The 14th World Conference on Earthquake Engineering
October 12-17, 2008, Beijing, China

SHEAR STRENGTH DEGRADATION INDUCED BY SEISMIC ACTION IN R.C. COLUMNS: EFFECTS ON PUSH-OVER ANALISYS

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ABSTRACT :

Unfortunately, most of the interaction models have been formulated for monotonic increments of the actions. Therefore they don't take into account the effect of strength degradation due to external cyclic actions induced by the earthquake. Recently, the authors extended a model for N-M-V interaction domain evaluation of r.c. elements having rectangular cross-sections, already formulated for static loads, to the case of seismic actions. The new model is obtained by limiting the range of variability of the deviation angle between the directions of the stress-fields and the cracks inclinations, as a function of the amplitude of the cyclic actions. In this paper the model is reformulated for r.c. elements having circular cross-sections. The evaluation of resistance domains for concrete bridge piers having current structural configuration highlight the increment of the risk level induced by shear strength degradation due to cyclic action. Lastly, a numerical example on actual motorway viaduct piers, where extensive experimental tests have been made for material characterization, is developed

KEYWORDS: r.c. circular section, N-M-V domains, plastic approach, cyclic degradation.

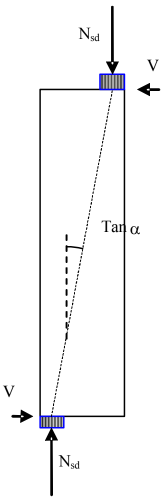
1. INTRODUCTION

The performance of reinforced concrete elements, subjected to normal force N, bending moment M and shear force V, has been object of many studies and reports. The models have been formulated for monotonic increasing of actions and, usually, they don't take into account the effect of degradation provided by external actions of cyclical nature as those produced by earthquakes.
Recent approaches for the analysis of the structures in seismic region aims to the evaluation of displacement/ductility demand rather than force demand. Unfortunately, the classical expressions for shear strength are independent of the magnitude of imposed deformation, leading to over-conservative estimates of capacity for low levels of displacement ductility demand, and becoming increasingly un-conservative as it increases. In literature the physical-mechanic models for cyclic action are by now well consolidated for bending moment and normal forces.
When the response is governed by tensile strength of concrete, the uncertainty of concrete cracking and the great sensitivity of the parameters that define the behavior of material make very complex the formulation of reliable assessment of the whole response. In this context, the approaches more frequently utilized in literature for the evaluation of the shear strength in r.c. elements are not always developed on the basis of valid physical-mechanical models and often they are not correlated to the entity of the corresponding deformation.
These difficulties stimulate a vigorous debate for the assessment of the response, already in the case of monotonic increment of the actions, while only few models are available for the shear strength prediction under the effect of cyclical actions. Several recent studies (Martin-Pérez & Pantazopoulou 1998 and 2001) have tried to resolve these drawbacks providing, with a smeared cracking non-linear model, an original light in the understanding of cyclic degradation of the shear strength capacity for imposed levels of deformation.
Smeared cracking approach has been developed initially for the analysis of elements loaded with monotonic load increases (MCFT - Vecchio & Collins (1986)), however it doesn't appear suitable to provide simplified relationships, necessary in the ordinary design.

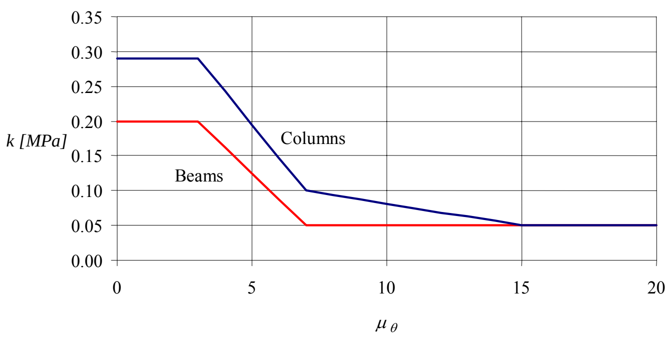
Therefore, the more refined design codes prefer to employ formulations based on simplified mechanic models, or on analytical formulations obtained through regression of experimental results.
The great majority of these models don't take into account, in evaluating the shear strength, the effects of interaction among the different internal forces, even if in literature numerous studies and searches exist on the behavior of r.c. elements under shear V, normal forces N and bending moment M.
Recently a new approach has been employed (Recupero et alt. (2003), Recupero et alt. (2005)) for the formulation of a model able to provide generalized resistance domains of structural elements in r.c., subject to monotonic increasing loads, for taking into account the internal force interaction effects (N-M-V).
The results, provided by the proposed approach, have been compared successfully against laboratory tests, proving the accuracy of the whole procedure of resistance evaluation. However it can be used for monotonic loads only, and it is not able to take in account the effect of strength degradation caused by alternate external actions, as those produced by the earthquake.
In this paper it is observed that the models, that use the stress-fields approach in non-seismic field, make possible an ample variation of the angle θ of inclination of stress-field of the concrete that is different from cracking surfaces with inclination β. When cyclic actions of large intensity occur, the range of variation of θ because of the progressive roughness reduction prevents the development of directions of yielding lines different from that of first cracking β.
In such context a new proposal is advanced that allows the drawing of interaction domains N-M-V for assigned ductility by limiting the range of variability of the angle of deviation between the directions of the yield line θ and the crack line β.

