\boldsymbol{P}}^m=\hat{\bar{P}}^m_1\otimes\hat{E}_1=[C^{me}\hat{\varepsilon}\otimes\hat{E}_1]$ be the material form of the elastic FPK stress tensor (see section 2.1) which is consistent with the kinematics of the present rod theory. The following split of the stress tensor is proposed:

$$\begin{aligned}\bar{\mathbf{P}}^{\mathrm{m}+} &= \langle\lambda_1\rangle\hat{e}_{\lambda 1}\otimes\hat{e}_{\lambda 1} = \hat{\bar{P}}_i^{\mathrm{m}+}\otimes\hat{E}_i,\\ \bar{\mathbf{P}}^{\mathrm{m}-} &= \bar{\mathbf{P}}^{\mathrm{m}} - \bar{\mathbf{P}}^{\mathrm{m}+} = \hat{\bar{P}}_i^{\mathrm{m}-}\otimes\hat{E}_i.\end{aligned} \tag{7}$$

Then, the Helmholtz free energy potential of the degrading model [37] is given by

$$\Psi^{\pm} = \frac{1}{2}(1-d^{\pm})\Big[(2\alpha_1+\alpha_2)\bar{P}^{\rm m}_{11}\bar{P}^{\rm m\pm}_{11} + \alpha_1 \bar{P}^{\rm m}_{\beta 1}\bar{P}^{\rm m\pm}_{\beta 1}\Big].$$

(8)

Damage criteria. Two (scalar) equivalent stresses are defined as

$$\bar{\tau}^{+} := \sqrt{\bar{\mathbf{P}}^{\rm m+} : \mathbf{C}_0^{-1} : \bar{\mathbf{P}}^{\rm m+}} = \begin{cases} \sqrt{\Psi_0^{+}} & \text{if} \quad \lambda_1 > 0 \\ 0 & \text{if} \quad \lambda_1 \leq 0 \end{cases},$$

$$\bar{\tau}^{-} := \sqrt{\sqrt{3}(K\bar{\sigma}^{-}_{\rm oct} + \bar{\tau}^{-}_{\rm oct})} = \begin{cases} 0 & \text{if} \quad \lambda_1 > 0 \\ \approx K\bar{P}^{\rm m}_{11} & \text{if} \quad \lambda_1 \leq 0 \end{cases},$$

(9)

where $\bar{\sigma}^{-}_{oct}$ and $\bar{\tau}^{-}_{oct}$ are the octahedral normal and shear stresses which depend on materials properties. Two separated *damage criteria* are defined:

$$g^{\pm}(\bar{\tau}^{\pm}, r^{\pm}) = \bar{\tau}^{\pm} - r^{\pm} \leq 0,$$

(10)

Constitutive relation and dissipation. Considering the free energy density one has that *Clausius-Duheim inequality* can be expressed as

$$\dot{\Phi} = (\hat{P}^{\rm m}_1 - \frac{\partial \Psi}{\partial \hat{\mathcal{E}}_n}) \cdot \dot{\hat{\mathcal{E}}}_n + \Psi_0^{\pm}\dot{d}^{\pm},$$

(11)

which establish that entropy always grows leading to an irreversible process. The following constitutive relation is obtained:

$$\hat{P}^{\rm m}_1 = (1-d^{\pm})\frac{\partial \Psi_0^{\pm}}{\partial \hat{\mathcal{E}}_n} = (1-d^{+})\hat{\bar{P}}^{\rm m+}_1 + (1-d^{-})\hat{\bar{P}}^{\rm m-}_1,$$

(12)

The material form of the *tangent stiffness tensor* is obtained as

$$\dot{\hat{P}}^{\rm m}_1 = (1-d^{\pm})\dot{\hat{\bar{P}}}^{\rm m\pm}_1 - \dot{d}^{\pm}\hat{\bar{P}}^{\rm m\pm}_1 = \mathcal{C}^{\rm mt}\hat{S}_n,$$

(13)

3.2. Constitutive relations for EDDs

The constitutive law proposed for EDDs is based on a previous work of the authors which provides a versatile strain-stress relationship with the following general form:

(13)

The model uncouples the total stress in viscous and non{viscous components, which correspond, in terms of *rheological* models, to a viscous dashpot device acting in parallel with a nonlinear hysteretic spring. The viscous component of the stress has the following form:

$$P^{\rm m}_{d_2}(\dot{\mathcal{E}}_{d_1}, t) = c_d(\dot{\mathcal{E}}_{d_1})\dot{\mathcal{E}}_{d_1},$$

(14)

