USE OF NEW EQUIVALENT NONLINEAR SDF SYSTEM OF PLANAR MULTI-STOREY R/C FRAMES IN STATIC PUSHOVER PROCEDURE

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ABSTRACT:

In the present paper, the issue of the approximate definition of a new equivalent Non-Linear Single Degree of Freedom (NLSDF) system on planar reinforced concrete (r/c) multistory frames is presented. It is known that, in order to estimate the demand seismic inelastic floor displacements the application on non-linear response history analysis (NLRHA) is suggested for accuracy reasons. However, due to its high computational cost the use of the simplified static pushover analysis is often preferred. Also, in the NLRHA there are difficulties in the qualitative comprehension, while in the static pushover procedure there is more natural inspection of the results of analysis. In order to perform the static pushover analysis an equivalent non-linear single degree of freedom system that represents the initial planar frame must be defined. In the present paper, the characteristics of an improved new equivalent non-linear single degree of freedom system are given and a suitable numerical example is presented. The definition of the NLSDF system has mathematically derived recently, considering suitable dynamic loadings on the masses of each r/c system and using simplified assumptions. The new equivalent NLSDF system in combination with the inelastic design spectra can give in many cases an approximate evaluation of the required seismic floor displacements for a known design earthquake. The present methodology is suitable for all types of planar frames that possess the required normality by the contemporary Seismic Codes in elevation, as well as in planar r/c frames where is impossible to appear soft-storey (yielding mechanism at all columns -top & bottom- of someone floor).

KEYWORDS: Non-linear SDF system, Pushover procedure, Inelastic seismic analysis.

1. INTRODUCTION

In order to estimate approximately the seismic demands at low seismic performance levels, such as life safety and collapse prevention, the application of the Non-linear Static Analysis (pushover procedure) is often preferred in the practice by Civil Engineers. In the case of planar multistory r/c frames, several non-linear single degree of freedom (NLSDF) systems were presented in the past. A first documented definition of the optimum equivalent nonlinear SDF system of planar r/c frames was given by Makarios (2005), while an improved documented definition of the optimum equivalent NLSDF system for the planar multi-storey frames has been given recently (Makarios 2008). In this last paper, the mathematic definition of the optimum equivalent NLSDF system of spatial asymmetric multistory r/c buildings has been given, too, while the coupling of translational and the torsional degrees-of-freedom of them has been taken into consideration, without dividing these asymmetric buildings into various separate subsystems. In the present paper, the above-improved definition of the optimum equivalent NLSDF system for the planar multi-storey frames is presented, as well as and a numerical example of five–storey r/c planar frame for illustrative reasons.

2. PROPERTIES OF THE EQUIVALENT NSDF SYSTEM OF PLANAR MULTI-STOREY SYSTEMS

Consider the N-story planar system of Figure 1a that have the N-dimension vector $\mathbf{u}$ of horizontal displacements according to its N-degrees of freedom as well as the diagonal mass matrix $\mathbf{M}$ (Figure 1c). At first, we should set an acceptance about the distribution $\mathbf{Y}$ in elevation of external static lateral floor forces P_i (i.e. triangularly or mode-shape's distribution, Figure 1b). Using the lateral static floor forces P_i with a known distribution

$\mathbf{Y} = \{Y_1 \quad Y_2 \quad \dots \quad Y_i \quad \dots \quad 1.00\}^T$ for $i=1,2,3,\dots,N$ a static pushover analysis is performed. After, consider an interval step of this pushover analysis near in the middle of the pushover curve, where the profile of horizontal static inelastic floor displacements $\mathbf{u}_O$ is noted (Figure 1d), therefore, vector $\mathbf{u}_O$ is written:

$$\mathbf{u}_O = \boldsymbol{\psi}\, u_N \tag{2.1}$$

from where the vector $\boldsymbol{\psi} = \{\psi_1 \quad \psi_2 \quad \dots \quad \psi_i \quad \dots \quad \psi_N\}^T$ with $\psi_N = 1.00$, is calculated directly (Figure 1d).

