Analysis of Seismic Energy Response and Distribution of RC Frame Structures

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ABSTRACT :

The paper reports the study on seismic energy response and distribution of a multi-story RC frame structure based on energy equation and nonlinear time history response analysis. The analysis used 1940 El Centro earthquake acceleration record as the input to the structural model. The energy response was analyzed in energy components. The component of strain energy was considered as the target of the study as it was the key component reflecting the damage extent of the structure. The strain energy of whole structure and its distribution in each story and among structural elements were further investigated and the relationship between the ductility of structural elements and their corresponding strain energy were examined to evaluate the seismic performance of the structural model. The energy analysis herein provides useful information and becomes a new way to understand the structural damage and deformation ductility in respect of strain energy. It also indicates the feasibility of its application in structural seismic performance design.

KEYWORDS: energy equation, dissipated energy, strain energy, nonlinear model, time history analysis, energy distribution, energy-ductility relation

1. INTRODUCTION

Usually the tremendous energy released in an earthquake is mostly worn off by diastrophism, tectonic movements, earth's surface cracks, etc. (Ergin & Recep, 2006). Even though a very small portion of the energy impacts structures, it could cause severe damage and collapse to the structures and result in gigantic catastrophe. This was sufficiently proved in many past earthquakes including the recent Wenchuan Earthquake (May 12, 2008) in China. The structure's responses to the earthquake motion, such as structural displacement, stress, and acceleration, are the representative phenomena of the energy (Xie, et al., 2003). The hysteretic dissipated energy is highly concerned as it relates to the damage of the structure. The dissipated energy of structures can be determined by numerical simulation, which is highly efficient if the calculating model is well built, and in dynamic test (Lestuzzi & Bachmann, 2007). There have been many methods developed and applied to avoid the structural damage, such as applying additional energy dissipation system to substitute the structural dissipation system (Symans, et al., 2008) and using base isolators to reflect and minimize input energy (Ye, et al., 2007; Ramallo, et al., 2002). However, the distribution characteristics of the hysteretic dissipated energy in structures are still unclear and hard to predict.

Many researchers are dedicated in structural nonlinear analysis and energy response studies (Ye, et al., 2000) for comprehensive knowledge on seismic response and earthquake-resistant design. As the results, the performance-based structural design (PBSD) becomes more and more highly stressed. Studies have shown that the ratio of hysteretic dissipated energy and damping dissipated energy in the total input energy only depends on structural dynamic characteristic, unrelated to the character of input earthquake wave (Liu, et al., 1996). To a simplified MDOF layer model, hysteretic dissipated energy was found to be trapezia-distributed, more in lower stories and less in upper stories (Shi, et al., 2005).

This paper put focus on the distribution of the strain energy among stories and in structural elements to reveal the energy transfer rule and the distribution characteristics of hysteretic energy. The relation of the ductility of a structural element to its corresponding strain energy was trial-studied as well.

2. ENERGY DEFINITION AND CASE STUDY

2.1. Energy Definition

There are some differences between relative energy equation and absolute energy equation (Gong & Xie, 2005). Although applying the absolute energy equation would be more physically meaningful (Uang & Bertero, 1990), in this paper the relative equation is used, considering the hardness to actually acquire the earthquake velocity record in calculation of absolute energy equation.

Integrating the MDOF equation of motion in time domain on relative displacement, we could draw the relative energy equation as

$$\int \mathbf{M}\ddot{\mathbf{u}}(t)d\mathbf{u}(t)+\int \mathbf{C}\dot{\mathbf{u}}(t)d\mathbf{u}(t)+\int \mathbf{r}(t)\mathbf{u}(t)d\mathbf{u}(t)=\int -\mathbf{M}\mathbf{I}\ddot{u}_g(t)d\mathbf{u}(t) \tag{2.1}$$

where $\mathbf{M}$ = system mass matrix; $\mathbf{C}$ = system damping matrix; $\mathbf{u}$(t)= relative displacement vector at time *t*; $\mathbf{r}$(t)= hysteretic force vector at time *t*; $\mathbf{I}$ = diagonal matrix consists of one or zero; and $\ddot{u}_g$(t)= ground input acceleration at time *t*. The equation (2.1) can be expressed as $E_k+E_d+E_s=E_I$, in which E_k = kinetic energy, E_d = damping energy, E_s = strain energy and E_I = input energy, respectively.