2. PRIESTLEY MODEL OF R.C. MEMBER SHEAR STRENGTH UNDER CYCLICAL ACTIONS

In many cases shear strength code equations for r.c. members are known be very conservative and prove large scatter when used to predict test results. Generally, they don't reflect the dependence of shear strength on flexural ductility. Except for a new formulation in recent Japanese Code (Architectural Institute of Japan (1994) and Watanabe F. & Ichinose T. (1991)) the effect of demand in deformation on shear strength has not been addressed in design codes.



a)



b)

Figure 2.1 a) contribution of axial force to column shear strength, b) degradation coefficient

In 1994 Priestley developed a model which provides close agreement with tests on simple r.c. members. In this model the nominal shear strength of columns is the sum of components due to transverse reinforcement, concrete contribution and axial load. Thus, in circular cross-section the shear strength V_{sd} can be evaluated as follows:

$$V_{sd} \leq \frac{\pi}{2} \cdot \frac{A_{sw}}{s_w} \cdot (D-x) \cdot \frac{f_{yk}}{\gamma_s} \cdot ctg\beta + k(\mu) \cdot 0.8\left(\frac{\pi D^2}{4}\right) \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} + N_{sd} \cdot \tan\alpha \qquad (2.1)$$

where x is the neutral axis depth, and the contribution of axial force to column shear strength and the significance of term (tan α) are described in Figure 2.1a. On the contrary Figure 2.1b shows the trend of $k(\mu)$ degradation coefficient curves versus the curvature ductility $\mu_\theta = \phi / \phi_y$ for beams and columns.
Unfortunately, analogous proposals have not been formulated yet for extending the Nielsen and Braestrup's (1978) stress fields model to the presence of cyclic seismic forces. Thus, the interaction resistance domains (N-M-V) proposed in Recupero et alt. (2003) and (2005), which was formulated on the basis of the Nielsen and Braestrup model, are not able to reproduce the strength degradation effects due to the bending ductility demand in the plastic hinge zones.

3. STRESS FIELDS MODEL AND RESISTANCE DOMAINS UNDER MONOTONIC ACTIONS

When concrete elements are simultaneously loaded by axial force N, bending moment M, and shear force V, the actual stress-fields distribution in the cross-section is very complex, and an analytical model that may determine their exact distribution cannot be easily derived. Nevertheless, the ultimate strength of the structural element can be evaluated under the following simplifying assumptions:

- classical contributions of strength due to dowel action, the aggregate interlock action and concrete traction resistance of the teeth are ignored;
- longitudinal and transverse reinforcements are subjected only to axial forces, including forces due to bending moments; their action is expressed by distributed stress-fields, assumed to be uniform;
- concrete in the external portion of cross-section is subjected to normal compressive stress-fields only, once again assumed to be uniform;
- stress-fields of central portion has a θ degree inclination angle (yield surface) on the longitudinal direction, which may differ from 45 degrees due to actions transmitted along the shear fractures;
- failure mode of the structural element occurs for concrete crushing or for reinforcements yielding or both.