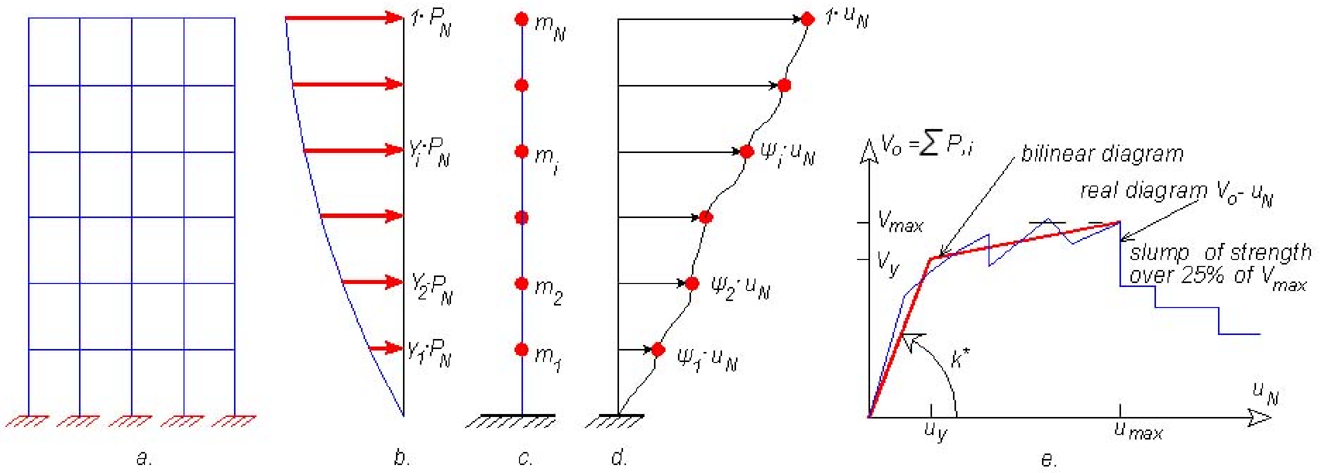

Figure 1 Planar N-storey frame and its diagram V_o-u_N .

After the static pushover analysis, the diagram V_o-u_N is easy done, where V_o is the base shear of the system and u_N is the horizontal displacement at the top of the system (Figure 1e). Following, two assumptions are proposed: First the $\boldsymbol{\psi}$ vector of above inelastic floor displacement distribution plays the role of a '*notional unique mode-shape*' of the system and second the **P** vector of lateral external floor forces P_i is a function of the time *t*, in other words $\mathbf{P}(t)$. Therefore, the vector $\mathbf{P}(t)$ is written:

$$\mathbf{P}(t) = \mathbf{Y}\, P_N \cdot f(t) \tag{2.2}$$

where $f(t)$ is a known suitable increasing monotonically uniformly time function, which is same for all forces at floor levels and P_N is the value of the load at the top level of the frame.
The system of dynamic equilibrium of *N* linear differential second order equations is written in matrix form:

$$\mathbf{M}\,\ddot{\mathbf{u}}(t) + \mathbf{C}\,\dot{\mathbf{u}}(t) + \mathbf{K}\,\mathbf{u}(t) = \mathbf{P}(t) \tag{2.3}$$

where $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, $\ddot{\mathbf{u}}(t)$ are the *N*-dimension vector of lateral floor displacements, velocities and accelerations respectively, while **M, C, K** are the *NxN* dimension, square matrices of masses, damping and lateral stiffness of the system, respectively. Inserting Eqn. 2.1 & 2.2 into Eqn 2.3, the last is written as:

$$\mathbf{M}\,\boldsymbol{\psi} \cdot \ddot{u}_N(t) + \mathbf{C}\,\boldsymbol{\psi} \cdot \dot{u}_N(t) + \mathbf{K}\,\boldsymbol{\psi} \cdot u_N(t) = \mathbf{Y}\, P_N \cdot f(t) \tag{2.4}$$

After mathematical analysis (Makarios 2008), Eqn. 2.4 is transformed in Eqn. 2.5 that means the equation of motion of a single-degree-of-freedom (SDF) system:

$$m^* \cdot \ddot{u}_N(t) + c^* \cdot \dot{u}_N(t) + k^* \cdot u_N(t) = L \cdot V_o(t) \tag{2.5}$$

where,

k^* is the lateral stiffness of the first branch (and $\alpha \cdot k^*$ of the second branch), which arise by the static pushover analysis of the system (by diagram V_o-u_N),