2.2. Analysis Model Establishment

A multi-story RC frame structure (Figure 1), conceived as an office building, was designed based on seismic fortification intensity 7 degree (basic earthquake acceleration 0.10 g) according to Chinese aseismic design code and other related codes. It was 7-story high, the first story height 3.6 m and others 3.3 m. Columns were designed constructionally, all columns having the same 550 mm square section and same longitudinal reinforcement (as shown in Figure 1). However, the hoop bar of the columns was made different between stories. The designed structural model was expected to be strong-column-weak-beam system.

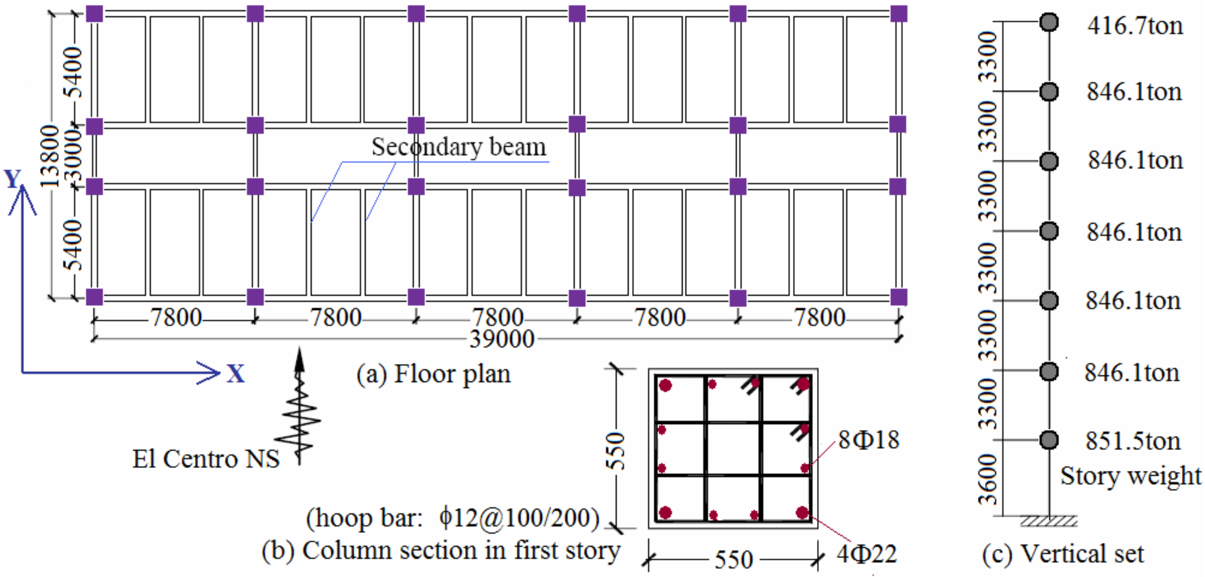


Figure 1. Scheme of the building model (dimension in mm)

Three-dimensional frame analysis model was created and the analysis used CANNY program (Li, 2004), which was verified in high reliability simulating the dynamic response of an instrumented building (Li, 1999). The energy response analysis based on the following assumptions.

① Beam element was treated in uniaxial bending and shear deformation in each frame plane, no axial deformation. The hysteresis model was in trilinear skeleton curve to allow for concrete crack and steel yielding, and the unloading and reloading was based on Takeda-hysteresis rules.
② Column element was idealized in multi-axial spring model (MS model) to incorporate the interactions among biaxial bending and fluctuation of axial load. The biaxial shear deformation of column element was simulated using multi-shear spring model (MSS model) with each shear spring in a trilinear skeleton-curve model including pinching effect after spring yielding. The potential plastic hinge length is assumed to be $0.1 \times L_{column}$.
③ Floor slab was approximated as infinite stiffness in horizontal plane (rigid assumption). The floor slab contribution to beam element resisting flexural deformation was roughly included. That is, the flexural stiffness of interior beam element (with double-side of slabs) was doubled, and of exterior beam element (with one-side slab only) was enlarged by 1.5 times.
④ For simplicity, the damping of the structural model was assumed to be time-independent and proportional to mass and stiffness matrix:

$$\mathbf{C} = 2h_m \omega \mathbf{M} + 2h_k / \omega \mathbf{K} \quad (2.2)$$

Where $\mathbf{C}$ and $\mathbf{M}$ were defined in equation (2.1), and $\mathbf{K}$ = the structural instantaneous stiffness matrix. The proportional factor h_m and h_k were assumed to be 4% and 1% respectively. ω = the elastic vibration frequency of the structural mode in the analysis direction. This damping assumption would be reasonable and acceptable for the change of damping or ductility had a small effect on the amount of energy imparted to a structure (Tony & William, 1984).

The acceleration record (NS component) obtained from the 1940 El Centro earthquake was used as input excitation and was scaled to be with the peak input-acceleration at 220 GAL, equivalent to the intensity 7 degree at hazardous earthquake defined in Chinese code for aseismic design of buildings (Chinese National Standard, 2001). The input was only made in the transverse Y-direction of the 3D structure model (Figure 1) in a consideration to compare the strain energy distribution among the beams in and out of the input direction.

3. ENERGY CALCULATION AND ANALYSES

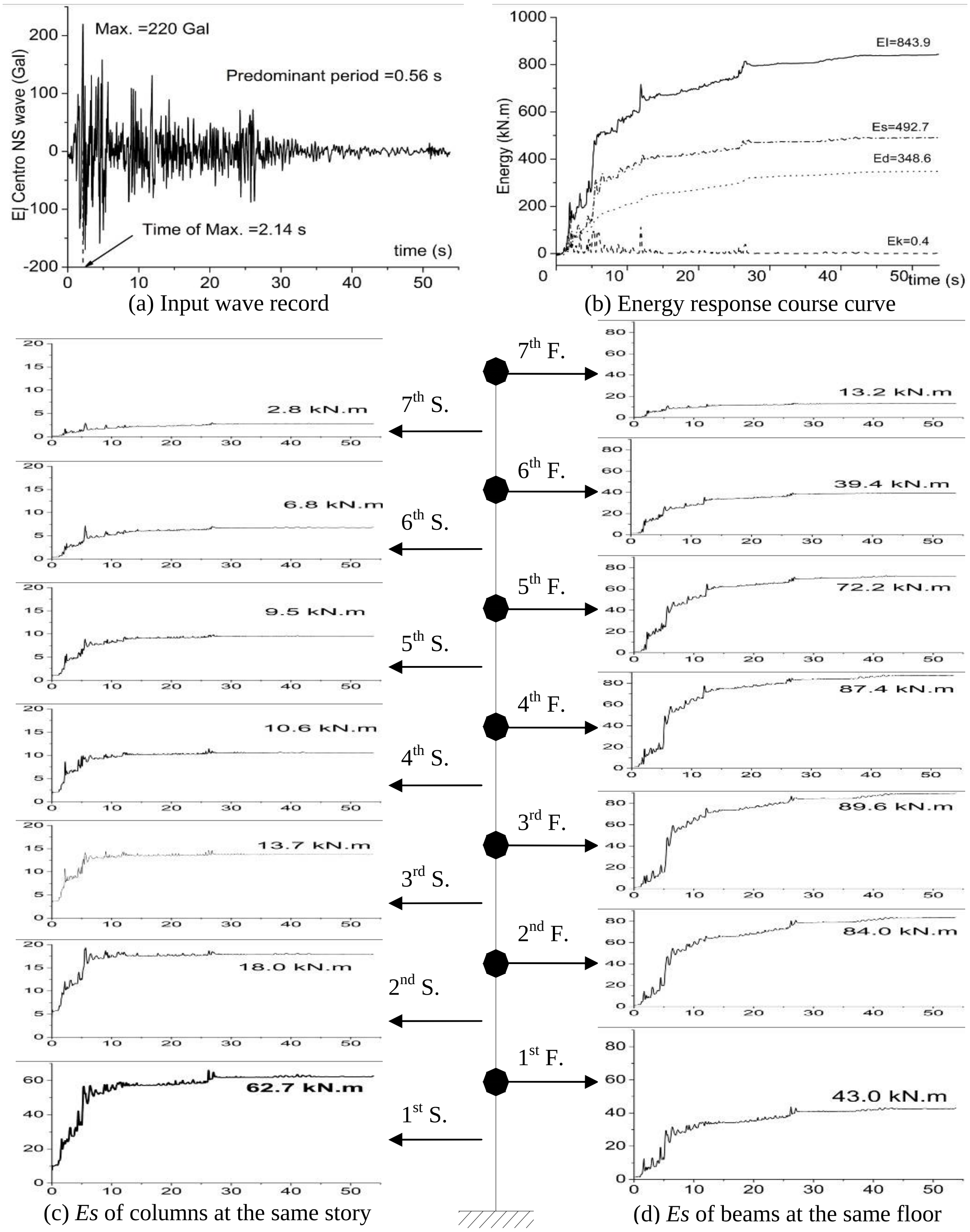
The energy response of whole structural model was calculated during the nonlinear time history analysis based on the restoring force vector and the increment of structural displacement vector, while the strain energy of individual structural elements (beams and columns) was calculated using element forces and element incremental deformation (both plastic deformation at hinge section and elastic distributed deformation along the element axial line). The energy response course curve was shown in Figure 2 (b). At the end of the time course the strain energy would be almost the accumulation of plastic hysteresis energy when the structure responded to hazardous earthquake and it was seriously damaged. The elastic strain energy would be little amount in the total strain energy. So it is meaningful to study on the relations between the strain energy and the damage extent (ductility) of structural elements.