The response of the nonlinear hysteretic spring is obtained solving the following system of nonlinear differential equations:

$$P_{d_1}^{\mathrm{m}}(\mathcal{E}_{d_1},t) = K_y(\mathcal{E}_{d_1}^{\flat},P_d^{\mathrm{m}\flat})\mathcal{E}_{d_1} + [K_e(\mathcal{E}_{d_1}^{\flat},P_d^{\mathrm{m}\flat}) - K_y(\mathcal{E}_{d_1}^{\flat},P_d^{\mathrm{m}\flat})]e,$$

$$\begin{array}{lll} \text{if} \quad \dot{\mathcal{E}}_{d_1} e \geqslant 0 & \longrightarrow & \dot{e} = \left[1 - \left|\frac{e}{d_y(\mathcal{E}_{d_1}^{\flat},P_d^{\mathrm{m}\flat})}\right|^{n(\mathcal{E}_{d_1}^{\flat},P_d^{\mathrm{m}\flat})}\right]\dot{\mathcal{E}}_{d_1}, \\ \text{else} & \longrightarrow & \dot{e} = \dot{\mathcal{E}}_{d_1}, \end{array}$$

(15)

where Ky is the post yielding stiffness, Ke the elastic stiffness, dy is the yielding strain of the material, and *e* represents an internal variable of plastic (hysteretic) strain, which takes values in the range [-dy; dy]. The parameter n in the associated flow rule describes the degree of smoothness exhibited by the transition zone between the pre and the post yielding branches of the hysteretic cycle. The parameters *Ke, Ky, dy, n* are nonlinear functions of the point where the strain rate changes of sign. Some examples can be seen in figure 3.

4. NUMERICAL EXAMPLES

4.1. Seismic response of a precast RC building with EDDs

The nonlinear seismic response of a typical precast RC industrial building shown in Fig. 4 is studied. The building has a bay width of 24 m and 12 m of inter-axes length. The storey height is 10 m. The concrete of the structure is H-35, (35 MPa, ultimate compression), with an elastic modulus of 29000 MPa.

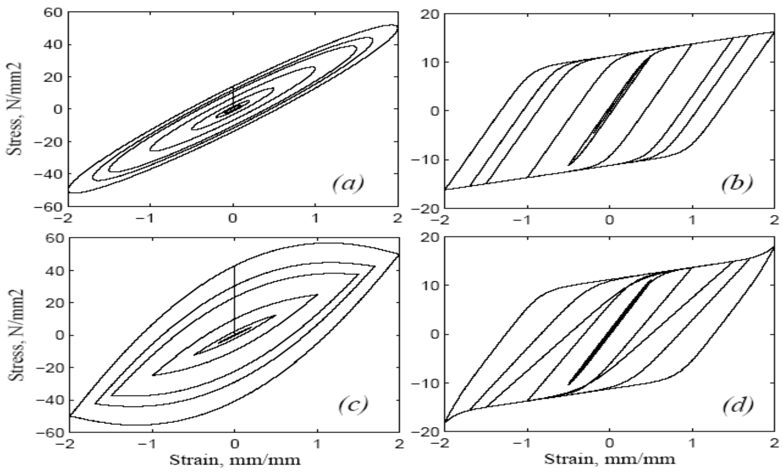

Figure 3. Examples of EDD's behaviors. (*a*): Maxwell model. (*b*): Bilinear inviscid plastic model. (*c*): Non linear dashpot. (*d*): Rubber model.

It has been assumed a Poisson coeFIcient of 0*:*2, a tension/compression relation of *n* =10, fracture energy of *G* + *f* =10 Nmm^2 (*G¡f* =1 Nmm^2). The ultimate tensile stress for the steel is 510 Mpa with ν = 0*:*15, *Gf* =500 Nmm^2, elastic modulus of 200000 Mpa. This figure also shows some details of the steel reinforcement of the cross sections. The dimensions of the columns are 60x60 cm2. The beam has an initial height of 40 cm on the supports and 140 cm in the middle of the span. The permanent loads considered are 1000 Nm^2 and the weight of upper half of the closing walls with 225.000 N. The input acceleration is the N-W component of the El Centro 1940 earthquake record.
First, a set of pushover analyze is performed considering the following cases: (i) The bare frame under small displacements assumption. (ii) The bare frame in finite deformation. (iii) The frame with EDDs and small deformation. (iv) Idem as (iii) but with finite deformation.

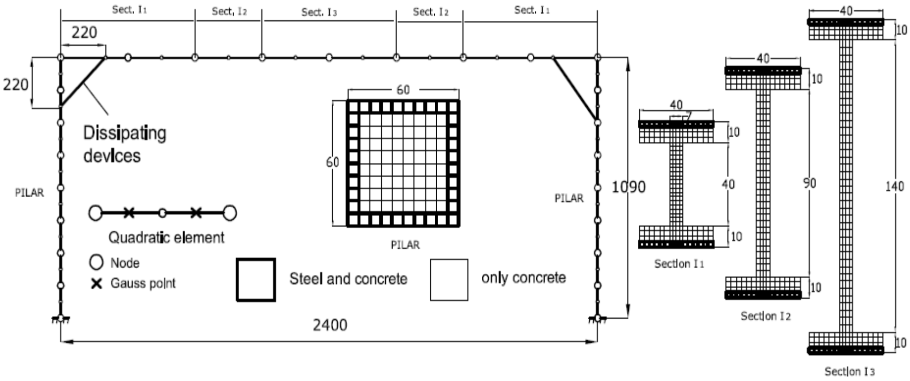

Fig. 4. Numerical model of the precast frame with energy EDDs and cross sectional meshes.