$k_o = \sum Y_i \cdot \boldsymbol{\psi}^T \mathbf{K}\, \boldsymbol{\psi} \big/ \boldsymbol{\psi}^T \mathbf{Y}$,

$L = k^* / k_o$ is the 'convergent coefficient',

$$m^* = \frac{k^* \cdot \sum Y_i}{k_o \cdot \boldsymbol{\psi}^T \mathbf{Y}} \cdot \boldsymbol{\psi}^T \mathbf{M}\, \boldsymbol{\psi} = \frac{k^* \cdot \left(\sum Y_i\right) \cdot \left(\sum m_i \psi_i^2\right)}{k_o \cdot \sum \psi_i Y_i} = \frac{L \cdot \left(\sum Y_i\right) \cdot \left(\sum m_i \psi_i^2\right)}{\sum \psi_i Y_i}$$

$$\boldsymbol{\psi}^T \mathbf{M}\, \boldsymbol{\psi} = m_1 \psi_1^2 + m_2 \psi_2^2 + ... + m_i \psi_i^2 + ... + m_N \psi_N^2 = \sum_{i=1}^{N} m_i \psi_i^2 \ ,$$

$$c^* = \sum Y_i \cdot \boldsymbol{\psi}^T \mathbf{C}\, \boldsymbol{\psi} \cdot L \big/ \boldsymbol{\psi}^T \mathbf{Y} = 2\, m^* \omega^* \zeta$$

because we consider and assume that, the SDF system possesses equivalent viscous damping, where $\omega^* = \sqrt{k^*/m^*}$ is the circular frequency of the SDF vibrator in linear range and ζ =0.05 is the equivalent viscous damping ratio corresponding to critical damping of the SDF system for reinforced concrete. In order to transform the diagram V_o-u_N of the multistory planar frame (Figure 2a) to capacity curve P^*-δ of the ideal equivalent NLSDF system (Figure 2b), the transformation factor ε is used:

$$\varepsilon = m^* / m_{tot} \tag{2.6}$$

where $m_{tot} = \mathbf{1}^T \mathbf{M}\, \mathbf{1}$, with $\mathbf{M}$ is the square mass matrix of the system and $\mathbf{1} = [1 \quad 1 \quad ... \quad 1]^T$ is the N-order vector with each element equal to unity. Therefore, the equivalent NLSDF system (Figure 1c) has been characterized by Eqn. 2.5 and it has the bilinear capacity curve P^*-δ according to Figure 2b.

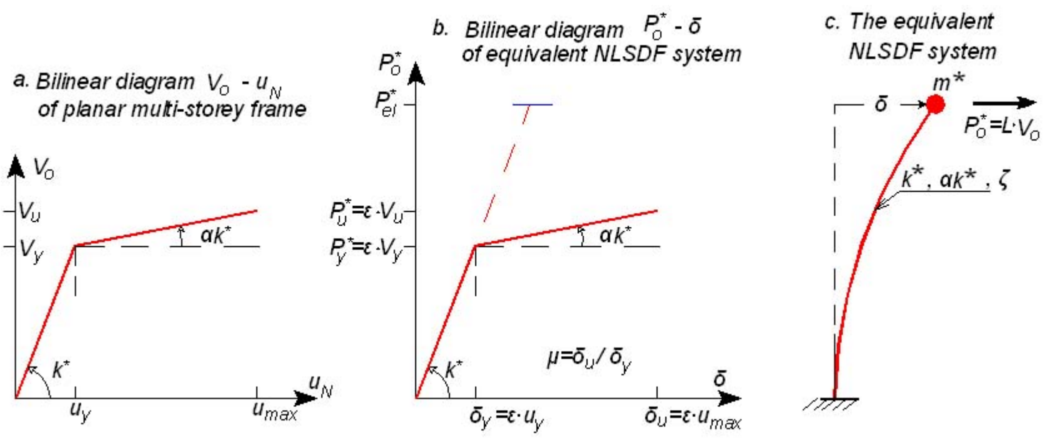

Figure 2 Characteristics of the improved optimum equivalent NLSDF system.