3.1. Distribution of strain energy in lower and upper stories

Figure 2 (c) and (d) showed clearly that the strain energy of columns decreased uniformly from lower to upper stories. Exception was the first story which dissipated more energy than the total of the above stories. However, the distribution of strain energy of beams was different. Most strain energy dissipated in the beams of 2nd floor to 5th floor. The beams of the first floor and of the top two floors held less energy. Apparently, the energy response courses of columns and beams were almost in the same accumulation tendency of time history.

The total energy (beam's + column's) was close to trapezoid-distributed along the stories (Figure 3). From the story-distribution of the strain energy, it showed clearly the weak story at the bottom of the structure, in which columns dissipated more energy than beams. It could predict that the first story would have yielded and invalidate if structure collapsed. As the analysis program was not expected to predict collapse, it did report most columns (141 out of 168) flexural cracked but none yielded, and 125 beams among total number of 266 yielded and the rest beam all racked (Table 1).

The damping energy was plotted in shadow-area in Figure 3, which was resulted from setting the total energy of the dissipated strain energy plus damping energy in uniform distribution along the stories, denoted as "Assumed total", giving each story 120.6 kN·m. It should be noted that the damping energy showed in Figure 3 was not corresponding to each story but just a schematic expression for damping energy.



(a) Input wave record (b) Energy response course curve

(c) *Es* of columns at the same story (d) *Es* of beams at the same floor

Figure 2. Time history of the input acceleration wave and the energy responses (S. =Story; F. =Floor)