By these assumptions, the analytical model of the structural element for rectangular, I and T shape cross-section, is a generalized truss model, where all the components (compressed chord, tension stringer, strut, and tie) are replaced by uniform stress-fields.
The adopted general criterion for resistance domain evaluation consists into dividing the basic structural element cross section in several layers having depth y_i not defined a-priori, subjected to uniform distributions of stresses (normal σ, shear τ, or both), so as to obtain as a whole the equilibrium with the internal actions N, M, V. This approach, typical for design of in plane loaded plates, has been proved to be equivalent to the stress-fields approach. Any number of subdivisions of the cross section may be assumed, provided that the basic structural element is in equilibrium.
For evaluating the shear strength for circular reinforced concrete columns, in literature several models have been proposed (Ang Beng Ghee et alt. (1989), Schwartz J. (2002)), but, they, generally, don't take into account the interaction effect among different internal actions. In the proposed model the transversal section has been divided in three regions of concrete (S_{c1}, S_{c2}, S_{c3}) and longitudinal steel arranged in continuous manner is considered, grouped in three different regions(S_{s1}, S_{s2}, S_{s3}) (see Figure (3.1c).
With reference to the concrete element of circular cross section, sectioned by a plane parallel to the web concrete stress field (Figure 3.1a), the following equilibrium equation in y direction is written:

$$V^* - q \cdot z = V_{sd} = 2\frac{A_{ws}}{s} ctg\theta \cdot R_c \; \sigma_{wt} \cdot \int_{\delta_1}^{\delta_2} \cos^2\alpha \cdot d\alpha = \frac{A_{ws}}{s} ctg\theta \cdot R_c \; \sigma_{wt} \cdot \left(\cos\delta_2 \sin\delta_2 + \delta_2 - \cos\delta_1 \sin\delta_1 - \delta_1\right) \quad (2.1)$$

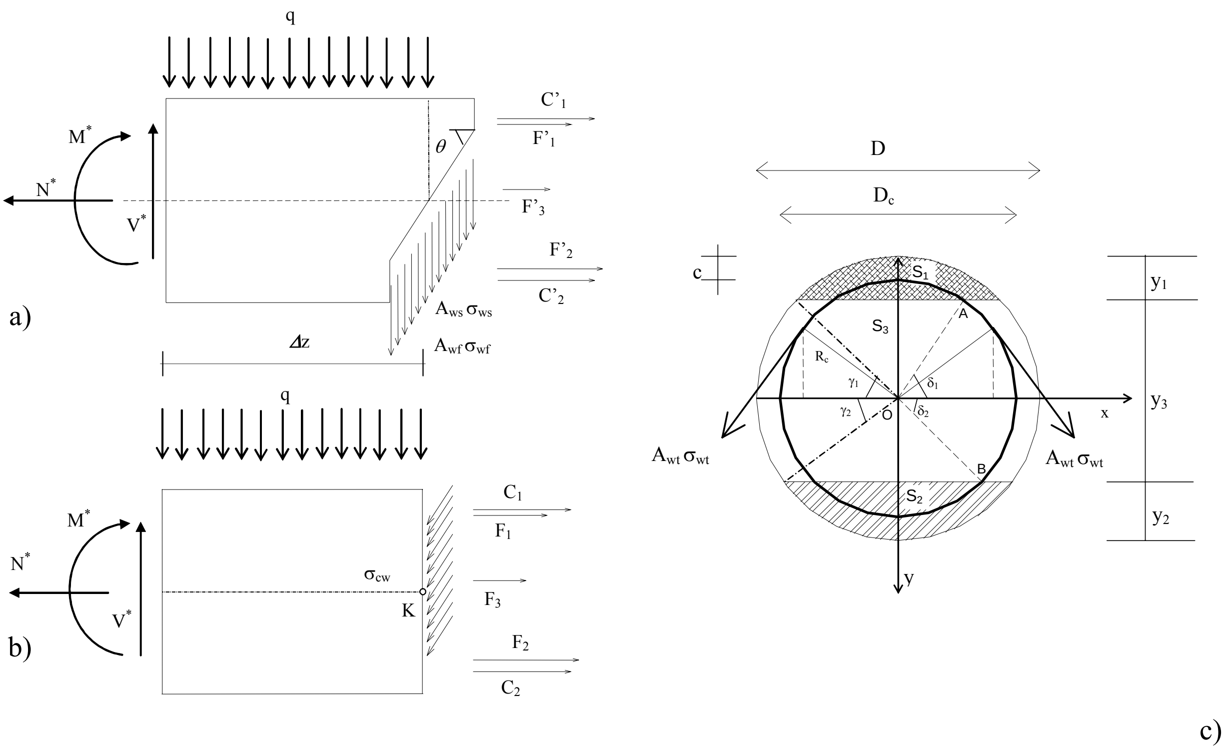


Figure 3.1 Circular shaped cross section.

where A_{ws} and s are the circular steel stirrup cross section area and spacing respectively. Now a new element segment is considered, obtained by two section planes with slope $\theta=90°$, at the abscissa z and $z+\Delta z$ (Figure 3.1b); thus the new equilibrium equation of the element segment in y direction reads:

$$V^* - q \cdot \Delta z = V_{sd} = \sigma_{cw}\, S_3 \cos\theta \sin\theta \tag{2.2}$$