The maximum elastic base shear P_{el}^* of the infinitely elastic SDF system and the yielding base shear P_y^* of the respective equivalent non-linear SDF system is:

$$P_{el}^* = m^* \cdot S_a^* \tag{2.7}$$

$$P_y^* = \varepsilon \cdot V_y \tag{2.8}$$

The elastic spectral acceleration S_a^* is calculated by the elastic response acceleration spectrum for the known period T^*. Consequently, the maximum required seismic displacement (seismic target-displacement) δ_t of the equivalent NLSDF system arises from the inelastic spectrum of a known earthquake, according to following equation (Figure 2c):

$$\delta_t = L \cdot \mu_d \cdot S_a^* \Big/ \left[R_y \cdot \left(\omega^*\right)^2 \right] \tag{2.9}$$

where $R_y = P_{el}^* \big/ P_y^*$ is the reduction factor of the system (Figure 2b) and the demand ductility μ_d of the non-linear SDF system is given according to Krawinkler & Nassar (1992):

$$\mu_d = 1 + \left(R_y^c - 1\right) \big/ c \tag{2.10}$$

with, the coefficient *c* is evaluated by :

$$c = \frac{\left(T^*\right)^{a_o}}{1 + \left(T^*\right)^{a_o}} + \frac{a_1}{T^*} \tag{2.11}$$

where the numerical coefficients depend on the slope αk^* of the yielding branch (Figure 2b), while in the practice, a mean value (i.e. for $\alpha = 2\%$) of the coefficient *c* is often preferred for all cases.
Note that, the demand ductility μ_d must be less than the available ductility $\mu = \delta_u \big/ \delta_y$. Next, the equivalent maximum required seismic displacement u_N at the top of the real planar system is directly given by the target displacement δ_t of the ideal NLSDF system as:

$$u_N = \delta_t \big/ \varepsilon \tag{2.12}$$

Finally, the maximum required seismic displacements of the other floors are taken by the step of the known pushover analysis, where the displacement u_N (that is just calculated by Eqn. 2.12 at the top of the system has been appeared. These displacements create the asked new vector $\mathbf{u}_1$, while if we ask a better approach then we can repeat the calculation of Eqn. 2.1 to Eqn. 2.12 using the new vector $\mathbf{u}_1$ instead of $\mathbf{u}_o$, but third approach is not needed.

3. INDICATIVE EXAMPLE OF A FIVE-STOREY R/C PLANAR FRAME

3.1 General

In order to present the improve methodology; consider the five-storey planar r/c frame of Figure 3. Each floor of the frame have concentrated mass equal *m=60.00* mass tons. In the beginning, the five-storey frame had analyzed according to the Hellenic Seismic Code (EAK/2003) using the inelastic design acceleration spectrum (with behavior factor *q=1.00*, but with all capacity design checks of r/c elements namely strong columns-low beams rule), while the response spectrum analysis was performed. Next, the r/c frame has been designed according to Hellenic Reinforced Concrete Code (EKOS/2003), while, in each plastic hinge, the flexural failure always is preceded of the shear failure. Next, the critical sections (at the ends of all structural elements) have been analyzed and the diagrams *M-c* of the moments *M (kN·m)* and the curvatures *c (rad/m)*, respectively, have been calculated. Afterwards, in each plastic hinge with length ℓ_p, the diagrams *M-θ* of the moments *M* and the rotations *θ (rad)*, respectively, are calculated. The plastic rotation θ_p of a plastic hinge (between the end section and the start section of the plastic hinge) is given by the known relationship $\theta_p = (c_u - c_y)\,\ell_p$, while the respective yielding rotation θ_y is given approximately from $\theta_y = c_y \cdot \ell_p$, where c_y & c_u the yielding curvature and ultimate one, respectively.