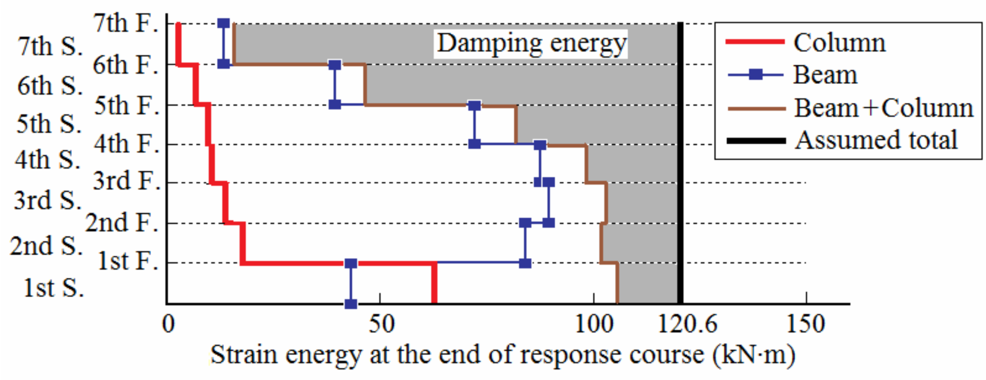


Figure 3. Disassembled strain energy distribution

Table 1. Damage distribution

Floor or Story Number	Beam		Column	
	Crack	Yield	Crack	Yield
7	All	17(7*)	24(2)	No
6		18	24	
5		18	24	
4		18	24(8)	
3		18	13(5)	
2		18	8(1)	
1		18	24(24)	
Sum	266	125(7)	141(40)	0

* Number in parentheses indicates elements with corresponding damage occurred in one end only.