Moreover, adopting the synthetic expression for bending moment M and normal force N at the abscissa z, the expressions of element internal forces in tension chord and in compressed chord are:

$$\sigma_{s1} \cdot \int_{S_{s1}} dS_s + \sigma_{s2} \cdot \int_{S_{s2}} dS_s + \sigma_{s3} \cdot \int_{S_{s3}} dS_s + \sigma_{c1} \int_{S_{c1}} dS_c + \sigma_{c2} \int_{S_{c2}} dS_c = F_1 + F_2 + F_3 + C_1 + C_2 = N_{sd} + V_{sd} \cdot ctg\theta \tag{2.3}$$

$$\sigma_{s1} \int_{S_{s1}} y_s dS_s + \sigma_{s2} \int_{S_{s2}} y_s dS_s + \sigma_{s3} \int_{S_{s3}} y_s dS_s + \sigma_{c1} \int_{S_{c1}} y_c dS_c + \sigma_{c2} \int_{S_{c2}} y_c dS_c = M_{sd} + V_{sd}\, ctg\theta \left(\int_{S_{c3}} y_c dS_c \Big/ \int_{S_{c3}} dS_c \right) \tag{2.4}$$

where y_c, y_s are the lever arms (algebraic values), that are calculated starting from the central axis of circular cross-section, and N_{sd}, M_{sd} and V_{sd} are the internal forces at the cross section at abscissa $z+\Delta z$. By using the static theorem of the plasticity theory, the so-called one "lower bound solution" can obtained by previous equations, once they satisfy both the following geometrical condition and mechanical inequalities for the concrete and steel stress fields:

$$\begin{aligned} & y_1 + y_2 + y_3 = D; \\ & -f_{ccd1} \le \sigma_{c1} \le 0; \quad -f_{ccd1} \le \sigma_{c2} \le 0; \qquad 0 \le \sigma_{wf} \le f_{fed} \\ & -f_{yd} \le \sigma_{s1} \le f_{yd}; \quad -f_{yd} \le \sigma_{s2} \le f_{yd}; \quad -f_{yd} \le \sigma_{s3} \le f_{yd}; \end{aligned} \tag{2.5}$$

in which f_{yd} is the yield strength of reinforcing steel, f_{fed} is the FRP effective design strength, f_{ccd1} is the

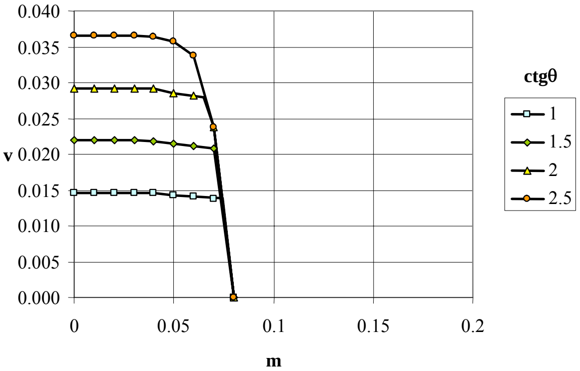
cylindrical strength of concrete that is different from f_{cod1} because it take into account the confinement effect of FRP reinforcement. This contribute can be estimated for example by classical formulation of Spoelstra and Monti (1999). Set the geometrical dimensions and longitudinal and transversal reinforcement values A_{ws} and A_{ls}, the N-M-V interaction domain may be determined by a optimization procedure based on static equalities and geometrical and static inequalities.
In the proposed model the following hypotheses are adopted:

- the slope of the yield surface θ and of the first crack surface β are different;
- the angular deviation between the two positions $\delta = \beta - \theta$ is limited by results of experimental tests under static load conditions (Eurocode 2, 1992) and by considerations on mechanical compatibility, i.e. angular deviation has to allow the stress transfer through the crack.