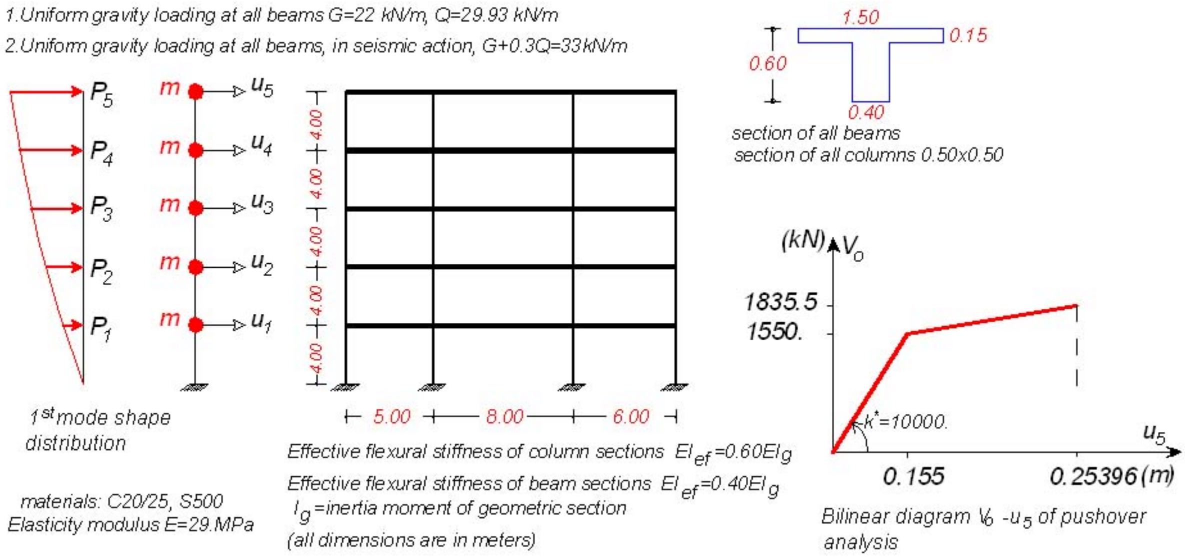

Figure 3 Planar five-storey r/c frame.

3.2 Static pushover analysis. Estimation of target-displacement

The lateral static forces of floors are distributed according to the first mode-shape of the system in elevation. Therefore, the vector **Y** is written:

$$\mathbf{Y} = \{0.217 \quad 0.488 \quad 0.724 \quad 0.900 \quad 1.000\}^T$$

Static pushover analysis of the five-storey r/c planar frame is performed, until a 25% reduction of the total strength of the system has been appeared. As we can see in the Figure 3, the diagram V_o-u_N of base shear vs displacement of the top of the frame is given, too. Consider an interval step of this pushover analysis in the end of the pushover analysis, the distribution of lateral floor inelastic displacements $\mathbf{u}_o$ of the planar frame is known and therefore the vector $\mathbf{\Psi}$ is calculated directly:

$$\mathbf{\Psi} = \{\psi_1 \quad \psi_2 \quad \psi_3 \quad \psi_4 \quad \psi_5\}^T = \{0.19775 \quad 0.46610 \quad 0.71098 \quad 0.89687 \quad 1.0000\}^T$$

Next, we are calculating the following parameters:

$$\mathbf{\Psi}^T\mathbf{Y} = \psi_1 Y_1 + \psi_2 Y_2 + \psi_3 Y_3 + \psi_4 Y_4 + \psi_5 Y_5 = \sum_{i=1}^{5} \psi_i \, Y_i = 2.592$$

$$\mathbf{\Psi}^T\mathbf{M}\,\mathbf{\Psi} = m_1\psi_1^2 + m_2\psi_2^2 + m_3\psi_3^2 + m_4\psi_4^2 + m_5\psi_5^2 = \sum_{i=1}^{5} m_i \, \psi_i^2 = 157.973$$

The lateral stiffness matrix **K** of the planar five-storey r/c frame, referring to the five lateral degree of freedom of the system (Figure 3), is calculated:

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} \end{bmatrix} = \begin{bmatrix} 183375.20 & -110255.75 & 24071.39 & -3864.77 & 730.69 \\ -110255.75 & 177135.37 & -109148.35 & 23610.42 & -2870.65 \\ 24071.39 & -109148.35 & 176747.15 & -107829.00 & 19595.95 \\ -3864.77 & 23610.42 & -107829.00 & 168338.83 & -80784.25 \\ 730.69 & -2870.65 & 19595.95 & -80784.25 & 63300.75 \end{bmatrix}$$