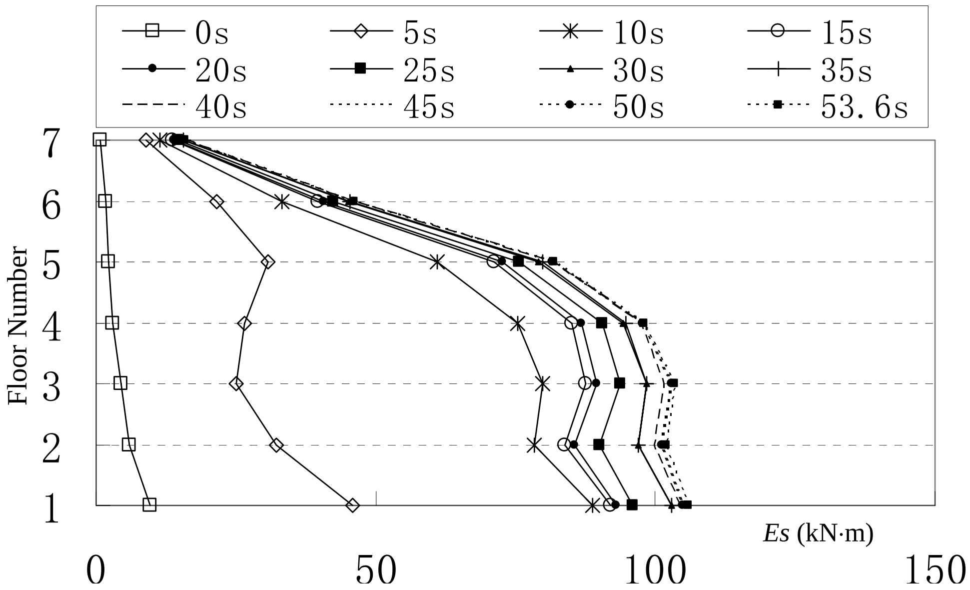


Figure 4. Strain energy distribution among floors at different time

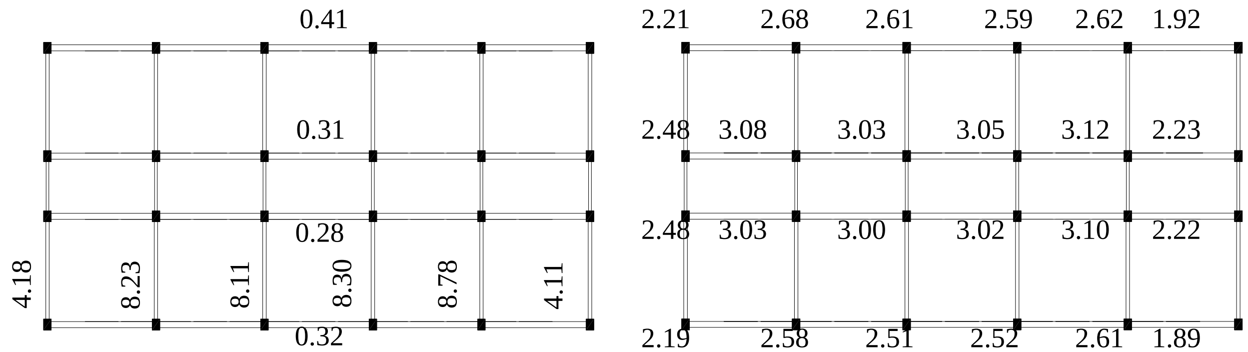
It could be expected that weaker beams or weaker columns in a story dissipated most of the energy and the rest elements did less energy, since the total strain energy should follow the near-trapezia-distributed rule (abbr. NTDR). It is well accepted that energy current flows into the structure from the base to top, and transfers the ground kinetic energy to the structure's strain energy, kinetic energy and dissipated energy. The structure dissipated the energy eventually from story to story so that each story in different energy distribution. The distribution of strain energy among floors at different time is shown in Figure 4. At beginning (time 0 sec) it was only elastic strain energy in columns caused by structural gravity load so that it was triangular-distributed. During the acceleration input up to 10 sec (including the peak excitation), the strain energy increased most fast. It resulted in NTDR extremely not followed at time 5 sec, which was just passed the peak excitation. Then it gradually turned to follow the NTDR in succession. It indicated that low frequency cyclic load tended to ensure the NTDR. Then the increase of the strain energy slowed down after passing the time of stronger excitation (after time 15 sec). Some considerable increasing of the energy was seen from 25 sec to 30 sec, which conformed to the relative strong input excitation as shown in Figure 2 (a). After 30 sec, the input excitation was relatively smaller so that the energy had little change. That is, the strain energy response showed the characteristics in a close relation to the change of input acceleration amplitude.

3.2. Horizontal distribution of E_s

Obviously, the distribution of strain energy depends on the element stiffness, deformation history, reaction force and damage extent and ductility of individual structural elements. As the acceleration wave was input in transverse Y-direction, beams in the longitudinal direction dissipated little energy at average 0.019 to that of Y-direction (Table 2). Details of the strain energy distribution of beams in the first floor and columns in the first story were shown in Figure 5. In the input direction, beams of interior frame dissipated almost double more energy than that of exterior frame. So to the columns, more energy of interior columns compared with that of exterior columns and even less in corner column. As no column in yielding, so the difference of strain energy among interior, exterior and corner columns was not so much as beam did.

Table 2. Strain energy distribution among beams in different direction (unit: kN·m)

Floor	7	6	5	4	3	2	1	Total
X-dir beam	0.39	1.19	1.26	1.29	1.31	1.36	1.32	8.12
Y-dir beam	12.66	38.29	70.99	86.16	88.33	82.63	41.71	420.77
X/Y ratio	0.031	0.031	0.018	0.015	0.015	0.016	0.032	0.019



(a) BEAM (the sum of E_s of the beams in one frame) (b) COLUMN

Figure 5. Detailed E_s distribution in first story (unit: kN·m)