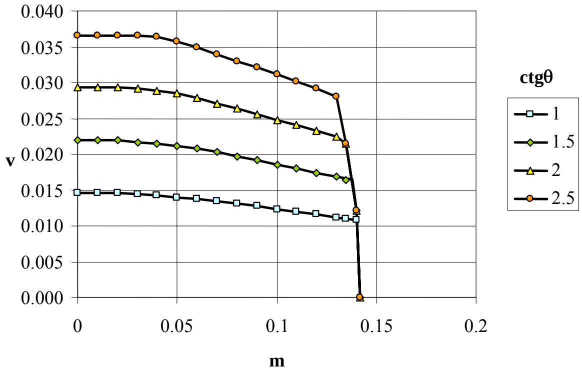
The value of β corresponds to the position of first cracking under the exercise loads. In particular for a structural element with pure bending (beam) the slope β is around 45°, instead for the columns it depends on the ratio between normal and shear force under exercise loads.
In presence of static loads, the inclination angle of yield surface is assumed by plastic approach, independently from the value β as the angle that allows the maximum shear strength in the range $0.4 \leq$ ctgθ ≤ 2.5, that corresponds to $22° \leq \theta \leq 68°$, as prescribed by the Eurocode 2, part.1.
A friendly model has been implemented for drawing the domains of resistance M-N-V. Useful references for details are reported in literature (Colajanni et alt. (2005), Recupero et alt. (2003) and Recupero (2006)).

4. RESISTENCE DOMAINS FOR ASSIGNED DUCTILITY

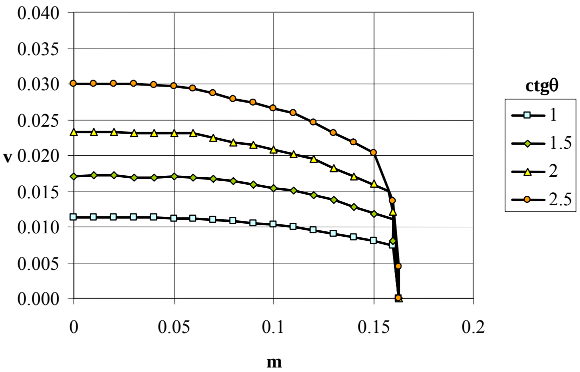
A procedure that provides a methodology for the evaluation of the resistance domains under seismic loads is proposed in this section.



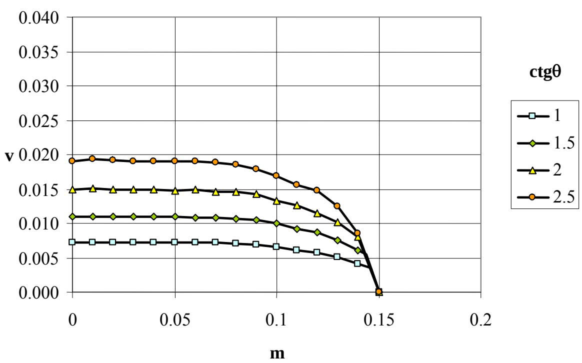
n = 0



n = 0.25



n = 0.50



n = 0.75

Fig. 4.1 Resistance domains with angular deviation δ and (ctg $\theta)_{max}$ fixed.

The procedure predicts the variation of the angular deviation in relationship both to geometric and mechanics characteristics and to the size of the plastic deformations shown by structural element under seismic action.
The formulated procedure aims to extending the suggestions provided by Priestley's proposal to the models that are based on diagonal stress fields. Priestley's models and its generalizations require that the resistant contribution to the truss models provided by the tensile strength of concrete is tuned as function of bending ductility demand and its extension to models based on the diagonal stress fields is apparently unfeasible.
In the models with diagonal stress fields, the inclination of the yield surface in comparison to the position of cracking surface is partly coupled to the effects of aggregate interlock that avoid the sliding of crack, and consequently is linked to the roughness of the crack surface in contact.
When the maximum deformations or/and the accumulated damage due to small amplitude cyclic action increase, the roughness of the sliding surfaces is reduced; thus the range of the deviation angle $\boldsymbol{\delta}$ formed by the cracks surface and the yield surface is limited. The proposed model fixes a limit value of the angle $\boldsymbol{\delta}$, that the more the ductility demand increases, the more $\boldsymbol{\delta}$, is reduced.
Initially, with the objective to stress how such an assumption modifies the resistance domains of r.c. structural elements, the effects of the progressive reductions of the deviation angle $\boldsymbol{\delta}$ on N-M-V domains are shown for a circular shape transversal sections with the following geometric and mechanic characteristics: f_{sd}/f_{cd}=320/20, $\rho_l = A_{sl}/A_c = 0.009$, $A_{sw}/s_w = 0.26$ *mmq/mm*). The resistance domains are shown in Figure 4.1 for four limit values: ctg$\boldsymbol{\theta}$ = 2.5 ($\boldsymbol{\theta}$≈22°), ctg$\boldsymbol{\theta}$=2 ($\boldsymbol{\theta}$≈26°), ctg$\boldsymbol{\theta}$=1.5 ($\boldsymbol{\theta}$≈34°) and ctg$\boldsymbol{\theta}$=1 ($\boldsymbol{\theta}$=45°) and for four normalized axial forces $n = N_{sd}/(f_{cd1} A_c)$ = 0, 0.25, 0.50, 0.75. The limitation on deviation angle δ becomes the corresponding limitation of the inclination angle $\boldsymbol{\theta}$ of stress fields. The Figure 4.1 show that the progressive reduction of the angle $\boldsymbol{\theta}$ of the yield surface position causes a noticeable reduction of the maximum shear strength; by contrast it doesn't have any influence on the ultimate bending moment.
Aiming at the characterization of the relation between angular deviation $\boldsymbol{\delta}$ and flexural ductility demand on the basis of the indications provided by Priestley et alt. (1996), it is observed that the limitation of the inclination of yield surface position influences the horizontal line of resistance domain corresponding to small values of the bending moment, in which failure of structural element is reached by attainment of maximum shear strength:

$$[V_{sd}]_{\max} = [ctg\theta]_{\max} \frac{A_{ws}}{s_w} R_c \; f_{yd} \cdot 2\int_{\delta_1}^{\delta_2} \cos^2 \varepsilon \cdot d\varepsilon = [ctg\theta]_{\max} \frac{A_{ws}}{s_w} R_c \; f_{yd} \left(\cos\delta \sin\delta + \delta\right)\Big|_{\delta_1}^{\delta_2} \tag{4.1}$$

where $\left(\cos\delta \sin\delta + \delta\right)\Big|_{\delta_1}^{\delta_2} = \left[\left(\cos\delta_2 \sin\delta_2 + \delta_2\right) - \left(\cos\delta_1 \sin\delta_1 + \delta_1\right)\right]$. If the contribution of axial force N_{sd} *tan* α can be neglected, the following relation hold:

$$[ctg\theta]_{\max} \frac{A_{ws}}{s_w} \left(\frac{D-2c}{2}\right) f_{yd} \left(\cos\delta \sin\delta + \delta\right)\Big|_{\delta_1}^{\delta_2} = \frac{\pi}{2}\frac{A_{sw}}{s_w}(D-x)\, f_{yd}\, ctg\beta + k_p(\mu)\cdot 0.8\left(\frac{\pi D^2}{4}\right)\frac{\sqrt{f_{ck}}}{\gamma_c} \tag{4.2}$$

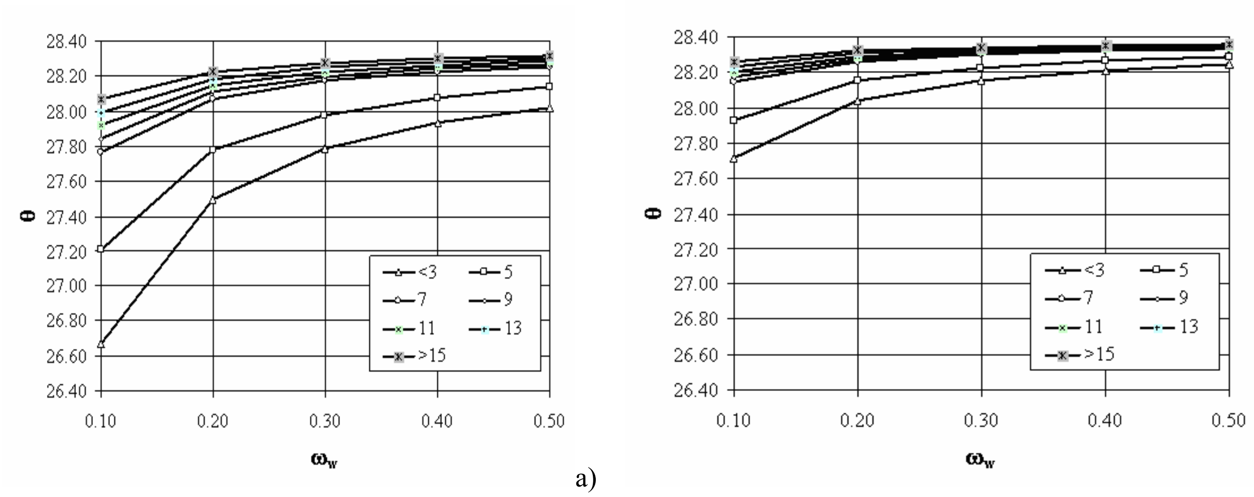
The degradation coefficient of shear strength provided by the concrete $k = k_p(\mu)$ (Figure 2.1b) is obtained by the Priestley's model, assuming $\boldsymbol{\beta}$ = 30° (***ctg*** $\boldsymbol{\beta}$ = 1.732). Thus, Equation 4.2 can be simplified as follows:

$$[ctg\theta]_{\max} = \frac{1}{\left(\cos\delta \sin\delta + \delta\right)\Big|_{\delta_1}^{\delta_2}} \left(\frac{\pi}{2}\frac{(D-x)}{R_c} ctg\beta + k_p(\mu)\cdot\left(\frac{s_w R_c}{A_{sw}}\frac{f_{cd2}}{f_{yd}}\right) 0.8\pi \left(\frac{R}{R_c}\right)^2 \frac{\sqrt{f_{ck}}}{f_{cd2}\cdot\gamma_c} \right) \tag{4.3}$$

where f_{cd1} and f_{cd2} are suggested in CEB-FIP (1993) and x, δ_1 and δ_2 must be evaluated according to the axial forces level.

5. NUMERICAL ANALYSIS

Some numerical analyses have been performed, in order to shows that different geometric and mechanics parameters and requested ductility can reduce the range of variability of the minimum inclination of the stress-fields into concrete. In Figure 5.1a the variation range of minimum inclination are reported for motorway viaduct circular-shape pier built with low strength concrete C18/15 and subjected to a normalized axial force n = 0.20, with variation in the mechanical ratio of stirrups $\omega_w = (s_w\, R_c\, f_{cd2}/A_{sw}\, f_{yd})$ and the required flexural ductility. For static loads (μ < 3), with variation of the mechanical ratio of stirrups in the range 0.1 <ω_w <0.5, the minimum inclination of the stress- fields is comprised in the range 26.67° < $\bar{\theta}$ <28.02° (1.87 > $ctg\,\bar{\theta}$ > 1.99). Therefore, the angular deviation in comparison to the conventional cracking angle of 30°that have been adopted by Priestley, results in the range 1.98° <δ< 3.33°.



a) b)

Figure 5.1 – Angle θ for piers with different stirrup percentages and required ductility

A wider deviation is required for small mechanical ratios of stirrups. In presence of a small density of stirrups, the maximum values of shear are gotten when the immersion of the stress-fields is reduced in order to cross within the yield line, on which the equilibrium is imposed, a larger number of stirrup arms.
With the increases of the required ductility, the range of $\bar{\theta}$ is reduced; for $\mu \geq 15$ and for the different values of mechanical percentage of stirrups it is gotten 28.07° <$\bar{\theta}$ <28.32° (1.85 > $ctg\,\bar{\theta}$ > 1.88).
Large ductility demand strongly penalizes the shear strength of elements with small amount of stirrups, while the reduction are small for members with large mechanical ratio of transversal reinforcement. In Figure 5.1b for the same circular-shape piers, subjected to a normalized axial force n = 0.20 too, cased with concrete having strength C50/40, the variation range of minimum inclination are shown. For low values of ductility demand(μ < 3), varying the mechanical percentage of stirrups in the range 0.1 <ω_w <0.5, the minimum inclination of the stress- fields is comprised in the range 27.71° < $\bar{\theta}$ <28.24° (1.86 > $ctg\,\bar{\theta}$ > 1.90); by contrast, for high value of the requested ductility, the range of $\bar{\theta}$ is reduced; for $\mu \geq 15$ and for the different values of mechanical percentage of stirrups the range 28.26° <$\bar{\theta}$ <28.35° (1.85 > $ctg\,\bar{\theta}$ > 1.86) is gotten. The increment of concrete strength provides a narrow range of variation for high mechanical percentage of stirrups than in the case of low concrete strength.
Moreover, with a higher concrete strength, the value of minimum inclination becomes closer to the limit of $\bar{\theta}$ = 30° conventionally provided by Priestley. For a first calculation this value can be used in the model of the stress-field without further investigations.