Therefore,

$$k_o = \frac{\sum Y_i}{\boldsymbol{\psi}^T \mathbf{Y}} \cdot \boldsymbol{\psi}^T \mathbf{K}\, \boldsymbol{\psi} = \frac{3.329}{2.592} \times 9234.07 = 11859.12 \;\; kN/m$$

$$m_o = \frac{\sum Y_i}{\boldsymbol{\psi}^T \mathbf{Y}} \cdot \boldsymbol{\psi}^T \mathbf{M}\, \boldsymbol{\psi} = \frac{3.329}{2.592} \times 153.973 = 197.753$$

The elastic lateral stiffness k^* of the equivalent NLSDF system arises directly by the diagram V_o-u_N of Figure 3 and is $k^* = 10000.\, kN/m$. Therefore, the convergent coefficient L and the mass m^* of the equivalent non-linear SDF system are given:

$$L = k^* / k_o = 10000./11859.12 = 0.843 \,, \qquad m^* = L \cdot m_o = 0.843 \times 197.753 = 166.7$$

The period T^* and circular frequency ω^* of the optimum equivalent non-linear SDF systems, according to its linear (on the first branch) response, are calculated, while the natural fundamental period of the five-storey frame has been calculated in *0.80* s:

$$T^* = 2\pi\sqrt{m^*/k^*} = 2\pi\sqrt{166.7/10000.} = 0.81\,\text{s} \,, \qquad \omega^* = 2\pi/T^* = 2\pi/0.81 = 7.757\ \text{rad/s}$$

The total mass m_{tot} of the planar multi-storey system is given in matrix-form:

$$m_{tot} = \mathbf{1}^T\, \mathbf{M}\, \mathbf{1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 60. & & & & \\ & 60. & & & \\ & & 60. & & \\ & & & 60. & \\ & & & & 60. \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 300.00$$

The ‘transformation factor’ ε, which using in order to transformed the diagram V_o-u_N of the real multi-storey system to the capacity curve P^*-δ of the ideal equivalent NLSDF system, is given:

$$\varepsilon = m^* / m_{tot} = 166.7/300. = 0.556$$

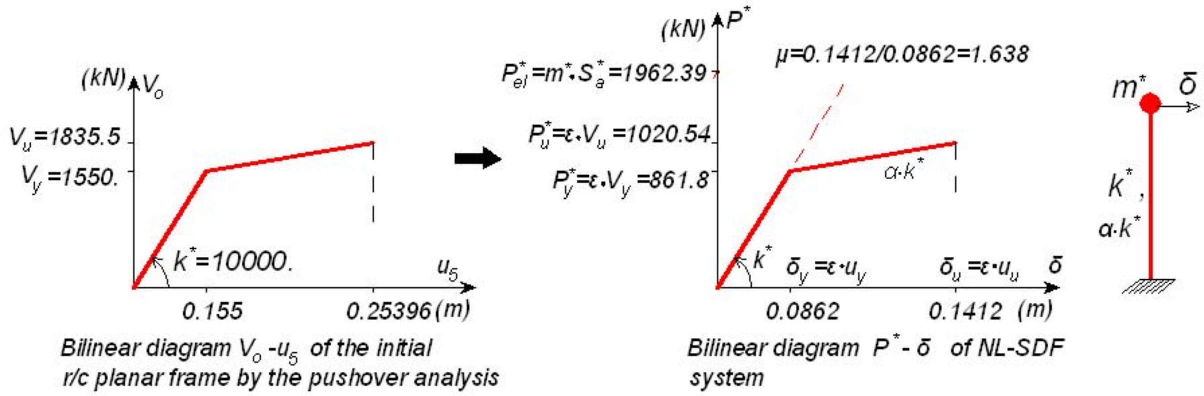

Figure 4 The bilinear diagram P^*-δ of the Non-Linear Single Degree of Freedom system.