4. ELEMENT DEFORMATION DUCTILITY AND CORRESPONDING STRAIN ENERGY

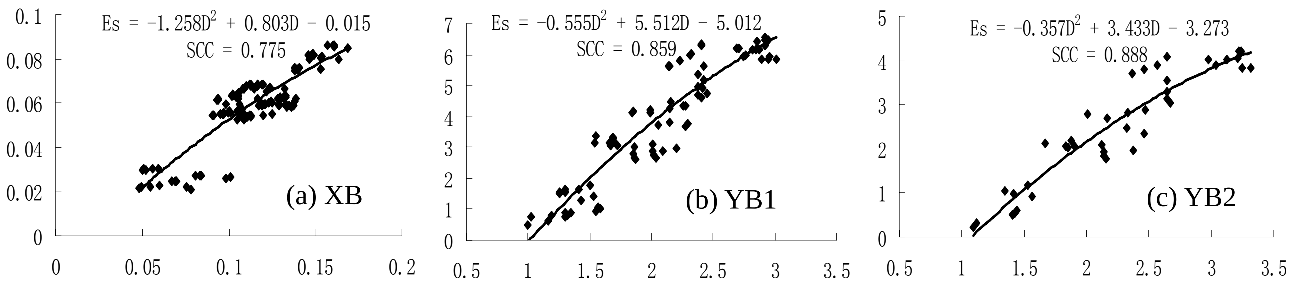
Considering no column yielded during the time history analysis, it was supposed that the energy was mainly dissipated in beam elements. So here we did to study the relation among the beams' flexural ductility and their corresponding strain energy (simply called as "*E_s-ductility relation*"). The mechanism of Es-ductility relation could be very complicated because of many factors affecting the element ductility. We tried to sort the beams into 3 categories: XB of all beams in longitudinal X-direction (out of input direction); YB1 of the beams of exterior longer-span and YB2 of the beams of interior shorter-span in the input Y-direction. Each category of the beams was equal dimension and similar reinforcement. The *Es-ductility relation* of each category was plotted in Figure 6, from which we could see that the ductility of XB was less than 1.0 (no yielding) so it generally dissipated less energy, and that of YB1 and YB2 yielded (ductility exceeded 1.0) and dissipated much more energy than that of XB. Regression analysis was done in several kinds of regressive function and the results were shown in Table 3 with the square of correlation coefficient (*SCC*). The results suggested that quadratic polynomial function was adoptable to describe the relation of element dissipated energy and its corresponding ductility (considering the element ductility depending on more factors further studies should be done to refine the regression function). Figure 6 showed the quadratic polynomial function curves of the three categories of beam elements. The category XB in Figure 6 (a) showed smallest *SCC* because of least plasticity developed (for no yielding). The suggested equation of E_s-*ductility relation* is:

$$E_s = -a\left(D - D^*_{\max}\right)^2 + E^*_s \tag{4.1}$$

Where, E_s = element strain energy; D = element ductility; a = coefficient representing element characteristics (a in positive value and unit of energy); $D^*_{\max}$ = the potential maximum ductility of element; E_s^* = element potential maximum strain energy. Noted that $D \le D^*_{\max}$ so the equation ensures the relation of E_s increasing with more ductility developed.

Table 3. Regression analysis results of element E_s-*ductility relation* (unit: kN·m)

Beam category	Q-polynomial equation		Linear equation		Exponential equation	
	E_s	SCC	E_s	SCC	E_s	SCC
XB	E_s=-1.258D^2+0.803D-0.015	0.775	E_s=0.532D-0.001	0.769	E_s=0.666D$^{1.120}$	0.750
YB1	E_s=-0.555D^2+5.512D-5.012	0.859	E_s=3.222D-2.813	0.852	E_s=0.687D$^{2.236}$	0.825
YB2	E_s=-0.357D^2+3.433D- 3.273	0.888	E_s=1.839D-1.635	0.875	E_s=0.351D$^{2.295}$	0.853



Horizontal axis: *ductility* D, Vertical axis: strain energy E_s (unit: kN·m)

Figure 6. Regression curves for E_s-*ductility relation*

5. SUMMARY

The energy responses, strain energy distribution and the relation between strain energy and element ductility of building structures were studied. A 3D RC frame structural model in strong-column-weak-beam system was designed according to the Chinese aseismic design code and was used in the study to calculate its responses to the input of strong ground motion from the 1940 El Centro earthquake acceleration record. The study mainly found the following facts that could be useful information in evaluating structural seismic performance and important in performance based structural design.

(1) The strain energy distribution along structural stories confirmed to the near-trapezia-distributed rule (NTDR). Weak story and damaged elements dissipated the most energy. Structural collapse could happen if the structural elements in a weak story could not afford sufficient energy dissipation ability to meet the NTDR.

(2) The strain energy is closely related to the element damage according to the element location, direction and damage extent. The structural elements of interior frame in lower and middle stories usually subjected to larger

inner forces and developed more plastic deformation hence dissipated more energy than that of others. So the structural engineers may have to pay more attention in the design of individual elements of frame structures.

(3) The preliminary study revealed that the polynomial function of regressive curve would be adoptable to describe the relation between element dissipated energy and its deformation ductility. As the element ductility depends on more factors further studies on different type of structural models and widely use of input excitations are expected to achieve extensively adapted conclusions and rules to prompt the application of energy method in structural design.

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