The purpose is to establish clearly the importance of considering second order effect coupled with inelasticity in the study of flexible structures. Fig. 5*a* shows the capacity curves obtained for the four mentioned cases. In this figure it is possible to see that for both, the passively controlled and uncontrolled cases, the small strain assumption overestimate the real load carrying capacity of the structure, due to the fact that the vertical load derived from the self-weight compress the columns, contributing to control the cracking and degradation due to the lateral loading. In the case of finite deformation, second order effects are taken into account, the so called P-Δ, and an anticipated strength degradation is observed for displacements over 60 mm which is a lateral displacement level expectable under strong seismic actions.
Additionally, the incorporation of EDDs increases the stiffness and the yielding point of the structure at global level without affecting the global ductility. Although at material point level, softening is always present for the damage model beyond the linear elastic limit, at global level only the simulation corresponding to case (iv) captures a small part of the softening post peak response.

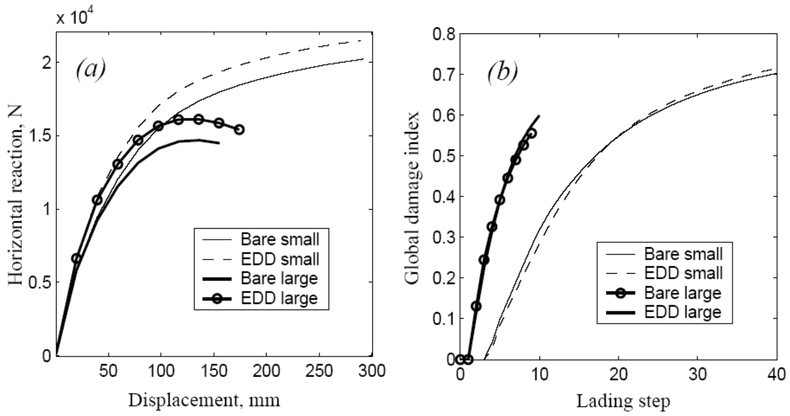

Figure 5. (*a*): Capacity curves. (*b*): Evolution of the global damage index.

Fig. 5*b* shows the evolution of the global damage index for the cases (i){(iv). Here it is possible to appreciate that the global damage index grows quickly for the cases when finite deformation is considered and the bene¯ts of adding EDDs are not visible due to the fact that the pushover analysis does not takes into account energy dissipation criteria. The results of the numerical simulations in the dynamic range allow seeing that the use of plastic EDDs contributes to improve the seismic behavior of the structure for the case of the employed acceleration record.
Fig. 6 shows the time history response of the horizontal displacement, velocity and acceleration of the upper beam-column joint for the uncontrolled and the controlled case. A reduction of approximately 57.5% is obtained for the maximum lateral displacement when compared with the bare frame.

Acceleration and velocity are controlled in the same way, but only 24.3% and 7.0% of reduction is obtained, respectively. A possible explanation for the limited effectiveness of the EDD is that the devices only contribute to increase the ductility of the beam-column joint without alleviating the base shear demand on the columns due to the dimensions of the device and its location in the structure.

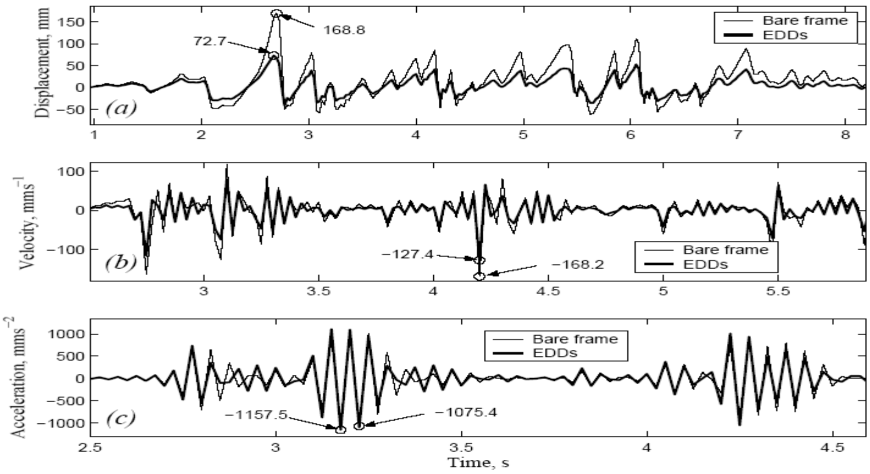

Fig. 6. Time history responses. (*a*): Horizontal displacement. (*b*): Velocity. (*c*): Acceleration.

5. NUMERICAL EXAMPLES

In this work, a geometrically exact formulation for curved and twisted beams has been extended for considering arbitrary distributions of rate dependent inelastic composite materials on the cross sections in the static and dynamic (even seismic) cases. The resulting model is implemented in a displacement based FE code. An iterative Newton-Raphson scheme is used for the solution of the discrete version of the linearized problem. An specific FE element for EDDs is developed (with only one integration point), based on the beam model but releasing the rotational degrees of freedom. Several examples confirm the ability of the model for simulating the nonlinear behavior of RC buildings with energy dissipating devices.

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