Therefore, the elastic spectral accelerations S_a^* and the elastic spectral displacements S_d^* are given for period $T^* = 0.81\,s$, by the threefold design elastic response acceleration spectrum of Hellenic Seismic Code (EAK/2003):

$$S_a^* = 3 \times (0.16g \times 2.50) = 11.772 \ m/s^2 \ , \qquad S_d^* = S_a^* \Big/ \left(\omega^*\right)^2 = 11.772 \big/ 7.757^2 = 0.20$$

The base shear P_{el}^* of the infinitely elastic SDF system and the yielding force P_y^* of the ideal equivalent NLSDF system are given:

$$P_{el}^* = m^* \cdot S_a^* = 166.7 \times 11.772 = 1962.39\ kN\ , \qquad P_y^* = \varepsilon \cdot V_y = 0.556 \times 1550.00 = 861.8\ kN$$

Therefore, the yield strength reduction factor R_y of the frame is calculated directly and the coefficients c and μ_d according to Eqn. 2.10-2.11 are given:

$$R_y = P_{el}^* \Big/ P_y^* = 1962.39/861.8 = 2.277\ , \qquad c=0.904, \qquad \mu_d = 2.22$$

However, the available ductility μ of the NLSDF system is given directly by:

$$\mu = \delta_u \big/ \delta_y = 0.1412/0.0862 = 1.638 \ < \ \mu_d = 2.22$$

Therefore, as we observe, the demand ductility μ_d is greater than the available ductility μ of the non-linear SDF system and it maybe means that this r/c frame is seismically inadequate for the above threefold design elastic response acceleration spectrum of EAK/2003. Next, the maximum demand inelastic displacement (target-displacement) δ_t of the ideal equivalent NLSDF system is:

$$\delta_t = L \cdot \mu_d \cdot S_d^* \big/ R_y = 0.843 \times 2.22 \times 0.20/2.277 = 0.164$$

Therefore, the maximum demand inelastic displacement $u_{N,t}$ at the top of the real frame is calculated:

$$u_{N,t} = \delta_t \big/ \varepsilon = 0.164/0.556 = 0.295$$

Furthermore, we are going to step of the static pushover analysis, which gave at the top of the frame the displacement $u_5=u_{N,t}$. In this step, the other floor displacements are known by the static pushover analysis, and these displacements are the asked seismic demand floor displacements.

5. CONCLUSIONS

A documented proposal of an improved optimum equivalent NLSDF system that represents the planar multi-storey r/c frame has been presented. For illustrative reasons, a numerical example of a five-storey planar frame is presented, too. The analytic definition of the improved NLSDF system has been mathematically derived recently (Makarios 2008) for three types of r/c structures (planar multi-storey frames, asymmetric single-storey buildings and asymmetric multi-storey buildings). It is known that in several cases, the static pushover procedure can give adequate results at maximum inelastic floor displacements, while in many other cases no. This improved optimum equivalent NLSDF system can be used in pushover analysis, in order to calculate the required seismic inelastic displacements using the inelastic design spectra. The basic characteristics of the equivalent NLSDF system are the generalized mass m^*, the generalized lateral bilinear stiffness k^* of the first branch and αk^* of the yielding branch of the bilinear diagram P^*-δ, the equivalent viscous damping ratio ζ, and the lateral degree of freedom δ. Also, the proposed equivalent NLSDF system possesses period near to the natural coupled period of the building, while it represents the planar multistory frame reliably. Thus, the natural meaning of the characteristics of the asymmetric building is not adulterated by the present methodology, because the total methodology is derived by mathematical way and it is highly significant.

6. REFERENCES

Makarios, T. (2005). Optimum definition of equivalent non-linear SDF system in pushover procedure of multistory r/c frames. *Engineering Structures Journal*; **27**, 5: 814-825.

Makarios, T. (2008). Equivalent non-linear single degree of freedom system of spatial asymmetric multi-storey buildings in pushover procedure. Theory & applications. *The Structural Design of Tall and Special Buildings Journal*, on-line downloading, http://www3.interscience.wiley.com, (in press TAL 459).

Krawinkler, H., Nassar, AA. (1992). Seismic design based on ductility and cumulative damage demands and capacities. *Nonlinear Seismic Analysis and Design of Reinforced Concrete Buildings*, eds P.Fajfar and H. Krawinkler. New York: Elsevier Applied Science, 23-39.