6. CONCLUSIONS

A new models is proposed for assessment of shear strength of r.c. elements with circular cross section, able to take into account the effects of interaction among the internal forces N_{sd} and M_{sd} and the strength degradation due to external cyclic actions, such as those produced by earthquakes. The model is derived on the basis of the stress-field approach, and it is able to predict the reduction of the shear strength related to the concrete damage

due to the attainments of large flexural curvature by linking the limitation of the angle of inclination of stress-fields to the flexural ductility demand.
The predict shear strength degradation are consistent with those predict by Priestley's model, where the shear strength reduction have been obtained by a reduction of the concrete tensile strength as function of bending ductility demand, while the effects of interaction among internal forces had been neglected.
The proposed method allows the drawing of the axial force-bending moment- shear force interaction resistance domains for assigned values of flexural ductility demand that should have to be used in a real push-over analysis instead of the classical axial force-bending moment resistance domains.
Numerical analyses have shown that, the more the ductility demand increases, the more the slope of θ approach the inclination of the first crack surface β, irrespective of the amount of stirrups. Moreover for high strength concretes the minimum value of the yield surface inclination becomes is close to the limit value $\bar{\theta}$ = 30°.Thus, a rough approximation of the shear strength can be obtained by assuming such an inclination of the stress field.

REFERENCES

Haddadin J., Hong S. T., Mattock A. H. (1971), Stirrup Effectiveness in Reinforced Concrete Beams with Axial Force, *Journal of the Structural Division, Proceedings ASCE*, V. 97, No ST9. Sept. 1971, pp. 2277-2297
Bach F., Nielsen M. P. Braestrup M.W. (1978), Rational Analysis of Shear in Reinforced Concrete Beams. ***IABSE Proc.* P-17/78, pp. 1-16**
Vecchio, F.J., Collins, M.P. (1986), The modified compression field theory for reinforced concrete elements subjected to shear. *ACI Structural Journal*; 83 (2): 59,61
Ang B.G., Priestley M.J.N., Paulay T. (1989), Seismic shear strength of circular reinforced concrete columns. *ACI Structural Journal*; 86 (1)
Watanabe, F., Ichinose T. (1991), Strength and ductility design of rc members subjected to combined bending and shear - *Concrete shear in earthquakes.* Houston: University of Houston,:429-38.
Eurocode No. 2 - Design of Concrete Structures – Part.1 : General Rules and Rules for Buildings – UNI ENV 1992-1-1
CEB-FIP (1993), Model Code for Concrete Structures for Buildings, *Comitè Eurointernational du Bèton, Lausanne*, May 1993
Hsu, T.T.C. (1993), *Unified Theory of Reinforced Concrete*. CRC
Wong, Y., Paulay, T., Priestley, M.J.N. (1993), Response of Circular Reinforced Concrete Columns to Multi-Directional Seismic Attack. *ACI Structural Journal*, vol. 90, n° 2, pp. 180 – 191.
Architectural Institute of Japan (1994), *AIJ structural design guidelines for reinforced concrete buildings*. Tokyo
Priestley M.J.N., Seible F., Calvi G.M. (1996), Seismic Design and Retrofit of Bridges, *John Wiley & Sons, Inc*
Priestley, M.J.N., Benzoni, G. (1996), Seismic Performance of Circular Columns with Low Longitudinal Reinforcement Ratios. *ACI Structural Journal*, vol. 93, n° 4, pp. 474 – 485.
Martin-Pérez, B., Pantazopoulou, S.J. (1998), Mechanics of concrete participation in cyclic shear resistance of reinforced concrete, *Journal of Structural Engineering, ASCE*; 124(6):633-41 June.
Spoelstra M.R., Monti G. (1999), FRP-Confined concrete model., *J. Compos. Constr*., ASCE, 3(3): 143-15
Martin-Pérez, B., Pantazopoulou S.J. (2001), Effect of bond, aggregate interlock and dowel action on the shear strength degradation of reinforced concrete, *Engineering Structures*, 23, 214-227
Schwartz J. (2002), Stress Field Design of Reinforced Concrete Members with Circular Cross Sections – *Structural Engineering International* – SEI Vol. 12 – Number 1 – February 2002
Recupero, A., D'Aveni, A., Ghersi, A. (2003), N-M-V Interaction Domains for Box I-shaped Reinforced Concrete Members, *ACI Structural Journal*, vol. 100, n° 1, pp. 113-119.
CSA Standard A23.3-04. Design of Concrete Structures, Ontario: CSA, 2004 (249 pp)
Recupero A., D'Aveni, A., Ghersi, A. (2005), N-M-V Interaction Domains for Prestressed Concrete Beams, *Journal of Structural Engineering*, vol. 131, n° 9.
P. Colajanni & L. La Mendola & A. Recupero (2005) Shear-Flexure Interaction of RC Elements Strengthened with FRP sheets - *ICCRRR - November 2005* – Cape Town, South Africa
Recupero A. (2006), A Proposal for a General N-M-V Design Method – *The Second International FIB Congress 2006 Naples*, 4- 8 June 2006